Republic of Iraq
Ministry of Education
General Directorate of Curricula

9

Series of Maths Books for Intermediate Stage

Mathematics

Third Intermediate

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This series (Maths for Intermediate stage) has been edited by a special team of specialists in Ministry of Education/ General Directorate of curricula with participation of specialists from universities professors in Ministry of higher Education according to international standards to achieve the goals of designing modern syllabus which helps the students to be:

Successful learners long life

Self-esteem individuals

Iraqi citizens feeling proud

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قسم التحضير الطباعي

INTRODUCTION

The Maths subject is considered one of the basic courses that helps students to acquire educational abilities to develop his thinking and solving problems and it helping to deal with difficult situations in his life.

As a starting point of attention by the Ministry of Education represented by the General Directorate of curricula to develop the curricula in general and specially of Maths in order to go a long with the technological and scientific development in different fields of life. A plan has set up to edit the series of Maths books for the three stages. Primary stage has been achieved and the work started to continue the series by editing the books of intermediate stage.

The series of new Iraqi Maths Books as a part of General frame work of curricula that reinforces the basic values as Iraqi identity, forgiveness, respecting different opinions, social justice and offering equal chance for creativity and it also reinforces abilities of thinking and learning, self-efficiency, action and citizen ship efficiency.

The series of Iraqi Maths books has been built on student- centered learning according to international standards.

The series of Iraqi maths books for intermediate stage has been built on six items: learn, make sure of your understanding, solve the exercises, solve the problems, think and write. The Maths book for third intermediate stage contains four basic fields: The numbers and the operations, Algebra, Geometry and Measurement, Statistics and Probabilities for each field. The books.

The maths books have distinguished by presenting material in modern styles that attract and help the student to be active through presenting drills, exercises and environmental problems in addition there are extra exercises at the end of the book that are different from the exercises and drills in the lessons because they are objective so the student can answer through multiple choices and that prepare the student to participate the international competitions.

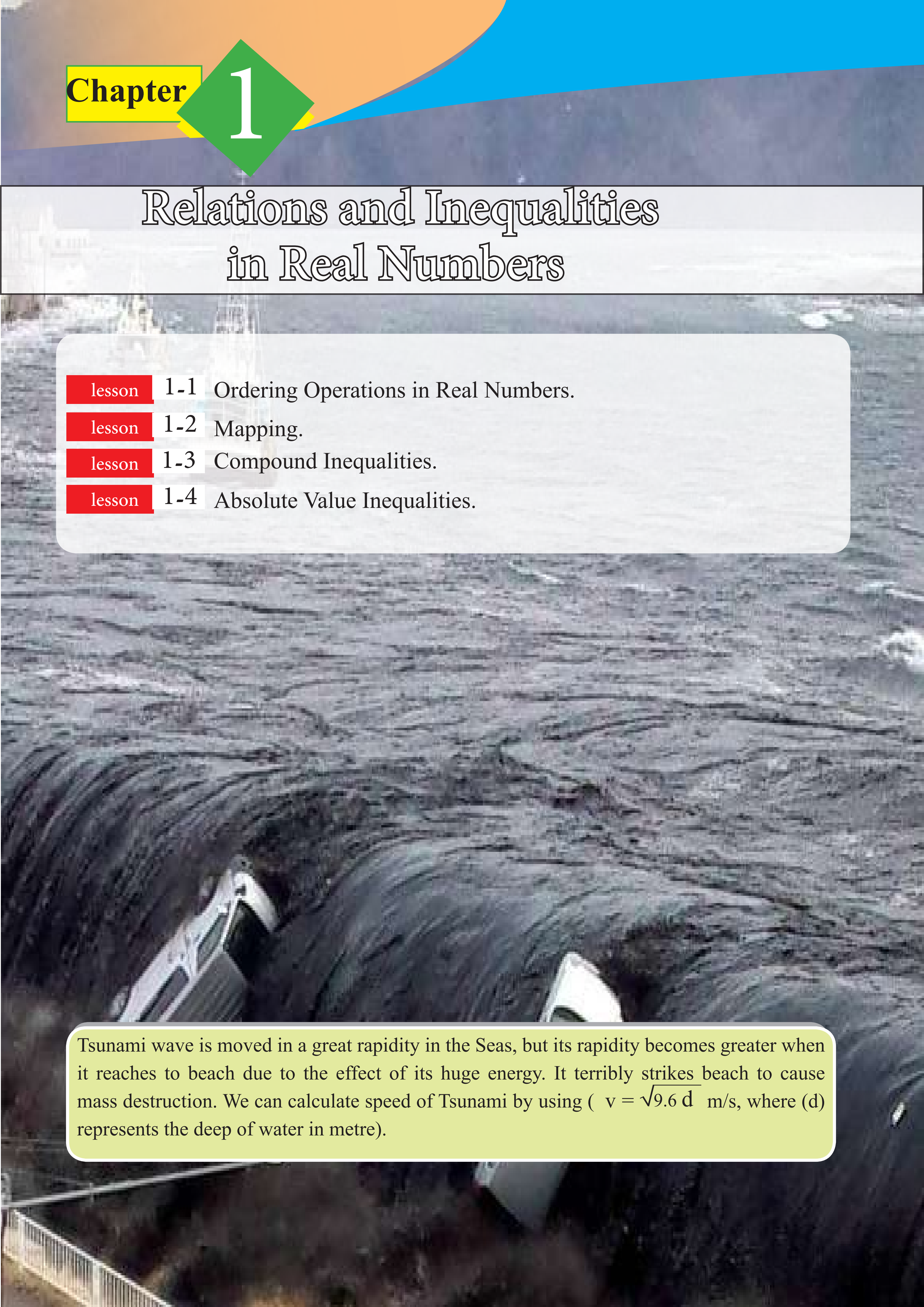
This book is an expansion for the series of developed Maths books for primary stage and it is also considered as support for the developed syllabus in maths and it also has a teachers book so we hope in applying them, the student will gain scientific and practical skills and develop their interest to study Maths.

We hope God help us to serve our country and our sons

Authors

Chapter 1

Relations and Inequalities in Real Numbers



Tsunami wave is moved in a great rapidity in the Seas, but its rapidity becomes greater when it reaches to beach due to the effect of its huge energy. It terribly strikes beach to cause mass destruction. We can calculate speed of Tsunami by using ($v = \sqrt{9.6\, d}$ m/s, where (d) represents the deep of water in metre).

Pretest

Classify the number if it is rational or irrtional number:

1 $\sqrt{25}$

2 $\sqrt{7}$

3 $\frac{0}{\sqrt{3}}$

4 $\sqrt{\frac{16}{25}}$

5 $\sqrt{\frac{49}{5}}$

6 $\frac{30}{4}$

7 $-6\frac{3}{2}$

8 $-\sqrt{8}$

Estimate the following square roots by near them to the nearest tenth, then represent them on the straight line of numbers:

9 $\sqrt{2}$ ≈.......

10 $-\sqrt{3}$ ≈......

11 $\sqrt{\frac{6}{25}}$ ≈......

12 $\sqrt{\frac{81}{49}}$ ≈......

Compare between the real numbers by using the symbols (< , > , =):

13 $\sqrt{5}$ ⬚ $2\frac{1}{3}$

14 1.25 ⬚ $\sqrt{2.25}$

15 $\sqrt{\frac{0}{3}}$ ⬚ $\frac{0}{6}$

16 $\frac{\sqrt{12}}{\sqrt{3}}$ ⬚ $\frac{\sqrt{5}}{\sqrt{20}}$

17 Ordering the following real numbers from the least to the greatest.

$\sqrt{7}$, 2.25 , $\sqrt{5}$

18 Ordering the following real numbers from the greatest to the least.

$-3\frac{1}{5}$, $-\frac{7}{3}$, -3.33

Solve the following inequalities in R by using properties of inequalities in the real numbers:

19 $3x + \frac{2}{5} \geq 4x - \frac{3}{5}$

20 $\frac{3}{7} > z - \frac{9}{14}$

21 $\frac{3y}{8} \geq \frac{2}{7}$

22 $\frac{-4m}{11} < \frac{9}{22}$

23 $6(z - 3) > 5(z + 1)$

24 $4\left(\frac{1}{2}v + \frac{3}{8}\right) > 0$

Simplify the following numerical sentences by using ordering operations on the real numbers:

25 $\sqrt{2}\,(1 - \sqrt{18}) =$ ....

26 $3\sqrt{12} + 2\sqrt{3} - 4\sqrt{3} =$ ....

27 $\frac{\sqrt{7} - 8\sqrt{7}}{2\sqrt{7}} =$ ....

28 $\frac{6\sqrt{44}}{\sqrt{5}} \div \frac{18\sqrt{11}}{\sqrt{5}}$

Lesson [1-1] Ordering Operations in Real Numbers

Idea of the lesson:

* Simplify the numerical sentences which contain real numbers by using ordering opertions.

Vocabulary:

* Real number.
* Rationalizing the Denominator
* Conjugate

Learn

Tsunami quake, which occurred in Japan in 2011 is considered one of the greatest quake which happened over the ages. Its speed can be calculated by using the law $v = \sqrt{9.6\,d}$ m/s, where d represents the deep of water. What's the approximate speed of Tsunami if the deep of water is 1000m?

[1-1-1] Using ordering operations to simplify numerical sentences .

You have previously learned the natural numbers, whole numbers, integers, rational numbers and real numbers. We can ordering them in the following:

$N \subset W \subset Z \subset Q \subset R$

You have also learned how to simplify the numerical sentences by using the ordering of operations in these numbers. We will develop your skills in simplifying the numerical sentences which contain different real numbers include real roots and perfect squares roots, and also simplifying fractions contain roots by applying the properties on them and by using the ordering of operations in the real numbers. We use also (Rationalizing) denominator to simplify sentences by multiplying the conjugate numbers (the result of multiplying two conjugate numbers is a rational number, (the number $2 - \sqrt{3}$ is conjugate to $2+\sqrt{3}$ because the product them is rational number).

Example (1) Find the approximate speed of Tsunami if the water deep is 1000m.

$v = \sqrt{9.6\,d}$ *Law of calculating Tsunami speed, where d represents the deep of water*

$= \sqrt{9.6 \times 1000} = \sqrt{9600} = 98$ m/sec *The approximate speed of Tsunami*

Example (2) **Simplifying the following numerical sentences by using the ordering of operations on the real numbers:**

i) $(\sqrt{12} - \sqrt{18})(\sqrt{12} + \sqrt{18}) = (2\sqrt{3} - 3\sqrt{2})(2\sqrt{3} + 3\sqrt{2})$ *by using the distribution*

$= 2\sqrt{3}(2\sqrt{3} + 3\sqrt{2}) - 3\sqrt{2}(2\sqrt{3} + 3\sqrt{2}) = 12 + 6\sqrt{6} - 6\sqrt{6} - 18 = -6$

ii) $\left(\sqrt[3]{\frac{8}{27}} - \sqrt{\frac{2}{3}}\right) \div \left(\frac{3\sqrt{2} - 2\sqrt{3}}{\sqrt{27}}\right) = \left(\frac{2}{3} - \sqrt{\frac{2}{3}}\right) \div \left(\frac{3\sqrt{2} - 2\sqrt{3}}{3\sqrt{3}}\right)$

$= \frac{2\sqrt{3} - 3\sqrt{2}}{3\sqrt{3}} \times \frac{-3\sqrt{3}}{2\sqrt{3} - 3\sqrt{2}} = -1$

Example (3) **Simplifying the following numerical sentences by using the ordering of operations on the real numbers, then write the result to nearest tenth:**

iii) $\sqrt{12}(\sqrt{3} - \sqrt{8}) - 6 = 2\sqrt{3}(\sqrt{3} - 2\sqrt{2}) - 6 = 2\sqrt{3} \times \sqrt{3} - 2\sqrt{3} \times 2\sqrt{2} - 6$

$= 6 - 4\sqrt{3 \times 2} - 6 = -4\sqrt{6} \approx -4 \times 2.4 = -9.8$

iv) $(-27)^{\frac{1}{3}}\left(\frac{1}{9}\sqrt{7} - \frac{1}{9}\sqrt{28}\right) = \sqrt[3]{-27}\left(\frac{1}{9}\sqrt{7} - \frac{2}{9}\sqrt{7}\right) = -3\left(\frac{1}{9}\sqrt{7} - \frac{2}{9}\sqrt{7}\right)$

$= -\frac{1}{3}\sqrt{7} + \frac{2}{3}\sqrt{7} = \frac{1}{3}\sqrt{7} \approx 0.9$

Note : $a^{\frac{n}{m}} = \sqrt[m]{a^n}$

Example (4) **Simplify the following numerical sentences by rationalizing the denominator and ordering operations on the real numbers.**

i) $\frac{7-\sqrt{5}}{\sqrt{5}} = \frac{7-\sqrt{5}}{\sqrt{5}} \times 1 = \frac{7-\sqrt{5}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}(7-\sqrt{5})}{\sqrt{5}\sqrt{5}} = \frac{7\sqrt{5}-\sqrt{5}\sqrt{5}}{5} = \frac{7\sqrt{5}-5}{5}$

ii) $\frac{\sqrt{21}}{2\sqrt{3}-\sqrt{7}} = \frac{\sqrt{21}}{2\sqrt{3}-\sqrt{7}} \times \frac{2\sqrt{3}+\sqrt{7}}{2\sqrt{3}+\sqrt{7}} = \frac{\sqrt{3}\sqrt{7}(2\sqrt{3}+\sqrt{7})}{(2\sqrt{3}-\sqrt{7})(2\sqrt{3}+\sqrt{7})}$ *Multiplying by conjugates*

$= \frac{6\sqrt{7}+7\sqrt{3}}{12-7} = \frac{6\sqrt{7}+7\sqrt{3}}{5}$ *The denominator is a difference between two squares*

1-1-2] Using calculator and approximation to Simplify Numerical Sentence

You have previously learned how to simplify the numerical sentences which contain integer negative powers and scientific form for number by using calculator. Now, you will develop your skills by simplify the numerical sentences which contain numbers raising to rational powers, in addition to the integers by using calculator to write the result in approximated way.

Example (5) **Calculate the powers for each of the following, then write the result which should be approximated to two decimal places, if it is not an integer:**

i) $9^{-\frac{3}{2}} = (3^2)^{-\frac{3}{2}} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27} \approx 0.04$

ii) $(\sqrt{7})^2 = (7^{\frac{1}{2}})^2 = 7$

iii) $2^{\frac{5}{3}} \times 2^{\frac{1}{3}} \times 2^{\frac{-3}{2}} = 2^{\frac{10+2-9}{6}} = 2^{\frac{1}{2}} = \sqrt{2} \approx 1.41$

iv) $5^2 \div 5^{\frac{3}{2}} = 5^{\frac{4}{2}-\frac{3}{2}} = 5^{\frac{1}{2}} = \sqrt{5} \approx 2.24$

Use the ordering of operations and write the result which should be approximated to two decimal places by using a calculator for each of the following:

v) $(\frac{1}{2})^2 + 3^{-2} - 2^{\frac{3}{2}} = \frac{1}{2^2} + \frac{1}{3^2} - \sqrt{2^3} = \frac{1}{4} + \frac{1}{9} - \sqrt{8} \approx 0.25 + 0.11 - 2.83 = -2.47$

vi) $8^{\frac{1}{3}} - (-8)^0 + 3^2 \times 3^{\frac{1}{2}} = \sqrt[3]{8} - 1 + 3^{\frac{5}{2}} = \sqrt[3]{8} - 1 + \sqrt{3^5} \approx 2 - 1 + 9 \times 1.73 = 16.57$

Example (6)

Use calculator to write the result in the scientific form for the number which should be aproximated to the nearest two decimal places:

i) $7.6 \times 10^{-4} - 0.41 \times 10^{-3} = 7.6 \times 10^{-4} - 4.135 \times 10^{-4} = 3.465 \times 10^{-4} \approx -3.47 \times 10^{-4}$

ii) $0.052 \times 10^4 + 7.13 \times 10^2 = 5.2 \times 10^2 + 7.13 \times 10^2 = 12.33 \times 10^2 \approx 1.23 \times 10^3$

iii) $(7.83 \times 10^{-5})^2 = (7.83 \times 10^{-5})(7.83 \times 10^{-5}) = 61.3089 \times 10^{-10} \approx 6.13 \times 10^{-9}$

iv) $4.86 \times 10^2 \div 0.55 \times 10^5 = (4.86 \div 0.55) \times 10^2 \times 10^{-5} \approx 8.84 \times 10^{-3}$

Make sure of your understanding

Simplify the following numerical sentences :

1. $(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3}) = ...$
2. $(\sqrt{7} - \sqrt{2})^2 = ...$
3. $(\sqrt{125} - \sqrt{20})(\sqrt[3]{\frac{8}{27}}) = ...$
4. $\frac{4\sqrt{12}}{5\sqrt[3]{-27}} \div \frac{2\sqrt{24}}{\sqrt{8}} = ...$

Questions 1-4 are similar to example 2

Simplify the following numerical sentences , and write the result to the nearest tenth:

5. $\sqrt{7}(\sqrt{28} - \sqrt{2}) - 5 \approx ......$
6. $(-125)^{\frac{1}{3}} (\frac{1}{10}\sqrt{3} - \frac{1}{4}\sqrt{12}) \approx .....$

Questions 5-6 are similar to example 3

Simplify the following numerical sentences by rationalizing the denominator and ordering operations on the real numbers:

7. $\frac{1-\sqrt{3}}{4\sqrt{3}} = ...$
8. $\frac{1-\sqrt{20}}{\sqrt{5}} = ...$
9. $\frac{\sqrt{50} - \sqrt{3}}{2\sqrt{3}} - \frac{10 - \sqrt{6}}{2\sqrt{6}} = ...$

Questions 7-9 are similar to example 4

Use the ordering of operations and write the result which should be approximated to two decimal places by using the calculator for each of the following:

10. $(\frac{1}{3})^2 + 3^{-3} - 3^{\frac{3}{2}} \approx ......$
11. $27^{\frac{1}{3}} - (-9)^0 + 3^2 \times 5^{\frac{1}{2}} \approx ......$

Questions 10-11 are similar to example 5

Use the calculator to write the result in the scientific form of the number which should be approximated to the nearest two decimal places:

12. $6.43 \times 10^{-5} - 0.25 \times 10^{-3} = ...$
13. $(9.23 \times 10^{-3})^2 = ...$

Questions 12-13 are similar to example 6

Solve the Exercises

Simplify the following numerical sentences:

14. $(\sqrt{18} - \sqrt{50})(\frac{-27}{64})^{\frac{1}{3}} = ...$
15. $\frac{\sqrt{12}}{3\sqrt[3]{125}} \div \frac{5\sqrt[3]{8}}{\sqrt{25}} = ...$

Simplify the following numerical sentences , and write the result to the nearest tenth.

16. $7\sqrt{\frac{2}{49}} - 3\sqrt{\frac{8}{81}} + \sqrt{\frac{18}{36}} \approx ......$

Simplify the following the numerical sentences by rationalizing the denominator and the ordering of operations in the real numbers :

17. $\frac{\sqrt{7} - 3\sqrt{5}}{\sqrt{7} + 3\sqrt{5}} = ...$
18. $\frac{\sqrt{33} - \sqrt{11}}{\sqrt{99}} - \frac{\sqrt{60} - \sqrt{5}}{5\sqrt{15}} = ...$

Solve the problems

19 Satellites: We essentialy use satellite in communications, such us TV signals, telephone calls in all over the world, weather forecasts and tracking of hurricanes. The satellites rotate around earth in limited speed and special orbits. The orbital speed of moon is calculated by the following relation $v = \sqrt{\frac{4\times10^{14}}{r}}$ m/sec, where r represents the radius of orbit (the distance of moon from the earth centre), what is the speed of moon if the orbit radius is 300 km?

20 Fighting fires: We can calculate the speed of flowing water which releases by fire trucks by using the following law $v = \sqrt{2hg}$ foot/sec, where h represents the maximum height of water, and (g) represents the acceleration speed of earth (32 foot/sec^2). To fire fighting in the forests, the firefighters in the Civil Defence need to huge bumper to pump water in height of 80 foot. Is it enough to use a pumper releases water in a speed of 72 foot/sec,?

1 foot = 30cm
Measuring unit in French system

21 Geometry: Find the area of triangle which topped a front of house if its height $(\sqrt{18} - \sqrt{3})$ meter and its base length is $(3\sqrt{2} + \sqrt{3})$ meter.

Think

22 Challenge: **Prove that the following is true:**

$$(7^{\frac{1}{3}} - 5^{\frac{1}{3}})\ (7^{\frac{2}{3}} + 7^{\frac{1}{3}}\, 5^{\frac{1}{3}} + 5^{\frac{2}{5}}) = 2$$

23 Correct the mistake: Shaker wrote the result of adding two numbers as follow:

$$8.4 \times 10^{-3} + 0.52 \times 10^{-2} = 1.36 \times 10^{-3}$$

Determin Shaker's mistake and correct it.

24 Numerical sense: Does the number $\sqrt{125}$ locate between the two numbers 10.28 and 11.28?

Write

The result of adding by approximation to the nearest tenth:

$$6^{\frac{3}{2}} + 5^{\frac{3}{2}} = ...$$

Lesson [1-2] Mappings

Idea of the lesson:
*Identify the mapping and its types and how it can be represented graphically in the coordinate plane and Identify the composition of mappings

Vocabulary:
*The relation.
*Ordered pair .
*Cartesian product .
*The mapping.
*Domain and Co-domain and the Range.
*The composition of mapping.

Learn

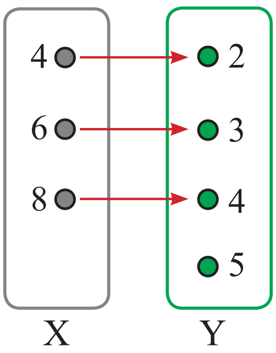
the group X represents the archaeological locations in Iraq x = {Ishtar gate, Awr , Al- Hadar}.Assume the group B represents some Iraqi cities y = {Baghdad, Al- Hila, Al-Nasiriya, Al-Mosul , Arbil}. The relation R:X→ y which represents the connection of each archaeological location with the city which it sits in:
R={(Al-Nasiriya, Awr),(Al-Mosul,Al-Hadar),(Babylon, Istar gate)}, called mapping where X represents its domain and Y represents its Co-domain.

[1-2-1] Mapping and its representation in the coordinate plane

You have previously learned the relation from set X to the set Y and it is subset (set of ordered pairs (x,y), where the first projection (the first coordinate) belongs to the set X and the second projection (the second coordinate) belongs to the set Y according to the Cartesian product X x Y. you will learn the mapping R:X→Y and how to represent it in an arrowy diagram and represent it (graphically) and identify its types.

The mapping: Let R relation from the set X to the set Y and each element in set X has one from in Y then the relation R can be called the mapping from X to Y ,R:X→Y . We called the set X the (domain) , and the set Y (Co-domain) , each elemeat in Y connected with element from X and represesents a from for it the set of all form in the Co-domain is called the (Range) , and rule which teansfers the element into its form is called the connection rule (mapping rule) and we refer to it by R (X) , (X,Y).

Example (1) If R: X→Y represents a mapping with rule ($y=\frac{1}{2}x$) from the set X= {4.6, 8} to the set, Y= {2,3,4,5}, and write the mapping in ordered pairs form ,then represent the mapping in an arrowy diagram and determine the domain and the range of the mapping.



The arrowy diagram explains the relation of connecting the elements of the two sets within the connection rule

Y=R(X)= $y = R(x) = \frac{1}{2}x$

4→2 , 6→3 , 8→4

So the set of mapping R = {(4,2),(6,3),(8.4)} Domain: is the set of first coordinates of the ordered pairs in R, and it is the set {4,6,8}

Range: is the set of the second coordinates of the ordered pairs in R, and it is the set {2,3,4}

Note: the range is a subset from the Co-domain of the mapping we see that the range ≠ Co-domain $(R_R) \neq (Y)$

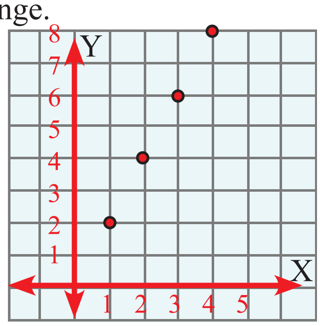
Example (2) The following table represents the relation between the weight (kg) and the price of fish (y = f(x)).

Does the relation represent a mapping?

If it is a mapping, then write the rule of mapping and Determine the domain and the range.

and represent in the coordinate plane.

x = weight (kg)	y = price in thousands dinars
1	2
2	4
3	6
4	8



The rule is $y = 2x$

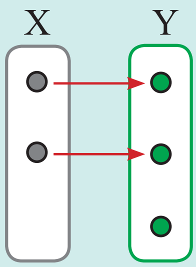
The domain = {1,2,3,4} the range ={2,4,6,8}

[1-2-2] The types of mappings

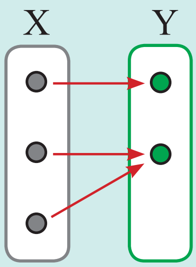
The mapping will be $f:X \rightarrow Y$

i) Surjective mapping

If the range=the co-domain



not Surjective mapping but it's injective

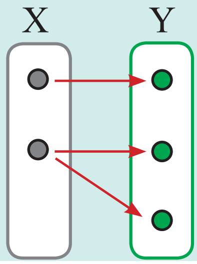


not Injective mapping but it's surjective

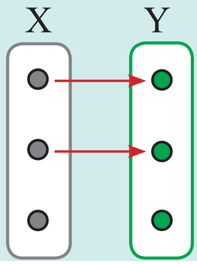
ii) Injective mapping : If

$\forall x_1, x_2 \in X ; x_1 \neq x_2 \Longrightarrow f(x_1) \neq (x_2)$

iii) Bijective mapping If the mapping is surjective and Injective at the same time



Relation not mapping



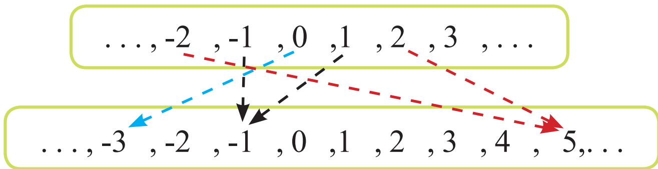
Bijective mapping surjective and injective

Example (3) If $f:Z \rightarrow Z$, where $f(x) = 2x^2 - 3$, show the type of the mapping, where Z represents the set of the integers.

$f(x) = 2x^2 - 3$ $f(-2)= 5$, $f(-1) = -1$, $f(0)= -3$, $f(1) = -1$, $f(2) = 5$

First: The mapping is not surjective because the range does not equal the co-domain.

Second: The mapping is not injective because $f(1) = f(-1) = -1$ while $1 \neq -1$

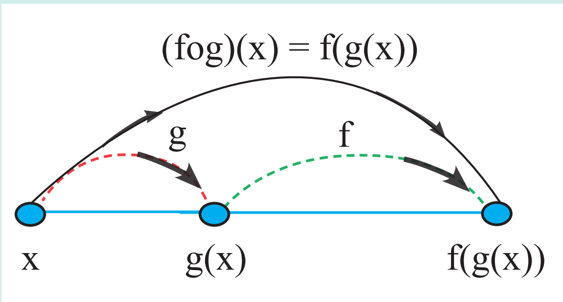


[1-2-3] Composition of mappings

We study a method to find a new mapping from two known mappings which are f(x),g(x) and they are:-

i) The mapping $(fog)(x)=f(g(x))$ and it can be read as f composite g (f after g), and it is a result of finding g(x) at first and then finding its image in the mapping f.

ii) The mapping $(gof)(x) =g(f(x))$ and it can be read as g composite f, and it is the result of finding f(x) at first, and then finding its image in the mapping g.



Example (4) **If $f: N \rightarrow N$, $f(x) = 2x + 1$, and $g: N \rightarrow N$, $g(x) = x^2$, Find the two mappings composition, i) $(fog)(3)$ ii) $(gof)(3)$, what do you conclude? iii) if $(fog)(x) = 33$, find the value of x**

i) Find $(fog)(3)$

$(fog)(3) = f(g(3)) = f(3^2)$
$= f(9) = 2 \times 9 + 1$
$= 19$

ii) $(gof)(3) = g(f(3))$
$= g(2 \times 3 + 1)$
$= g(7) = 7^2 = 49$

Note: $(fog)(3) \neq (gof)(3)$

iii) $(fog)(x) = f(g(x)) = f(x^2) = 2x^2 + 1$

$2x^2 + 1 = 33 \Rightarrow 2x^2 = 32 \Rightarrow x^2 = 16 \Rightarrow x = 4$ or $x = -4 \notin N$ neglect

Make sure of your understanding

Write a connection rule for the mapping and represent it in an arrowy diagram and write the domain and the range of it:

Questions 1-2 are similar to example 1

1. $f = \{(1,2), (2,3), (3,4), (4,5)\}$
2. $g = \{(1,3), (2,5), (3,7), (4,9)\}$

Write the rule for the following mappings and represent them in the coordinate plane and write their domain and range:

Questions 3-4 are similar to example 2

3. $f = \{(1,0), (2,0), (3,0), (4,0)\}$
4. $g = \{(0,0), (1,-1), (2,-2), (3,-3)\}$

Questions 5 are similar to example 3

5. If the mapping $f: N \rightarrow N$, where $f(x)=3x+2$. Show if the mapping is surjective or not.

Questions 6-7 are similar to example 4

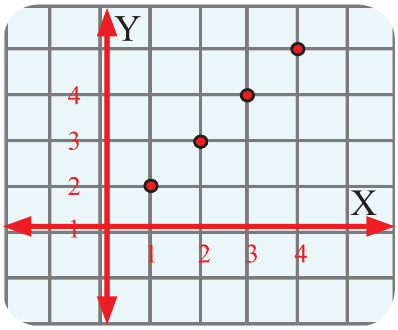
6. Assume the two mappings $f: Z \rightarrow Z$, where $f(x)= 3x+1$ and $g: Z \rightarrow Z$, where $g(x)=2x+5$. Find the value of x if $(fog)(x)=28$.
7. If $f: N \rightarrow N$, where $f(x)=5x+2$ and $g: N \rightarrow N$, where $g(x)=x+3$. Write the mapping (fog) by writing its ordered pairs.

Solve the Exercises

8. If $A=\{1,2,3\}$ and $B=\{4,5,6\}$, and $f: A \rightarrow B$ is defined as follow: $f = \{(1,4), (2,5), (3,6)\}$, Draw the arrowy diagram of the mapping and represent it in the coordinate plane.
9. If $f: A \rightarrow Z$, where $f(x) = x^2$ and the set $A=\{-2,-1,0,1,2\}$. Represent the mapping in the coordinate plane, and show if the mapping is injective or not?
10. Assume $f: N \rightarrow N$, and $g: N \rightarrow N$, $f(x)=x^2$, and $g(x)=x+1$. It is required to find:
i) $(gof)(x)$, $(fog)(x)$, ii) $(fog)(2)$, $(gof)(2)$

Solve the problems

11 Temperatures: In a day of winter, the temperatures recorded as shown in the following relation $R=\{(6,-2), (9,-3), (12,-4), (15,-5)\}$, where the first coordinate represents the time in hours , and the second coordinate represents the temperature in celsius degrees. Represent the relation in a table and represent it in the coordinate plane . Does the relation represent a mapping or not? Explen your answer.



12 Coordinate plane: The nearby graphic figure represents the mapping f:N→N.
Write the coordinates of the ordered pairs which can be represented by the mapping points in the graphic. Write a connection rule of the mapping, is the mapping an injective or not?

13 Health: The relation $W_r = 2(\frac{W_b}{3})$ represents the mass of water in human body, and W_b represents the mass of human body. Hassan's mass is 150 kg, he follows a diet to reduce the mass for three months, he lost 6kg in the first month and 12kg in the second month and 12kg in the third month. Write all the ordered pairs for the relation between Hassan's mass and the mass of water in his body. Does it represent a mapping or not ?

Think

14 Challenge: If A={1,2,3,} and g:A→A is a defined mapping, as follow:

g = {(3,1),(1,2), (2,3)} , f = {(1,3),(3,3), (2,3)}
show does fog = gof ?

15 Correct the mistake: Yaseen saide that the relation f:Z→Z, where $f(x) = x^3$ does not represent an injective mapping. Determine Yaseen's mistake and correct it.

16 Numerical sense: **Determine if each of the following relations f:X→Y represents a mapping or not? Explain that.**

X	1	2	3	4	5
Y	3	5	7	9	11

Write

The value of x, if the mapping f: N → N , and where $f(x) = 4x - 3$, $(fof)(x) = 33$.

Lesson [1-3] Compound Inequalities

Idea of the lesson:
*Solving the inequalities which contain connecting tools (and) , (or), then representing the solution on the numbers line

Vocabulary:
*Compound inequalities
*Intersection
*Union
*Solution set

Learn

We use the minimum and maximum celsius degrees to measure the weather temperature during a day because it is variable from time to time. If the minimum celsius temperature in Baghdad is $8^{\circ}c$ and the maximum one is $15^{\circ}c$ during December month, write an ineqalities reperesents the tempereathure in Baghdad , then find its solution

[1-4-1] Compound inequalities which contain "and"

You have previously learned the algebraic inequalities and their properties. You have also learned how to find the solution set and how to represent it in the line of numbers. Now, you will learn the compound inequalities which contain the connecting tool "and" and how to find their solution set and how to represent them in the line of the real numbers. As the compound inequality contains the connecting tool "and" and it consists of two inequalities, so it will be true only when the two inequalities are true. According to that, its solution set will be a set of intersection for solution of the two inequalities. We can do that by two methods, the first is graphically by representing the solution of the two inequalities in the line of numbers and then determine the intersection area. The second method is algebraically by finding the solution set for each inequality, then take their intersection set ($S = S_1 \cap S_2$).

Example (1) Write the compound inequality which represents the minimum and maximum celsius temperatures in Baghdad, then find its solution.

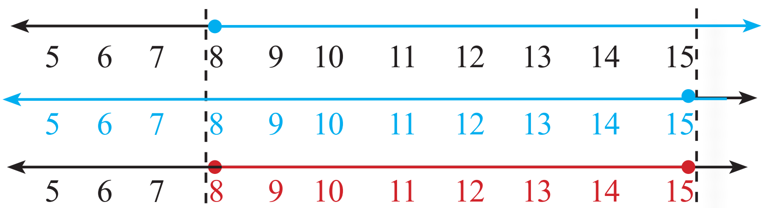
The temperature (minimum) is not less than 8^0 ($8 \leq x$), while the temperature (maximum) is not greater than 15°($x \leq 15$). The temperature is not less than 8 and not greater than 15° ($8 \leq x$ and $x \leq 15$). It can be solved in any of the two methods:

The first method : graphically

$x \geq 8$

$x \leq 15$

$8 \leq x \leq 15$



It can be read x greater than or equals to 8 and less than or equals to 15

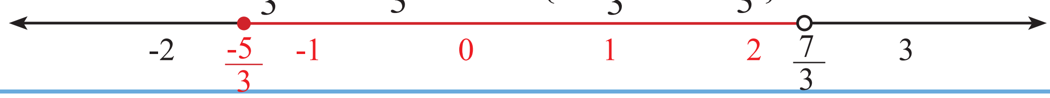
The second method: Algebraically : $8 \leq x \leq 15 \Rightarrow 8 < x$ and $x \leq 15$

$$\Rightarrow S = S_1 \cap S_2 = \{x: x \geq 8\} \cap \{x: x \leq 15\} = \{x: 8 \leq x \leq 15\}$$

Example (2) Solve the compound inequality which contains (and) $-3 \leq 3x+2 < 9$ algebraically, then represent the solution on the straight line of numbers.

$$-3 \leq 3x+2 < 9 \Rightarrow -3 - 2 \leq 3x+2 - 2 < 9 - 2 \Rightarrow -5 \leq 3x < 7 \Rightarrow \frac{-5}{3} \leq \frac{3x}{3} < \frac{7}{3}$$

$$\Rightarrow \frac{-5}{3} \leq x < \frac{7}{3} \Rightarrow S = \{x: \frac{-5}{3} \leq x < \frac{7}{3}\}$$



[1-4-2] Compound inequalities which contain “or”

After you have already learned the compound inequality which contains the connecting tool (and), you will learn the compound inequality which contains the connecting tool (or) and it will be true only when, at least one of its two inequalities is true. Accordingly its solution set is a set of the two qualities solution union. It can be found in two methods, first graphically by representing the two inequalities solution in the line of numbers, then determining union area. Second method, algebraically by finding the solution set for each inequality, then taking their union set ($S = S_1 \cup S_2$).

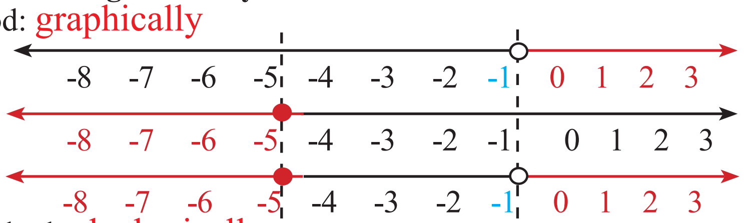
Example (3) **Solve the compound inequality $2 < x+3$ or $x+3 \leq -2$ graphically and algebraically.**

First method: graphically

$x+3 > 2$ $\qquad x > -1$

$x+3 \leq -2$ $\qquad x \leq -5$

$x \leq -5$ or $x > -1$



Second method: algebraically

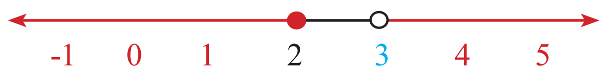
$x+3 \leq -2$ or $x+3 > 2 \Rightarrow \left\{ \begin{array}{l} x+3 > 2 \text{ or } x+3 \leq -2 \\ x > -1 \text{ or } x \leq -5 \end{array} \right\} \Rightarrow S = S_1 \cup S_2$

$= \{x: x > -1\} \cup \{x: x \leq -5\}$

Example (4) **Solve the compound inequality which contains (or) algraphically and represent the solution on the line of numbers.**

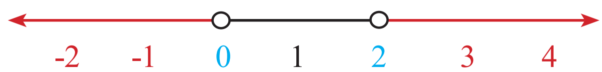
i) $y-3 \leq -1$ or $y+3 > 6 \Rightarrow y \leq 2$ or $y > 3$

$\Rightarrow S = S_1 \cup S_2 = \{y: y \leq 2\} \cup \{y : y > 3\}$



ii) $\frac{2v+1}{3} > \frac{5}{3}$ or $\frac{2v+1}{3} < \frac{1}{3} \Rightarrow v > 2$ or $v < 0$

$\Rightarrow S = S_1 \cup S_2 = \{ v : v > 2 \} \cup \{ v : v < 0 \}$



[1-4-3] Triangular Inequality

One of the subjects which connects algebra to geometry is the triangular inequality “in each triangle, the sum of two sides length is greater than the length of the third side” , it is used in the geometrical constructions and designs. If the lengths side of a triangle is (A B C), then the following three inequalities should be true : $A+B > C$, $A+C > B$, $B+C > A$

Example (5) i) Can the three sides of a triangle with length 13cm ,10cm and 2cm compose a triangle ?

No, they can’t because: $2 + 10 \not> 13$ is false , $10 + 13 > 2$ is true , $2 + 13 > 10$ is true .

ii) Write a compound inequality which shows the length of the third side in a triangle which has two sides with length 8cm and 10cm.

Suppose that the length of the third side is x, then:

$8 + 10 > x \Rightarrow 18 > x \Rightarrow$ The third side is less than 18

$8 + x > 10 \Rightarrow x > 2 \Rightarrow$ The third side is greater than 2

$10 + x > 8 \Rightarrow x > -2 \Rightarrow$ Doesn’t give any useful data

$\Rightarrow$ So the length of this side must be less than18 and greater than 2 and by the compound inequality, we see that the range of the third side length is $2 < x < 18$

Make sure of your understanding

Solve the compound inequalities which include (and) graphically:

Questions 1-2 are similar to example 1

1. $-4 \le y - 1 < 3$
2. $-4 \le x+2 \le 8$

Solve the compound inequalities which include (and) algebraically, then represent the solution set on the line of numbers:

Questions 3-4 are similar to example 2

3. $12 \le x+6$ and $x+6 < 15$
4. $-9 < 2x - 1 \le 3$

Solve the compound inequalities which includes (or) graphically:

Questions 5-6 are similar to example 3

5. $8y \le 32$ or $8y \ge 64$
6. $\frac{2z}{3} < \frac{2}{3}$ or $\frac{2z}{3} \ge \frac{8}{9}$

Solve the compound inequalities which include (or) algebraically, then represent the solution on the line of numbers:

Questions 7-8 are similar to example 4

7. $3n-7 > -5$ or $3n-7 \le -9$
8. $x+15 < 22$ or $x+15 \ge 30$

Can the three sides, which shown below, compose a triangle?

Questions 9-12 are similar to example 5

9. 1cm, 2cm, $\sqrt{3}$ cm
10. 5cm ,4cm, 9cm
11. 1cm, $\sqrt{2}$ cm, $\sqrt{2}$ cm
12. 3cm, 4cm, $2\sqrt{3}$ cm

Solve the Exercises

Solve the compound inequalities which include (and) graphically:

13. $-12 < x$ and $x \le -7$
14. $2 \le y + 4 < 6$

Solve the compound inequalities which include (and) algebraically, then represent the solution set on the line of numbers:

15. $14 \le 3x+7$ and $3x+7 < 26$
16. $\frac{1}{15} \ge \frac{z+3}{5} \ge \frac{1}{25}$

Solve the compound inequalities which includes (or) graphically:

17. $z-2 < -7$ or $z-2 > 4$
18. $x-6 \le -1$ or $x-6 > 4$

Solve the compound inequalities which include (or) algebraically, then represent the solution set on the line of numbers:

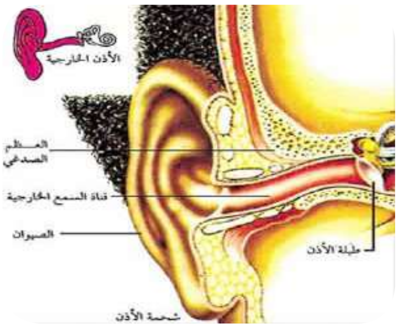
19. $\frac{y}{2} < 3\frac{1}{2}$ or $\frac{y}{2} > 7\frac{1}{2}$
20. $5x \le -1$ or $5x \ge 4$

Write the compound inequality which shows the length of the third side in the triangle which has two known-length sides:

21. 3cm , 10cm
22. 6cm, 4cm
23. 1cm, 3cm

Solve the problems

24 Sound: Human's ear can hear sound which its frequency is not less than 20 Hz and not more than 20000 Hz. Write a compound inequality represents the frequencies which human's ear can not hear them, then represent on line of numbers.



25 Cars tyre: The ideal air pressure which is recommended for tyres of saloon cars is not less than 28 pascal (N/m^2) and not more than 36 pascal. Write a compound inequality which represents the pressure, then represent on line of numbers.

Note: Pascal is unit for measuring the pressure of air which is (N/m^2) .

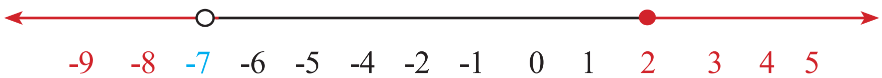
26 Magnetic train: Hanging magnetic train which operates in the magnetic lifting force, briefly it is called (Maglev). Different types of the magnetic trains were designed all over the world, the speed of those train is not less than 300 k/h and not more than 550 k/h. Write an equality represents the speed of train, then represent on line of numbers.

Think

27 Challenge: Write a compound inequality shows the range of the third side length in each triangle:

7 cm , 12 cm , x cm

28 Correct the mistake: Sawsen said that the compound inequality $-4 < x+3$ and $x+3 \leq 5$ represents the set of solution in the following line of numbers.



Show Sawsen's mistake, then correct it.

29 Numerical sense: **Mention if the three lengths are for a triangle or not? Explain that.**

i) 3.2cm, 5.2cm, 6.2cm　　　ii) 1cm, 1cm, $\sqrt{2}$ cm

Write

A compound inequality represents the minimum degree temperature of which is 18° and the maximum degree temperature of which is 27°.

Lesson [1-4] Absolute Value Inequalities

Idea of the lesson:
*Solving inequalities which contain an absolute value.

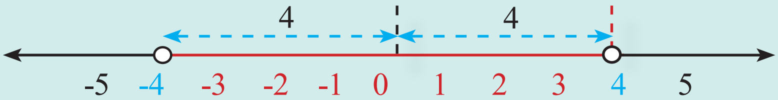
Vocabulary:
* Absolute value

Learn

Babylon hotel is one of the tourist hotels in Baghdad. It locates in Al-jadriya area. The ideal temperature of water in the swimming pool is 25 celsius, with increas or decrease of one degree. Write an absolute value inequality represents the range of water temperature in the swimming pool.

[1-5-1] Absolute value Inequalities $|g(x)| \leq a$, $|g(x)| < a$, where $a \in R^+$

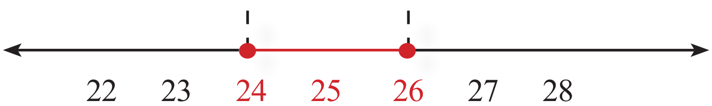
You have previously learned about the compound inequalities which contain (and) and (or), and how to solve them graphically and algebraically, and how to represent the solution set in the line of numbers. Now, you will learn the absolute value inequality with from $|g(x)| \leq a$, $|g(x)| < a$, $a \in R^+$
for example :$|x| < 4$ which means: What are the values of x which is far of zero in less than 4 units? they include all numbers between -4 and 4, then represent them in the line of numbers which is:



We note that the solution of this quality is:$\{-4 < x$ and $x < 4\}$
That means that the absolute value inequality was connect to relation of less than (less or equals to) represents a compound inequality which includes (and).
In general form : $|x| \leq a \Rightarrow -a \leq x \leq a$, $a > 0$

Example (1) Write the absolute value inequality which represents the temperature of water in the swimming pool, then represent it graphically.

Assume that the temperature of water is (x) celsius, so the inequality which represents the temperature of pool when it is not more than 26 celsius is:
$x \leq 25 + 1 \Rightarrow x - 25 \leq 1$
and the inequality which represents the temperature of pool when it is not less than 24 celsius is:
$x \geq 25 - 1 \Rightarrow x - 25 \geq -1$
So the absolute value inequality is the compound inequality which represents the range of water temperature in the swimming pool. $x - 25 \geq -1$ and $x - 25 \leq 1 \Rightarrow -1 \leq x - 25 \leq 1 \Rightarrow |x-25| \leq 1$
The representation of the solution set and the line of numbers is:



Example (2) **Solve the absolute value inequalities, then represent the solution in the line of numbers.**

i) $|x + 6| < 3 \Rightarrow -3 < x + 6 < 3 \Rightarrow -3-6 < x < 3-6$
$\Rightarrow -9 < x < -3$

ii) $|y| - 5 \leq 1 \Rightarrow |y| \leq 1 + 5 \Rightarrow |y| \leq 6$
$\Rightarrow -6 \leq y \leq 6$

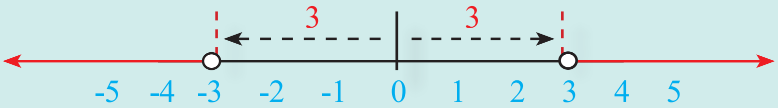
[1-5-2] Absolute value Inequalities which is in from of $| g (x) | \geq a$, $| g (x) | > a$ where $a \in R$

After you have learned the absolute value inequality which contains the form of $| g (x) | \leq a$, $| g (x) | < a$ where $x \in R$. Now, you will learn the absolute value inequality which contains the form of $| g (x) | \geq a$ $| g (x) | > a$ where $x \in R$ for example: $| x | > 3$, which means: distance between x and zero greater than 3 that is

$x > 3$ or $x < -3$ and the inequality solution set is $\{ x:x < -3 \} \cup \{ x:x > 3\}$

So the absolute value inequality with the relation greater than (greater or equal) is a compound relation which includes (or).

In general form : $| x | \geq a \Rightarrow | x \geq a$ or $x \leq -a$, $a > 0$



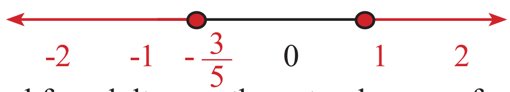
Example (3) **Solve the absolute value inequality, then represent the solution on the line of numbers.**

i) $| x + 4 | > 2 \Rightarrow -2 > x+4$ or $x+4 > 2 \Rightarrow -6 > x$ or $x > -2$

$$\Rightarrow S = S_1 \cup S_2 = \{x: x < -6 \} \cup \{x: x > -2\}$$

ii) $| 5y -1| \geq 4 \Rightarrow - 4 \geq 5y-1$ or $5y-1 \geq 4 \Rightarrow -\frac{3}{5} \geq y$ or $y \geq 1$

$$\Rightarrow S = S_1 \cup S_2 = \{y: y \leq -\frac{3}{5} \} \cup \{y: y \geq 1\}$$



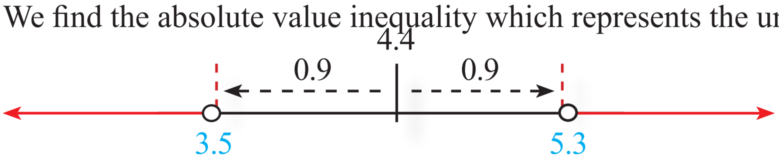
iii) In analysis of a blood for adult men, the natural range of potassium is (3.5 – 5.3) mmol/L . Write the absolute value inequality which represents the unnatural range of potassium in human blood.

The inequality which represents the unnatural quantity of potassium and less than the lowest value of average is: $x < 3.5$

The inequality which represents the unnatural quantity of potassium and more than the greatest average is: $x > 5.3$

The unnatural range of potassium is the compound inequality solution: $x > 5.3$ or $x < 3.5$

We find the absolute value inequality which represents the unnatural range of potassium:



we take the mid point between two points , then we substract and add the radius of the distance.

$3.5 > x$ or $x > 5.3 \Longleftrightarrow 4.4 - 0.9 > x$ or $x > 4.4 + 0.9$

$$\Longleftrightarrow -0.9 > x- 4.4 \text{ or } x- 4.4 > 0.9 \Longleftrightarrow | x- 4.4 | > 0.9$$

Example (4) **Find the solution set for the following absolute value inequalities:**

i) $| 2x - 5 | +3 < 11 \Rightarrow | 2x - 5| < 8 \Rightarrow -8 < 2x - 5 < 8 \Rightarrow -3 < 2x < 13$

$$\Rightarrow -\frac{3}{2} < x < \frac{13}{2} \Rightarrow \{x: x > -\frac{3}{2} \} \cap \{x: x < \frac{13}{2}\} \Rightarrow \{ x : \frac{-3}{2} < x < \frac{13}{2} \}$$

ii) $|7 - y | < 8 \Rightarrow - 8 < 7- y < 8 \Rightarrow -15 < -y < 1 \Rightarrow -1 < y < 15 \Rightarrow \{y: y > -1\} \cap \{y: y < 15\}$

iii) $| \frac{2t-8}{4} | \geq 9 \Rightarrow | \frac{2(t-4)}{4} | \geq 9 \Rightarrow | \frac{t-4}{2} | \geq 9 \Rightarrow | t -4 | \geq 18$

$t - 4 \leq -18$ or $t -4 \geq 18 \Rightarrow t \leq -14$ or $t \geq 22 \Rightarrow \{t : t \leq -14\} \cup \{t : t \geq 22\}$

iv) $| \frac{5 - 3v}{2} | \geq 6 \Rightarrow | 5 - 3v| \geq 12 \Rightarrow -12 \geq 5- 3v$ or $5- 3v \geq 12 \Rightarrow - 3v \geq -17$ o r $-3v \geq 7$

$$\Rightarrow v \geq \frac{17}{3} \text{ or } v \leq \frac{-7}{3} \Rightarrow \{v : v , \frac{17}{3}\} \cup \{v: v \leq \frac{-7}{3}\}$$

Make sure of your understanding

Write the absolute value inequality which represents the following problems:

Questions 1 are similar to examples 1,3

1 The ideal temperature inside flats is 22°celsius with increase or decrease of 2°celsius.

Solve the absolute value inequalities, then represent the solution on the line of numbers.

Questions 2-5 are similar to example 2

2 $|x+1|<5$

3 $|3z-7|\leq 2$

4 $|x|+8<9$

5 $|5y|-2\leq 8$

Questions 6-9 are similar to example 3

6 $|x+4|>6$

7 $|5z-9|>1$

8 $|2x|+7\geq 8$

9 $|4y|-2>3$

Questions 10-13 are similar to example 4

10 $|5-x|<10$

11 $|4z-14|>2$

12 $\left|\frac{x-12}{4}\right|\leq 9$

13 $\left|\frac{6-2y}{4}\right|\geq 9$

Solve the Exercises

Write the absolute value inequality which represents the following problems:

14 The temperature inside fridge should be 8°celsius with increasing or decreasing of 0.5°celsius. Write the range of ideal temperature inside fridge.

15 The boiling degree of water is 100°celsius at the sea surface level. It increases or decreases in the mountainous areas and valleys in no more than 20°celsius. Write the range of vipration in the boiling degree of water.

Solve the following absolute value inequalities:

16 $|x+3|<6$

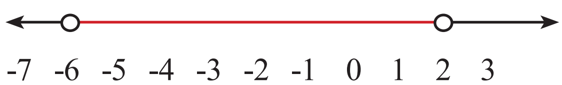
17 $|2z|-5<2$

18 $2|x|-7\geq 1$

19 $|11z|-2\geq 9$

20 $\left|\frac{4}{5}z-1\right|>\frac{4}{5}$

Write an inequality includes an absolute value for all of the following graphic inequalities:

21


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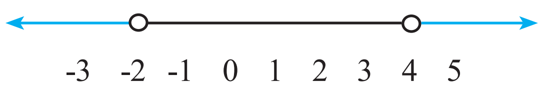

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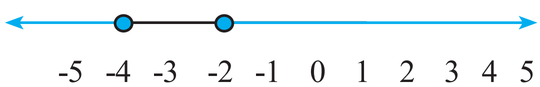


Solve the problems

Write the absolute value inequality which represents each of the following problems:

25 Badger: The animal, Badger is one of mammals which belongs to the division of preanants. It has short legs. It lives in holes which the Badger itself made. The length of its body, from head to tail, is from 68cm to 76cm. Write the range of Badger length.

26 Health: The natural pulse rate (number of heart (beats) for adult men is from 60 to 90 beats in minute. Write the range of unnatural heart beats of human.

27 Transportation: The civilian plane flies in height from 8 km to 10 km where it is considered a moderate area. Write the range of the civilian aviation area.

Think

28 Challenge: **Solve the absolute value inequalities and represent the solution on the line of numbers .**

i) $\left| \dfrac{\sqrt{3}\,(x+1)}{\sqrt{2}} \right| \le \sqrt{6}$ ii) $\left| \dfrac{\sqrt{12} - \sqrt{3}\,y}{\sqrt{5}} \right| \ge \sqrt{15}$

29 Correct the mistake: Khulood said that the absolute value inequality $|6 - 3y| \ge 7$ represents a compound inequality with a relation (and). and with its solution : $\{ y : -\frac{1}{3} \le y \le \frac{13}{2} \}$
Show the mistake of Khulood, then correct it.

30 Numerical sense: **Write the solution set for the following absolute value inequalities in the real numbers set:**

i) $|z| - 1 < 0$ ii) $|x - 1| > 0$

Write

An absolute value inequality represents a situation from life, then represent the solution set on the straight line of numbers.

Chapter Test

Simplify the following numerical sentences by using the ordering of operations in the real numbers:

1 $(\sqrt{3}+\sqrt{5})\ (\sqrt{3}+\sqrt{5}) = ...$

2 $\frac{\sqrt{3}-\sqrt{6}}{\sqrt{3}} - \frac{\sqrt{8}-5}{3\sqrt{2}} = ...$

Use the ordering of operations and the calculator to write each of the following which should be written to the nearest tenth.

3 $(\frac{1}{125})^{\frac{1}{3}} - (-\frac{1}{2})^{0} + (121)^{\frac{1}{2}} \times (\frac{1}{9})^{\frac{1}{2}} = ...$

4 If $f : Z \to R$, where $f(x) = x^2$. Draw an arrowy diagram for the mapping, then show if the mapping is injective, surjective or bijective ?

5 If the mapping $f: N \to N$, where $f(x) = 3x + 1$, $g : N \to N$, where, $g(x) = x^2$
Find: (gof)(5) , (fog)(5) , (gof)(2) , (fog)(2) .

6 If the mapping $f: R \to R$ where $f(x)= 3x+1$ and the mapping $g: R \to R$, since $g(x)= 2x+5$.
Does (fog) (x) = (gof) (x) ? find the value of x if (fog) (x) = 28

Solve the compound inequalities, then represent the solution set on the line of numbers.

7 $12 \leq x + 6$ و $x+6 < 20$

8 $\frac{1}{8} \geq \frac{z+2}{2} > \frac{1}{16}$

9 $x-3 \leq -5$ or $x-3 > 5$

10 $7t-5 > -1$ or $7t-5 \leq -14$

11 $y \leq 0$ or $y + 7 \geq 16$

12 $\frac{y}{3} < 1\frac{1}{3}$ or $\frac{y}{3} > 9\frac{1}{3}$

Write the compound inequality shows the range of the third side in each triangle:

13 4cm , 9cm

14 5cm ,12cm

15 7cm ,15cm

Solve the following abslute value inegualities:

16 $|x - 6| \leq 3$

17 $|3z| - 5 < 4$

18 $|x + 1| > \frac{1}{2}$

19 $6|x| - 8 \geq 3$

20 $|3y| - 2 > 9$

21 $|8z| - 1 > 7$

22 $|4 - 3y| \geq 14$

23 $|\frac{6-3y}{9}| \geq 5$

Chapter 2

Algebraic Expressions

Al-Mustansiriya school is an ancient one .It was established in the reign of Abbassiyn in Baghdad in 1233 .It was an important cultural and scientific center. It locates in AL- Risafa side of Baghdad . There is a rectangular area in the middle of school which has a great fountain and o'clock of school .If we assume that the length of the internal area of school is (x+14) meters and its width is(x+2)meters,then we can calculate the area by multiplying the tow algebraic expressions (x+14) (x+2).

Pretest

Find the result of adding or subtracting the following algebraic expressions:

1. $(3x^2 + 4x - 12) + (2x^2 - 6x + 10)$

2. $(\frac{1}{2} zy + 5z - 7y) - (\frac{1}{4} zy - 3z + 2y)$

Find the result of multiplying the following algebraic terms:

3. $7x^2 \times \frac{1}{14x}$

4. $\sqrt{2}\ yz \times \sqrt{2}\ yz^2$

5. $\frac{3}{4} v^2 t \times \sqrt{12}\ t^{-1}$

6. $3h\ (\frac{1}{6} v - \frac{1}{3} h^{-2})$

Find the result of multiplying two algebraic expressions:

7. $(x+2)\ (x- 2)$

8. $(5- 2z)\ (3+ 3z)$

9. $(\frac{1}{2} x^2 + 6)\ (\frac{4}{3} x^2 +12)$

10. $(2\sqrt{3}\ t -4)^2$

11. $(x+3)\ (x^2- 3x + 9)$

12. $(xy + 1)\ (x^{-1}y - xy^{-1} - 1)$

Find the result of multiplying by using the vertical method:

13. $(y-1)\ (y+1)$

14. $(2x+3)\ (4x^2 - x - 5)$

15. $(3-z)\ (3+ 5z - z^2)$

Find the result of dividing the following algebraic expressions:

16. $\frac{3xy^2}{15x^2y}$

17. $\frac{-47z^{-2}}{7z^2}$

18. $\frac{8x^3 + 4x^2 - 2x}{2x}$

19. $\frac{21 - 14a + 7a^2}{7a}$

Factoring the algebraic expressions by using the greater common factor :

20. $3y^3 + 6y^2 - 9y$

21. $\frac{1}{2} zx^2 - 2z^2 x + 4zx$

Lesson [2-1] Multiplying Algebraic Expressions

Idea of the lesson:
* Multiplying an algebraic expression by other algebraic expression which represents special cases.

Vocabulary:
* Square of sum
* Square of difference
* Cubic of sum
* Cubic of difference

Learn

A square-shaped home garden was surrounded by a fence. The length of its side is h meter with an aisle of one meter width. What is the area of aisle according to h ?

[2-1-1] Multiplying two Algebraic Expressions each one Contains two terms.

You have previously learned how to multiply two algebraic terms with each other and how to multiply an algebraic expression by another one. Now, you will learn how to multiply two algebraic expressions which each one of them has two terms with each other , and they represent a square of sum or a square of difference or sum multiply by difference, by using the properties that you previously studied which are distributing ,substituting and ordering.

Example (1) Find the area of the aisle which surrounds the square-shaped garden.

The area of the aisle is the difference between the two area of the big square (garden with aisle) and the small square (the garden)

$(h+2)^2 = (h+2)(h+2) = h^2 + 2h + 2h + 4 = h^2 + 4h + 4$ *The area of garden with aisle*

$h \times h = h^2$ *The area of garden*

$(h^2 + 4h + 4) - h^2 = h^2 - h^2 + 4h + 4 = 4h + 4$ *The area of aisle*

Example (2) **Find the result of multiplying an algebraic expressions by another algebrail expression where each one has two terms :**

i) $(x + y)^2 = (x + y)(x + y) = x^2 + xy + yx + y^2 = x^2 + 2xy + y^2$ *Square of sum for two terms*

ii) $(x - y)^2 = (x - y)(x - y) = x^2 - xy - yx + y^2 = x^2 - 2xy + y^2$ *Square of difference between two terms*

iii) $(x + y)(x - y) = x^2 - xy + yx - y^2 = x^2 - y^2$ *Sum of two terms × the difference between them.*

iv) $(x + 3)(x + 5) = x^2 + 5x + 3x + 15 = x^2 + 8x + 15$ *Sum of two terms × sum of two terms.*

v) $(x + 2)(x - 6) = x^2 - 6x + 2x - 12 = x^2 - 4x - 12$ *Sum of two terms ×the difference between two terms.*

vi) $(x - 1)(x - 4) = x^2 - 4x - x + 4 = x^2 - 5x + 4$ *difference between two terms×difference between two terms.*

Example (3) **Find the result of multiplying the following algebraic expressions:**

i) $(z + 3)^2 = z^2 + 6z + 9$

ii) $(h - 5)^2 = h^2 - 10h + 25$

iii) $(2x - 7)(2x + 7) = 4x^2 - 49$

iv) $(3y + 1)(y + 2) = 3y^2 + 7y + 2$

v) $(v + \sqrt{2})(v - \sqrt{2}) = v^2 - 2$

vi) $(n - \sqrt{3})(5n - \sqrt{3}) = 5n^2 - 6\sqrt{3}\,n + 3$

[2-1-2] Multiplying algebraic expressions from two terms by another three terms

You have previously learned the multiplying of algebraic expression which have many terms. Now, you will learn special cases of multiplying an algebraic expression which consists of two terms by another algebraic expression which consists of there terms by using the properties that you studied in distributing , substituting and ordering.

Example (4) **Find the result of multiplying an algebraic expression which consists of two terms by an algebraic expression which consists of three terms:**

i) $(x+2)(x^2-2x+4) = x^3-2x^2+4x+2x^2-4x+8 = x^3+8 = x^3+2^3$ *The result of multiplying is the sum of two cubes*

ii) $(y-3)(y^2+3y+9) = y^3+3y^2+9y-3y^2-9y-27 = y^3-27 = y^3-3^3$ *The result of multiplying is the difference between two cubes*

iii) $(y+2)^3 = (y+2)(y+2)^2 = (y+2)(y^2+4y+4)$ *Cube of two terms sum*

$= y^3+4y^2+4y+2y^2+8y+8 = y^3+6y^2+12y+8$

iv) $(z-3)^3 = (z-3)(z-3)^2 = (z-3)(z^2-6z+9)$ *Cube of the difference between two terms*

$= z^3-6z^2+9z-3z^2+18z-27 = z^3-9z^2+27z-27$

Example (5) **Find the result of multiplying the following algebraic expressions :**

i) $(2v+5)(4v^2-10v+25) = 8v^3-20v^2+50v+20v^2-50v+125 = 8v^3+125 = (2v)^3+5^3$

ii) $(\frac{1}{3}-z)(\frac{1}{9}+\frac{1}{3}z+z^2) = \frac{1}{27}+\frac{1}{9}z+\frac{1}{3}z^2-\frac{1}{9}z-\frac{1}{3}z^2-z^3 = \frac{1}{27}-z^3 = (\frac{1}{3})^3-z^3$

iii) $(x-\sqrt[3]{2})(x^2+\sqrt[3]{2}x+\sqrt[3]{4}) = x^3+\sqrt[3]{2}x^2+\sqrt[3]{4}x-\sqrt[3]{2}x^2-\sqrt[3]{4}x-\sqrt[3]{8}$

$= x^3+\sqrt[3]{2}x^2-\sqrt[3]{2}x^2+\sqrt[3]{4}x-\sqrt[3]{4}x-2 = x^3-2$

iv) $(x+\frac{1}{2})^3 = (x+\frac{1}{2})(x+\frac{1}{2})^2 = (x+\frac{1}{2})(x^2+x+\frac{1}{4}) = x^3+x^2+\frac{1}{4}x+\frac{1}{2}x^2+\frac{1}{2}x+\frac{1}{8}$

$= x^3+x^2+\frac{1}{2}x^2+\frac{1}{4}x+\frac{1}{2}x+\frac{1}{8} = x^3+\frac{3}{2}x^2+\frac{3}{4}x+\frac{1}{8}$

v) $(y-5)^3 = (y-5)(y-5)^2 = (y-5)(y^2-10y+25)$

$= y^3-10y^2+25y-5y^2+50y-125 = y^3-15y^2+75y-125$

Make sure of your understanding

Find the result of multiplying an algebraic expression by another algebraic expression where both of them have two terms:

1 $(x + 3)(x - 3)$

2 $(\sqrt{7} - h)^2$

3 $(z + \sqrt{5})(z - \sqrt{5})$

4 $(v + 5)(v + 1)$

5 $(x - 3)(x - 2)$

6 $(3x - 4)(x + 5)$

7 $(\frac{1}{3}y + 3)(\frac{1}{3}y + 2)$

Questions 1- 7 are similar to examples 2-3

Find the result of multiplying an algebraic expression which consists of two terms by another algebraic expression which consists of three terms:

8 $(y+2)(y^2 - 2y+4)$

9 $(2z + 4)(4z^2 - 8z + 16)$

10 $(v - \sqrt[3]{3})(v^2 + \sqrt[3]{3}\ v + \sqrt[3]{9})$

11 $(\sqrt[3]{\frac{2}{7}} + m)(\sqrt[3]{\frac{4}{49}} - \sqrt[3]{\frac{2}{7}}\ m + m^2)$

12 $(x + 5)^3$

13 $(y - 4)^3$

Questions 8-13 are similar to examples 4-5

Solve the Exercises

Find the result of multiplying an algebraic expression by another algebraic expression where both of them have two terms:

14 $(n - 6)^2$

15 $(2x - 3)(x + 9)$

16 $(y + \sqrt{6})(y - \sqrt{6})$

17 $(4 - y)(5 - y)$

Find the result of multiplying an algebraic expression which consists of two terms by another algebraic expression which consists of three terms:

18 $(x+6)(x^2 - 6x+36)$

19 $(z - 3)^3$

20 $(x - \sqrt[3]{4})(x^2 + \sqrt[3]{4}\ x + \sqrt[3]{16})$

21 $(\sqrt[3]{\frac{1}{5}} + n)(\sqrt[3]{\frac{1}{25}} - \sqrt[3]{\frac{1}{5}}\ n + n^2)$

Solve the problems

22 **Swimming pool:** Baghdad hotel is one of the important tourist hotels in Baghdad, the capital of Iraq. The length of the swimming pool is (x+9) meter and the width is (x+1) meter.
It is surrounded by an aisle which its width is 1 meter.
Write the area of the swimming pool with the aisle in simplest form.

23 **History:** Babylon city locates to the north of Al-Hila city in Iraq. Babylonians lived there since about 3000 years BC. In 575 , they built the gate of Ishtar which considers the eighth gate of Babylon wall. Wa·el drew a painting represents the gate of Ishtar. The dimensions of the painting was (y+7), (y-4) cm.
Write the painting area which was drawn by Wa·el in simplest form by y .

24 **Ornament at fish :** A cubic-shoped aquarium, the length of its side is (v+3) cm.
Write the volume of the aquarium in simplest form.

Think

25 **Challenge:** Find the result of the following in simplest form:

$(x + 1)^2 - (x - 2)^2$

26 **Correct the mistake:** :Nisreen wrote the result of multiplying the two algebraic expressions, as follow:

$(\sqrt{5}\ h - 4)(h - 6) = 5h^2 + 10h - 24$

Determine Nisreen·s mistake , then correct it.

27 **Numerical sense:** Which of the following two numbers is greater.

$(\sqrt{3} - \sqrt{2})^2$ or the number $(\sqrt{3} + \sqrt{2})^2$? Clarify your answer.

Write

The result of multiplying the two algebraic expressions :

$(2z + \frac{1}{2})(2z - \frac{1}{2})$

Lesson [2-2] Factoring the Algebraic Expression by using a Greater Common Factor

Idea of the lesson:
*Factoring the algebraic expression by using the greater common factor.

Vocabulary:
*Factoring the algebraic expression
*The greater common factor
*Binomial terms
*Inverse
*Checking the correction of solution.

Learn

The monument of Kahrmana square in the middle of Baghdad is one of the distinctive civilizational landmarks in Iraq . It locates in the center of the square in Al-Karada. The radius of the circular statue is (r) meter. It is surrounded by a basin which is like a circular aisle. If the radius and the basin of the statue is r +2 meter, find the basin area.

[2-2-1] Factoring the algebraic expression by using a greater common factor

You have previously learned how to find the greater common factor for numbers . You have also learned how to factor the algebraic expression by using the greater common factor (GCF) . Now, you will increase your skills by learning the factoring of algebraic expression which consist of two or three terms by using the greater common factor, then checking the correction of solution.

Example (1) The radius of the base of kahrmana statue is r meter, and the radius of the statue with the basin is r+2 meter. Find the basin area.

$A_1 = r^2\pi$ *the area of statue .*

$A_2 = (r+2)^2\pi = (r^2 + 4r + 4)\pi = r^2\pi + 4r\pi + 4\pi$ *the area of statue with basin*

$A = A_2 - A_1 = r^2\pi + 4r\pi + 4\pi - r^2\pi$ *the area of basin*

$= 4r\pi + 4\pi = 4\pi(r+1)$ *(4π)the greater common factor*

The area of basin which surrounds the statue is $4\pi(r+1)$ squre meters

Example (2) **Factoring each expression by using the greater common factor (GCF), then checking the correction of solution :**

i) $6x^3 + 9x^2 - 18x = 3x(2x^2 + 3x - 6)$ *The greater common factor is 3x*

Checking :

$3x(2x^2 + 3x - 6) = 3x(2x^2) + 3x(3x) - 6(3x)$ *To check by using the multiplication of algebraic expressions.*

$= 6x^3 + 9x^2 - 18x$

ii) $\sqrt{12}\, y^2z + \sqrt{2}(\sqrt{6}\, yz^2 - \sqrt{24}\, yz)$ *Open the bracket with simplify the numerical roots.*

$= 2\sqrt{3}\, y^2z + 2\sqrt{3}\, yz^2 - 4\sqrt{3}\, yz$

$= 2\sqrt{3}\, yz(y + z - 2)$ *The greater common factor is $2\sqrt{3}\, yz$*

$2\sqrt{3}\, yz(y+z-2) = 2\sqrt{3}\, y^2z + 2\sqrt{3}\, yz^2 - 4\sqrt{3}\, yz$

Checking :
To check , we use the multiplication of the algebraic expression .

We see that the variables are equaled in terms with the original. expression and it is also with numerical factors because:

$2\sqrt{3} = \sqrt{12}$, $2\sqrt{3} = \sqrt{2}\sqrt{6}$, $4\sqrt{3} = \sqrt{2}\sqrt{24}$

Example (3) **Factoring each expression by using the binomial as a greater common factor :**

i) $5x(x+3) - 7(x+3) = (x+3)(5x-7)$ — *The greater common factor is* $(x+3)$

ii) $\frac{1}{2}(y-1) + \frac{1}{3}y^2(y-1) = (y-1)(\frac{1}{2} + \frac{1}{3}y^2)$ — *The greater common factor is* $(y-1)$

iii) $\sqrt{3}\,v^2(z+2) - \sqrt{5}\,v(z+2) = v(z+2)(\sqrt{3}\,v - \sqrt{5})$ — *The greater common factor is* $v(z+2)$

[2-2-2] Factoring an algebraic expression by using the property of grouping

You have previously learned in the previous how to factor the algebraic expression which consists of two or three terms by using the greater common factor. Now, you will learn how to factor an algebraic expression which consists of four terms or more by using the grouping of the terms ,where the grouping terms have common factors.

Example (4) **Factoring each algebraic expression by using the grouping, then check the correction of the solution:**

i) $4x^3 - 8x^2 + 5x - 10 = (4x^3 - 8x^2) + (5x - 10)$ — *Grouping terms which have common factors.*

$= 4x^2(x-2) + 5(x-2)$ — *Factoring the grouping terms*

$= (x-2)(4x^2+5)$ — *The greater common factor is(x-2)*

checking:

$(x-2)(4x^2+5) = x(4x^2+5) - 2(4x^2+5)$ — *Using the property of distributing*

$= 4x^3 + 5x - 8x^2 - 10 = 4x^3 - 8x^2 + 5x - 10$ — *Using multiplication and ordering*

ii) $\sqrt{2}\,h^2t + \sqrt{3}\,t^2v - \sqrt{8}\,h^2v - \sqrt{12}\,v^2t$

$= (\sqrt{2}\,h^2t - \sqrt{8}\,h^2v) + (\sqrt{3}\,t^2v - \sqrt{12}\,v^2t)$ — *Grouping terms*

$= \sqrt{2}\,h^2(t-2v) + \sqrt{3}\,tv(t-2v)$ — *Factoring the grouping terms*

$= (t-2v)(\sqrt{2}\,h^2 + \sqrt{3}\,tv)$ — *The greater common factor is(t- 2v)*

checking:

$(t-2v)(\sqrt{2}\,h^2 + \sqrt{3}\,tv) = t(\sqrt{2}\,h^2 + \sqrt{3}\,tv) - 2v(\sqrt{2}\,h^2 + \sqrt{3}\,tv)$ — *Using the property of distributing*

$= \sqrt{2}\,h^2t + \sqrt{3}\,t^2v - \sqrt{8}\,h^2v - \sqrt{12}\,v^2t$ — *Using multiplication and ordering*

Example (5) Factoring the algebraic expression by using the grouping with the inverse:

$14x^3 - 7x^2 + 3 - 6x = (14x^3 - 7x^2) + (3 - 6x)$ — *Grouping the terms*

$= 7x^2(2x-1) + 3(1-2x)$ — *Factoring the grouping terms*

$= 7x^2(2x-1) + 3(-1)(2x-1)$ — *Using the inverse*

$= 7x^2(2x-1) - 3(2x-1)$ — *Writing +3(-1) as -3*

$= (2x-1)(7x^2-3)$ — *The greater common factor is (2x-1)*

Make sure of your understanding

Factor each expression by using the greater common factor (GCF) , then check the correction of solution:

1. $9x^2 - 21x$
2. $10 - 15y + 5y^2$
3. $14z^4 - 21z^2 - 7z^3$
4. $\sqrt{8}\ t^2r + \sqrt{2}\ (tr^2 - \sqrt{3}\ tr)$

Questions 1- 4 are similar to example 2

Factor each expression by using the binomial as a greater common factor:

5. $3y(y - 4) - 5(y - 4)$
6. $\frac{1}{4}(t + 5) + \frac{1}{3}t^2(t + 5)$
7. $\sqrt{2}\ n(x + 1) - \sqrt{3}\ m(x + 1)$
8. $2x(x^2 - 3) + 7(x^2 - 3)$

Questions 5-8 are similar to example 3

Factor each expression by using the property of grouping , then check the correction of solution:

9. $3y^3 - 6y^2 + 7y - 14$
10. $21 - 3x + 35x^2 - 5x^3$
11. $2r^2k + 3k^2v - 4r^2v - 6v^2k$
12. $3z^3 - \sqrt{18}\ z^2 + z - \sqrt{2}$

Questions 9 -12 are similar to example 4

Factor each expression by using the property of grouping with the inverse:

13. $21y^3 - 7y^2 + 3 - 9y$
14. $\frac{1}{2}x^4 - \frac{1}{4}x^3 + 5 - 10x$
15. $6z^3 - 9z^2 + 12 - 8z$
16. $5t^3 - 15t^2 - 2t + 6$

Questions 13-16 are similar to example 5

Solve the Exercises

Factor each expression by using the greater common factor(GCF), then check the correction of solution

17. $12y^3 - 21y^2$
18. $6v^2(3v - 6) + 18v$

Factor each expression by using the binomial as a greater common factor:

19. $\frac{1}{7}(y + 1) + \frac{1}{3}y^2(y + 1)$

Factor each expression by using the property of grouping, then check the correction solution:

20. $5x^3 - 10x^2 + 10x - 20$
21. $3t^3k + 9k^2s - 6t^3s - 18s^2k$

Factor the expression by using the property of grouping with inverse:

22. $12x^3 - 4x^2 + 3 - 9x$

Solve the problems

23 Solar energy : The solar panels are the main component in the solar energy systems which generate electricity .The cells are manufactured from semiconducting materials such as silicon. They absorb the light of sun . What are the dimensions of the solar panel , if its area was $3x(x-4)-22(x-4)$ square meter.

24 Flamenco bird: Flamenco bird is one of the migratory birds which has beautiful shape . Its color is pink. These birds travel for long distances during the season of the annual migration passing by the marshes in the south of Iraq to get food from the water pools. If the area of the water pool which was covered by the flamenco birds in one of the Iraqi marshes is $4y^2 + 14y + 7(2y+7)$ square meter.
What is the shape of that pool, and what are its dimensions ?

25 Baghdad o·clock : It is a high building which has a Four_ Faces o·clock at the top of it.This building locates in the celebration park in Baghdad . It was established in 1994 .What is the radius of the internal circle of the o·clock if you know that its area is $z^2 \pi - 3z \pi - (3z - 9)\pi$.

Think

26 Challenge: Factor of the following expressions in a simplest form:

$5x^5 y + 7y^3 z - 10x^5 z - 14z^2 y^2$

27 Correct the mistake: Ibtisam had written the result of factoring the following expression, as follow:

$\sqrt{2}\, t^4 - \sqrt{24}\, t^3 + t^2 - \sqrt{12}\, t = (t + 2\sqrt{3}\,)(\sqrt{2}\, t^2 - t)$

Discover the mistake of Ibtisam and correct it.

28 Numerical sense: What is the unknown number in this expression.

$x^2 + 3x + 5x + 15 = (x + 3)(x + \square)$

Write

The result of subtraction the expression (x + y) (x - y) from the expression (x + y) (x + y) in simplest form .

Lesson [2-3]	Factoring the Algebraic Expression by using Special Identities

Idea of the lesson:
*Factoring the algebraic expression as a difference between two squares and a complete squre.

Vocabulary:
*The difference between two squares
*Perfect square
*Complet square
*The missing term

Learn

Al-Shaab international stadium in Baghdad is one of the important stadiums in Iraq. It was established in1966 . If the area of the playground, which was allocated for playing football , is x^2 -400 square meter , what are the dimensions of the playground ?

[2-3-1] Factoring the algebraic expression by the difference between two squares.

You have previously learned how to find the result of multiplying an algebraic expression which represents the sum of two terms by another algebraic expression which represents the difference between them , and the result represents the difference between their two squares . Now, you will learn the inverse operation of multiplication which is factoring the algebraic expression which is as a difference between two squares $(x^2 - y^2) = (x + y)(x - y)$.
The expression $x^2 + y^2$ can not be factored in this stage .

Example (1) Find the dimensions of football playground which its area is x^2 - 400 square meter ?

$x^2 - 400 = (x)^2 - (20)^2$ *Write each terms as a perfect square*
$= (x + 20)(x - 20)$ *Write the factoring*
The first bracket: the square root of the first term + the square root of the second term.
The second bracket: the square root of the first term – the square root of the second term.

So the length of the football playground is x+20 meter and its width is x-20 meter.

Example (2) **Factoring each of the following algebraic expressions as a difference between two squares**

i) $x^2 - 9 = (x + 3)(x - 3)$

ii) $36y^2 - z^2 = (6y + z)(6y - z)$

iii) $49 - v^2 = (7 + v)(7 - v)$

iv) $2x^2 - z^2 = (\sqrt{2}x + z)(\sqrt{2}x - z)$

v) $5h^2 - 7v^2 = (\sqrt{5}h + \sqrt{7}v)(\sqrt{5}h - \sqrt{7}v)$

vi) $12 - t^2 = (2\sqrt{3} + t)(2\sqrt{3} - t)$

vii) $8x^3y - 2xy^3 = 2xy(4x^2 - y^2)$ *Factoring by using the common factor*
$= 2xy(2x + y)(2x - y)$ *Factoring by using the difference between two squares.*

viii) $\frac{1}{16}z^4 - \frac{1}{81} = (\frac{1}{4}z^2 + \frac{1}{9})(\frac{1}{4}z^2 - \frac{1}{9}) = (\frac{1}{4}z^2 + \frac{1}{9})(\frac{1}{2}z + \frac{1}{3})(\frac{1}{2}z - \frac{1}{3})$

[2 -3-2] Factoring the Algebraic Expression by the Perfect square

you have previously learned how to find the result of multiplying a square of sum two terms and a square of difference between two terms , the result was consisted of three terms. Now , you will learn the inverse operation of multiplication which is factoring an expression consists of three terms in form of perfect square .

$x^2 + 2xy + y^2 = (x + y)^2$, $x^2 - 2xy + y^2 = (x - y)^2$ The alagebric expression $a x^2 \pm bx + c$ will be a perfect square if $bx = \pm 2\sqrt{(ax^2)(c)}$ where $a \neq 0$

Example (3) **Factor each of the following algebraic expressions which are in a form of a perfect square.**

i) $x^2 + 6x + 9 = (x)^2 + 2(x \times 3) + (3)^2$

$= (x + 3)(x + 3)$

$= (x + 3)^2$

ii) $y^2 - 4y + 4 = (y)^2 - 2(y \times 2) + (2)^2$

$= (y - 2)^2$

iii) $16z^2 - 8z + 1 = (4z)^2 - 2(4z \times 1) + (1)^2 = (4z - 1)^2$

Write the first and last terms as a perfect square
Write the middle term as a double of the first term root multiplying by the root of the last term.
Write factoring of expression.
The final factoring as (root of last term +first term root) ^{2}
Note the sign between the two numbers is the sign of the middle term.

Example (4) **Determine which of the following algebraic expressions represents a perfect square and factor it**

i) $x^2 + 10x + 25$

$(x)^2$ $(5)^2$

$2(x)(5) = 10x$ perfect square

$x^2 + 10x + 25 = (x+5)^2$

ii) $y^2 + 14x + 36$

$(y)^2$ $(6)^2$

$2(y)(6) = 12y \neq 14y$ Not perfect square

iii) $4 - 37v + 9v^2$

$(2)^2$ $(3v)^2$

$-2(2)(3v) = -12v \neq -37v$

Not perfect square

iv) $9h^2 - 6h + 3$

$(3h)^2$ $(\sqrt{3})^2$

$-2(3h)(\sqrt{3}) = -6\sqrt{3}h \neq -6h$

Not perfect square

Example (5) **Write the missing term in the algebraic expression $a x^2 + bx + c$ to make it a perfect squre , then factor it**

To become a perfect square, , we apply the law of the middle term.

$bx \pm \sqrt{(ax^2)(c)}\,2$

i) $25x^2 - \ldots\ldots + 49$

$bx = 2\sqrt{(ax^2)(c)}$ $bx = 2\sqrt{(25x^2)(49)}$ $bx = 70x$

$25x^2 - 70x + 49 = (5x - 7)^2$

ii) $\ldots\ldots + 8x + 16$

$bx = 2\sqrt{(ax^2)(c)}$ $8x = 2\sqrt{(ax^2)(16)}$ $64x^2 = 4 \times 16 \times ax^2$ $ax^2 = x^2$

$x^2 + 8x + 16 = (x + 4)^2$

iii) $y^2 + 14y + \ldots\ldots$

$by = 2\sqrt{(ay^2)(c)}$ $14y = 2\sqrt{(y^2)(c)}$ $196y^2 = 4 \times y^2 \times c$ $c = 49$

$y^2 + 14y + 49 = (y + 7)^2$

Make sure of your understanding

Factor each of the following algebraic expressions as a difference between two squares:

1 $x^2 - 16$

2 $36 - 4x^2$

3 $h^2 - v^2$

4 $9m^2 - 4n^2$

5 $27x^3z - 3xz^3$

6 $\frac{1}{4}y^2 - \frac{1}{16}$

Questions 1-6 are similar to example 2

Factor each of the following algebraic expressions as a perfect square:

7 $y^2 - 8y + 16$

8 $9z^2 - 6z + 1$

9 $v^2 + 2\sqrt{3}\ v + 3$

10 $4h^2 - 20h + 25$

Questions 7-10 are similar to example 3

Determine which one of the following algebraic expressions represents a perfect square, then factor it.

11 $x^2 + 18x + 81$

12 $16 - 14v + v^2$

13 $64h^2 - 48h - 9$

14 $3 - 4\sqrt{3}\ t + 4t^2$

Questions 11 -14 are similar to example 4

Write the missing term in the algebraic expression ax^2+bx+c to become a perfect square, then factor it.

15 $\ldots + 14y + 49$

16 $z^2 + 4z + \ldots\ldots$

17 $3 - \ldots\ldots + 9x^2$

18 $4x^2 + 2\sqrt{5}\ x + \ldots$

Questions 15-18 are similar to example 5

Solve the Exercises

Factor each of the following algebraic expressions in simplest form :

19 $25 - 4x^2$

20 $y^2 - 121$

21 $12 - 3t^2$

22 $8y^3x - 2x^3y$

23 $\frac{1}{3}z^5 - \frac{1}{12}z$

24 $4x^2 + 20x + 25$

25 $16n^2 + 8\sqrt{3}\ n + 3$

26 $4t^3 - 12t^2 + 9t$

Determine which of the following algebraic expressions represents a perfect square, then factor it:

27 $4x^2 + 18x + 16$

28 $y^2 + 10y + 25$

29 $2h^2 - 12h - 18$

30 $4v^2 + 4v + 4$

Write the missing term in the algebraic expression ax^2+bx+c to become a perfect square, then factor it:

31 $y^2 + \ldots + 36$

32 $25 - 20x + \ldots$

33 $5 - \ldots\ldots + 16x^2$

34 $81 + 18z + \ldots$

Solve the problems

35 **Al-Malwiya minaret :** It locates in Samara city, Iraq . It is one of the Iraqi distinctive landmark because of its unique shape. It is also one of the Iraqi famous ancient landmarks which belongs to the reign of Abbassiyyn . It based on a square base which its area is $x^2 - 8x + 16$ square meter What is the length of the base side which the minaret based on according to x?

36 **Farm for breeding cows :** Saad has a squre-shaped farm for breeding cows. The length of its side , is X meter. He extended his farm to became in a rectangular shped farm According to that, the area of the farm became $x^2 - 81$ square meter, What are the length and width of the farm after extension according to x?

37 **Painting :** Bashar drew a painting represents the marshes in the south of Iraq. The expression which represents the area of painting was $4x^2 - 8x + 9$ cm^2. Does the expression of the painting area represent a perfect square or not ?

Think

38 **Challenge:** Determine which the following algebraic expressions represent a perfect square and factor it:

$$\frac{1}{9}x^2 - \frac{1}{6}x + \frac{1}{16}$$

39 **Correct the mistake:** Muntaha said that the expression $(2x+1)(2x-1)$ is a factoring to the perfect squre $4x^2 - 4x + 1$. Determine the mistake of Muntaha and correct it.

40 **Numerical sense:** Dose the expression $9x^2 + 12x - 4$ represent a perfect square or not ? Clarify your answer.

Write

A factoring for the algebraic expression $4x^2 - 8x + 4$.

Lesson [2-4] Factoring the Algebraic Expression of three terms by Probe and Error (Experiment).

Idea of the lesson:
*Factoring the algebraic expression which consists of three terms by using
*Probe and Error.

Vocabulary:
*The two middles.
*The two parties.
*The middle term.

Learn

Assyrian winged bull (shido lamas). It is the way in which this name is written in the Assyrian writings. The origin of the word lamas is derived from the Summerian word Lammu.
There is a statue of it in AL-Moosal,s monument. What are the dimensions of the painting of winged bull which its area is $x^2 + 10x + 21$ centimeter square ?

[2-4-1] Factoring the algebraic expression x^2+bx+c

You have previously learned how to find the result of multiplying an algebraic expression by another algebraic expression which both of them consists of two terms:
i) $(x+2)(x+3) = x^2+5x+6$, ii) $(x+3)(x-5) = x^2-2x-15$, iii) $(x-1)(x-4) = x^2-5x+4$
Now you will learn the inverse operation of multiplication which is factoring the algebraic expression which consists of three terms x^2+bx+c by using the probe and error (experiment) for factoring the algebraic expression, We find two real.
m, n, where $b = m + n, c = nm$ and write $x^2 + bx + c = (x + n)(x + m)$.

Example (1) What are the dimensions of the painting of winged bull which its area is $x^2 + 10x + 21$ centimeter squar?

For factoring the algebraic expression, we follow the following steps:
Factoring the algebraic expression:

Factors of number 21	Sum of the two factors
(1) (21)	1 + 21 = 22
(3) (7)	3 + 7 = 10
(-3) (-7)	(-3) + (-7) = -10

Result of multiplying the two parties $+7x$
Result of multiplying the two middles $+3x$
The middle term $+10x$

Two parties
$x^2 + 10x + 21 = (x + 3)(x + 7)$
Two middles

The width of the painting is $(x + 3)$cm
The length of the painting is $(x + 7)$cm

Example (2) Factoring the following algebraic expression $y^2 + y - 12$

Factors of number 12	Sum of two Factors
(1) (-12)	1 - 12 = -11
(12) (-1)	12 - 1 = 11
(2) (-6)	2 - 6 = -4
(6) (-2)	6 - 2 = -4
(3) (-4)	3 - 4 = -1
(4) (-3)	4 - 3 = 1

The result of multiplying the two parties $+4y$
The result of multiplying $-3x$
The middle term $+y$

$y^2 + y - 12 = (y - 3)(y + 4)$

Example (3) **Factoring the following algebraic expressions :**

i) $z^2 - z - 6 = (z - 3)(z + 2)$	The middle term	$2z - 3z = - z$
ii) $x^2 - 9x + 18 = (x - 3)(x - 6)$	The middle term	$- 6x - 3x = - 9x$
iii) $y^2 + 6y - 27 = (y + 9)(y - 3)$	The middle term	$- 3y + 9y = + 6y$
iv) $15 - 8z + z^2 = (5 - z)(3 - z)$	The middle term	$- 5z - 3z = - 8z$

[2-4-2] Factoring the algebraic expression ax^2+bx+c where $a \neq 0$

Now, you will learn how to factor an algebraic expression which consists of three terms as in the form of $a x^2 + bx + c$ and that $a \neq 0$.

Example (4) **Factoring each of the following algebraic expressions :**

i) $\mathbf{6x^2 + 17x + 7}$

$6 = \begin{cases} (1)(6) \\ (2)(3) \end{cases}$, $7 = (1)(7)$

We find the factors of 6, 7, as follow

(1) (6) (1) (7) $\Rightarrow$ $(1)(1) + (6)(7) = 43$

$(1)(6) + (7)(1) = 13$

(2) (3) (1) (7) $\Rightarrow$ $(2)(1) + (3)(7) = 23$

$(2)(7) + (3)(1) = 17$

$6x^2 + 17x + 7 = (2x + 1)(3x + 7)$

The result of multiplying the two parties	$+14x$
The result of multiplying the two middles	$+3x$
The middle term	$+17x$

ii) $\mathbf{7y^2 - 26y - 8}$

We find the factors of 7, 8, as follow

$8 = \begin{cases} (1)(8) \\ (2)(4) \end{cases}$, $7 = (1)(7)$

$(1)(1) - (8)(7) = - 55$

$(1)(7) - (8)(1) = - 1$

$(2)(1) - (4)(7) = - 26$

$(2)(7) - (4)(1) = - 10$

$7y^2 - 26y - 8 = (7y + 2)(y - 4)$

The result of multiplying the two parties	$-28y$
The result of multiplying the two middles	$+2y$
The middle term	$- 26y$

Example (5) **Factor each of the following algebraic expressions in a simplest form:**

i) $3z^2 - 17z + 10 = (3z - 2)(z - 5)$	The middle term	$- 15z - 2z = - 17z$
ii) $4v^2 - v - 3 = (4v + 3)(v - 1)$	The middle term	$- 4v + 3v = - v$
iii) $15 + 11h + 2h^2 = (5 + 2h)(3 + h)$	The middle term	$+ 5h + 6h = 11h$
iv) $6x^2 - 51x + 63 = 3(2x^2 - 17x + 21) = 3(x - 7)(2x - 3)$	The middle term	$- 3x - 14x = -17x$

Make sure of your understanding

Factor each of the following algebraic expressions in simplest form:

Questions 1-6 are similar to examples 1,3

1. $x^2 + 6x + 8$
2. $1 - 2z + z^2$
3. $x^2 - 13x + 12$
4. $3 + 2z - z^2$
5. $x^2 - 2x - 3$
6. $15 - 8z + z^2$

Factor each of the following algebraic expressions in simplest form:

Questions 7-12 are similar to examples 4-5

7. $2x^2 + 5x + 3$
8. $3y^2 - 14y + 8$
9. $3x^2 - 10x + 8$
10. $8 - 25z + 3z^2$
11. $5y^2 - y - 6$
12. $6 + 29z - 5z^2$

Put the signs between the terms in brackets to make the factoring of the algebraic expression correct:

Questions 15-18 are similar to example 4

13. $x^2 + 9x + 20 = (x\ldots4)(x\ldots5)$
14. $y^2 - 12y + 20 = (y\ldots2)(y\ldots10)$
15. $6x^2 - 7x + 2 = (2x\ldots1)(3x\ldots2)$
16. $20 - 7y - 3y^2 = (5 \ldots 3y)(4 \ldots y)$

Solve the Exercises

Factor each of the following algebraic expressions in simplest form:

17. $x^2 + 9x + 14$
18. $y^2 - 5y + 6$
19. $x^2 - 2x - 3$

Factor each of the following algebraic expressions in simplest form:

20. $2x^2 + 12x - 14$
21. $4y^2 - 6y + 2$
22. $10 + 9z - 9z^2$
23. $2x^2 + 3x + 1$
24. $13y^2 - 11y - 2$

Put the signs between the terms in brackets to make the factoring of the algebraic expression correct:

25. $x^2 + x - 20 = (x\ldots4)(x\ldots5)$
26. $35 + 3y - 2y^2 = (5\ldots y)(7\ldots 2y)$

Solve the problems

27 Al-Ikhidher Castle: It is an ancient castle locates in Karbala governorate in the middle of Iraq. The ruins of the castle are still existed nowadays. It represents a unique defensive fortress which surrounded by a great rectangular -shaped wall. What are the dimensions of external wall by x , if the castle area with wall represents by
$6x^2 - 39x + 60$ square meter .

28 Entertaining games: Discovery swing considers one of the dangerous games in the fun city. The expression $5t^2 + 5t - 30$ represents the path of Discovery in the fun city , where (t) represents the time of movemment , and the factoring of expression helps to know the time which the swinging takes in the first time. Factor the expression.

29 The subway: The subway considers a system of under ground railway. It is one of the fast means for transportation in the big cities and in those cities which have a high density of population. Each train consists of several vehicles. If the expression $14y^2 - 23y + 3$ represents the ground area of the vehicle in square meter. What are the dimensions of the vehicle by y ?

Think

30 Challenge: Factor of the following algebraic expressions in simplest form:
$4x^3 + 4x^2 - 9x - 9$

31 Correct the mistake: Sa,ad factored the expression $6z^2 - 16z - 6$, as follow:

$6z^2 - 16z - 6 = (3z - 1)(2z + 6)$
Discover Sa,ad,s mistake, then correct it.

32 Numerical sense: Can we determine if the signs of the two bracket in factoring the expression $x^2 - 12x + 35$ are different or similar and without factoring the expression ?
Clarify you answer.

Write

The signs between the terms inside brackets to make the factoring of the algebraic expression correct.
$6z^2 + 5z - 56 = (3z \dots 8)(2z \dots 7)$

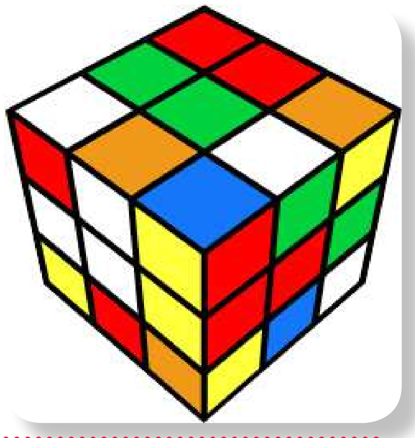
Lesson [2-5] Factoring the Algebraic Expression, sum of two cubes or difference between two cubes.

Idea of the lesson:
* Factoring the algebraic expression of three terms which represents sum (difference between) two cubes.

Vocabulary:
* Sum of two cubes
*Difference between two cubes

Learn

Rubik's cube, which was invented by the Hungarian sculptor and architecture
Arnw Rubik in 1974, is a three – dimensions mechanical puzzle. What is the sum volume of two cubes of Rubik's , the length of the first cube side is 3 dcm, and the length of the side of the second cube is 4 dcm?



[2-5-1] Factoring the algebraic expression, sum of two cubes.

You have previously learned in the first lesson of this chapter the multiplication of two –terms algebraic expression by three-terms algebraic expression . The result of their multiplication represents an expression as a sum of two cubes, like $((x + 2)(x^2 - 2x + 4) = x^3 + 8 = x^3 + 2^3)$ Now, you will learn the inverse operation which is factoring the two- terms algebraic expression which represents the sum of two cubes: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ where $x = \sqrt[3]{x^3}$, $y = \sqrt[3]{y^3}$

Example (1) What is the sum of two volumes of two cubes of Rubik . The side length of the first cube is 3dcm, while the side length of the second one is 4dcm.

$v_1 + v_2 = 3^3 + 4^3$ — The cube volume = length x width x hight = (length of side)^3

$= (3 + 4)(3^2 - 3 \times 4 + 4^2)$ — Law of factoring sum of two cubes.

$= 7(9 - 12 + 16) = 7 \times 13 = 91\text{dcm}^3$

Example (2)

Factor each of the following expressions in simplest form:

i) $x^3 + 5^3 = (x + 5)(x^2 - 5x + 5^2)$

ii) $y^3 + 8 = y^3 + 2^3 = (y + 2)(y^2 - 2y + 4)$

iii) $8z^3 + 27 = 2^3z^3 + 3^3 = (2z)^3 + 3^3 = (2z + 3)(4z^2 - 6z + 9)$

iv) $\frac{1}{a^3} + \frac{1}{64} = \frac{1}{a^3} + \frac{1}{4^3} = (\frac{1}{a} + \frac{1}{4})(\frac{1}{a^2} - \frac{1}{4a} + \frac{1}{16})$

v) $\frac{27}{x^3} + \frac{8}{125} = \frac{3^3}{x^3} + \frac{2^3}{5^3} = (\frac{3}{x})^3 + (\frac{2}{5})^3 = (\frac{3}{x} + \frac{2}{5})(\frac{9}{x^2} - \frac{6}{5x} + \frac{4}{25})$

vi) $\frac{1}{2}t^3 + 4 = \frac{1}{2}(t^3 + 8) = \frac{1}{2}(t^3 + 2^3) = \frac{1}{2}(t + 2)(t^2 - 2t + 4)$

vii) $0.008 + v^3 = (0.2)^3 + v^3 = (0.2 + v)(0.04 - 0.2v + v^2)$

[2-5-2] Factoring the Algebraic Expression difference between two cubes

You have learned in the first lesson of this chapter the multiplying of an algebraic expression which consists of two terms by another one which consists of three terms

$(x - 3)(x^2 + 3x + 9) = x^3 - 27 = x^3 - 3^3$, and the result of their multiplying represents an expression as a difference between two cubes,like

Now , you will learn the inverse operation which is factoring the algebraic expression which consists of two terms and in the form of difference between two cubes $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ where $x = \sqrt[3]{x^3}$, $y = \sqrt[3]{y^3}$.

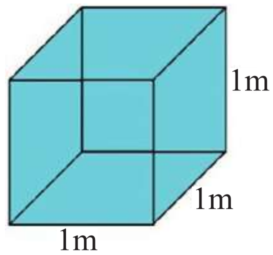
Example (3) Cube –shaped basin filled with water, the length of its side is 1m .
The water was transferred to another cube – shaped basin which is bigger than the first one, the length of its side is 1.1 m.
What is the additional quantity of water that we need to fill the big basin ?

the additional quantity of water = the volume of big cube - the volume of small cube.

$v_2 - v_1 = (1.1)^3 - 1^3$

$= (1.1 - 1)((1.1)^2 + 1.1 \times 1 + 1^2)$ Law of factoring the difference between two cubes

$= 0.1(1.21 + 1.1 + 1) = 0.1 \times 3.31 = 0.331 \text{ m}^3$



Example (4)

Factor each of the following algebraic expressions in simplest form:

i) $x^3 - 3^3 = (x - 3)(x^2 + 3x + 3^2) = (x - 3)(x^2 + 3x + 9)$

ii) $y^3 - 64 = y^3 - 4^3 = (y - 4)(y^2 + 4y + 16)$

iii) $27z^3 - 8 = 3^3z^3 - 2^3 = (3z)^3 - 2^3 = (3z - 2)(9z^2 + 6z + 4)$

iv) $\frac{1}{b^3} - \frac{1}{125} = \frac{1}{b^3} - \frac{1}{5^3} = (\frac{1}{b} - \frac{1}{5})(\frac{1}{b^2} + \frac{1}{5b} + \frac{1}{25})$

v) $\frac{1}{3}t^3 - 9 = \frac{1}{3}(t^3 - 27) = \frac{1}{3}(t^3 - 3^3) = \frac{1}{3}(t - 3)(t^2 + 3t + 9)$

vi) $0.216 - n^3 = (0.6)^3 - n^3 = (0.6 - n)(0.36 + 0.6n + n^2)$

vii) $1 - 0.125z^3 = 1 - (0.5)^3z^3 = (1 - 0.5z)(1 + 0.5z + 0.25z^2)$

viii) $32 - \frac{1}{2}m^3 = \frac{1}{2}(64 - m^3) = \frac{1}{2}(4^3 - m^3) = \frac{1}{2}(4 - m)(16 + 4m + m^2)$

Make sure of your understanding

Factor each of the following algebraic expressions in simplest form :

Questions 1- 8 are similar to examples 1-2

1. $y^3 + 216$
2. $x^3 + z^3$
3. $125 + 8z^3$
4. $\frac{1}{27}x^3 + \frac{1}{8}$
5. $\frac{1}{a^3} + \frac{1}{64}$
6. $\frac{1}{3}t^2 + 9$
7. $0.125 + v^3$
8. $1 + 0.008z^3$

Factor each of the following algebraic expressions in simplest form :

Questions 9-16 are similar to examples 3-4

9. $a^3 - 8^3$
10. $8y^3 - 64$
11. $\frac{1}{c^3} - \frac{1}{8}$
12. $\frac{1}{2}v^3 - 4$
13. $0.125 - m^3$
14. $25 - \frac{1}{5}n^3$
15. $3b^3 - 81$
16. $0.216v^3 - 0.008t^3$

Solve the Exercises

Factor each of the following algebraic expressions in simplest form :

17. $6^3 + x^3$
18. $125y^3 + 1$
19. $\frac{1}{64} + \frac{8}{125}y^3$
20. $\frac{1}{5}v^3 + 25$
21. $0.125x^3 + 0.008y^3$

Factor each of the following algebraic expressions in simplest form :

22. $y^3 - 64$
23. $\frac{1}{x^3} - \frac{27}{8}$
24. $9 - \frac{1}{3}n^3$
25. $25c^3 - \frac{1}{5}$
26. $0.01x^3 - 0.0084y^3$

Solve the problems

27 Library: Shtotgart's library, in Germany, is one of the most beautiful libraries in the world. It is also one of the largest libraries in line with the requirements of the modern education in Germany the shape of the building was cubic. The side length is $\frac{1}{2}y^3 - 13\frac{1}{2}$ meter. Factor the expression which represents the side length.

28 Aquarium: The volume of aquarium of ornamental fish is $25x^3$ cubic meter.
A cube-shaped stone was put inside the aquarium. The size of stone was $\frac{1}{5}$ cubic meter . The aquarium was filled with water . write the expression which represents the volume of water then factor it .

29 Residence : The designs of new house – buildings start to take different shapes which are more complicated in architecture. These houses were designed in shape of cubes If the volume of the first house is $\frac{8}{a^3}$ cubic meter and the second house is $\frac{27}{b^3}$ cubic meter. Write is the volume of the two houses?

Think

30 Challenge: Factor of the following algebraic expressions in simplest form:
$0.002z^3 - 0.016y^3$

31 Correct the mistake: Bushra factored the expression $8v^3 - 0.001$,as follow
$8v^3 - 0.001 = (2v + 0.1)(4v^2 - 0.4v + 0.01)$
Discover Bushra's mistake, then correct it..

32 Numerical sense: Is it possible to add 27 , 8 by using the method of factoring the sum of two cubes ? clarify your answer.

Write

The signs between the terms inside brackets to make the factoring of algebraic expression correct:
$125 - x^3 = (5\ldots x)(25 \ldots 5x \ldots x^2)$

Lesson [2-6] Simplifying Rational Algebraic Expressions

Idea of the lesson:
*Multiplying and dividing the rational algebraic expressions then write them in simplest form.
*Adding and subtracting the rational algebraic expressions then write them in simplest form.

Vocabulary:
* Ratio
*Fraction

Learn

Hassan had bought a group of flowers bouquets in $x^2 - x - 6$ dinars.
The cost of one bouquet was $2x - 6$ dinars. Write the ratio of one bouquet cost to the total cost of all bouquets, then write it in simplest form.

[2-6-1] Simplifying the multiplying and dividing of rational algebraic expressions

You have previously learned the properties of the rational and real numbers, you have also learned how to simplify the numerical sentences by using the least common multipler (L.C.M) and ordering operations. Now, you will learn how to simplify the rational algebraic expressions (Fractional) by dividing each of numerator and dominator by a common factor, and repeat it so that no way stay for that, and then, we can say that the expression is in a simplest form.

Example (1) Write the cost ratio of one flower bouquet to the total cost of the bouquets in a simplest form.

$$\frac{\text{cost of one bouquet}}{\text{cost of total bouquet}} = \frac{2x - 6}{x^2 - x - 6} = \frac{2(x - 3)}{(x - 3)(x + 2)}$$ Factor the numerator and dominator

$$= \frac{2\cancel{(x - 3)}}{\cancel{(x - 3)}(x + 2)} = \frac{2}{x + 2}$$ By dividing the numerator and dominator on the common factor

Example (2) **Write each of the following expressions in simplest form:**

i) $$\frac{x^2 - 4}{(x^2 - 4x + 4)} = \frac{(x + 2)(x - 2)}{(x - 2)^2} = \frac{(x + 2)\cancel{(x - 2)}}{(x - 2)\cancel{(x - 2)}} = \frac{x + 2}{x - 2}$$

ii) $$\frac{5z + 10}{z - 3} \times \frac{z^3 - 27}{(z^2 + 6z + 8)} = \frac{5\cancel{(z + 2)}}{\cancel{z - 3}} \times \frac{\cancel{(z - 3)}(z^2 + 3z + 9)}{\cancel{(z + 2)}(z + 4)} = \frac{5(z^2 + 3z + 9)}{z + 4}$$

iii) $$\frac{16 - x^2}{3x + 5} \times \frac{(3x^2 + 2x - 5)}{(x^2 + 3x - 4)} = \frac{\cancel{(4 + x)}(4 - x)}{\cancel{(3x - 5)}} \times \frac{\cancel{(3x + 5)}\cancel{(x - 1)}}{\cancel{(x + 4)}\cancel{(x - 1)}} = 4 - x$$

iv) $$\frac{8 + t^3}{4 - 2t + t^2} \div \frac{(2 + t)^3}{t^2 + 9t + 14} = \frac{8 + t^3}{4 - 2t + t^2} \times \frac{t^2 + 9t + 14}{(2 + t)^3}$$ Multiply the first by the second which is inverted

$$= \frac{\cancel{(2 + t)}\cancel{(4 - 2t + t^2)}}{\cancel{4 - 2t + t^2}} \times \frac{\cancel{(t + 2)}(t + 7)}{(2 + t)^{\cancel{3}}} = \frac{t + 7}{2 + t} = \frac{t + 7}{t + 2}$$ Factoring the numerator and dominator and dividing by the common factor

[2-6-2] Simplyfing adding and subtracting of the rational algebraic expressions.

You have previously learned how to factor the algebraic expressions and how to find the least common multipler (L.C.M): represents the result of multiplying the common factors by the biggest power and unjoined factors), when simplifying fractional numerical sentences.

Now, you will learn how to simplify the adding and subtracting of the (fractional) rational algebraic expressions by factoring each of the numerator and dominator of the faction to simplest form, then adding and subtracting the fractional expressions by using the common multiplier and simplify the expression to simplest form.

Example (3) Write the rational algebraic expression in simplest form.

$$\frac{y^2}{(y+2)} - \frac{4}{(y+2)}$$

The least common multipler (x + 2)

$$= \frac{y^2 - 4}{(y+2)} = \frac{(y+2)(y-2)}{(y-2)} = y - 2$$

Factoring the numerator as a form of a difference between two squares by dividing each of numerator and dominator by y + 2

Example (4) **Write each of the following expressions in simplest form:**

i) $$\frac{7x - 14}{x^2 - 4} + \frac{5}{(x+2)} = \frac{7(x-2)}{(x+2)(x-2)} + \frac{5}{x+2}$$

By factoring the numerator and dominator

$$= \frac{7}{x+2} + \frac{5}{x+2}$$

The least common multipler (x + 2)

$$= \frac{7+5}{x+2} = \frac{12}{x+2}$$

ii) $$\frac{4z}{2z-5} - \frac{z}{z+3} = \frac{4z}{2z-5} \times \left(\frac{z+3}{z+3}\right) - \frac{z}{z+3} \times \left(\frac{2z-5}{2z-5}\right)$$

The least common multipler

$$= \frac{4z(z+3) - z(2z-5)}{(2z-5)(z+3)} = \frac{2z^2 + 17z}{(2z-5)(z+3)} = \frac{z(2z+17)}{(2z-5)(z+3)}$$

iii) $$\frac{t^2 + 2t + 4}{t^3 - 8} + \frac{12}{3t - 6} = \frac{t^2 + 2t + 4}{(t-2)(t^2 + 2t + 4)} + \frac{12}{3(t-2)} = \frac{1}{(t-2)} + \frac{4}{(t-2)} = \frac{5}{(t-2)}$$

iv) $$\frac{8}{v+4} + \frac{2}{v-4} - \frac{2}{v^2 - 16} = \frac{8}{v+4} + \frac{2}{v-4} - \frac{1}{(v+4)(v-4)} = \frac{8(v-4) + 2(v+4) - 1}{(v+4)(v-4)}$$

$$= \frac{8v - 32 + 2v + 8 - 1}{(v+4)(v-4)} = \frac{10v - 25}{(v+4)(v-4)} = \frac{5(2v-5)}{(v+4)(v-4)}$$

Make sure of your understanding

Write each of the following expressions in simplest form:

Questions 1-6 are similar to examples 1-2

1. $\frac{2z^2 - 4z + 2}{z^2 - 7z + 6}$

2. $\frac{y^3 + 27}{y^3 - 3y^2 + 9y}$

3. $\frac{5x + 3}{x + 3} \times \frac{x^2 + 5x + 6}{25x^2 - 9}$

4. $\frac{z^2 + 7z - 8}{z-1} \times \frac{z^2 - 4}{z^2 + 6x - 16}$

5. $\frac{x^2 - 9}{x^3 + 4x + 4} \times \frac{x^2 - 4}{x^2 - x - 6}$

6. $\frac{2y^2 - 2y}{y^2 - 9} \div \frac{y^2 + y - 2}{y^2 + 2y - 3}$

Write each of the following expressions in simplest form:

Questions 7-12 are similar to examples 3-4

7. $\frac{2}{x^2 - 9} + \frac{3}{x^2 - 4x + 3}$

8. $\frac{2y^3 - 128}{y^3 + 4y^2 + 16y} - \frac{y - 1}{y}$

9. $\frac{z^2 + z + 1}{z^4 - z} - \frac{z + 3}{z^2 + 2z - 3}$

10. $\frac{x^2 - 1}{x^2 - 2x + 1} - 1$

11. $\frac{3}{z - 1} + \frac{2}{z + 3} + \frac{8}{z^2 + 2z - 3}$

12. $\frac{y - 3}{y - 1} + \frac{5y - 15}{(y - 3)^2} - \frac{3y + 1}{y^2 - 4y + 3}$

Solve the Exercises

Write each of the following expressions in simplest form:

13. $\frac{x + 5}{12x} \times \frac{6x - 30}{x^2 - 25}$

14. $\frac{3 - x}{4 - 2x} \times \frac{x^2 + x - 6}{9 - x^2}$

15. $\frac{y^2 - 7y}{y^3 - 27} \div \frac{y^2 - 49}{y^2 + 3y + 9}$

Write each of the following expressions in simplest form:

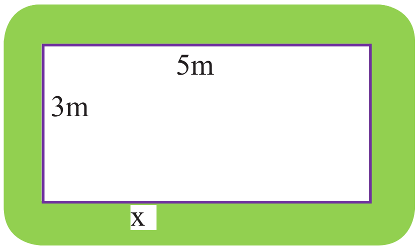
16. $\frac{5}{x^2 - 36} - \frac{2}{x^2 - 12x + 36}$

17. $\frac{3}{x - 2} - \frac{2}{x - 2} - \frac{4 + 2x + x^2}{x^3 - 8}$

Solve the problems

18 **Library:** If the algebraic expression $x^2 - 4$ represents the number of scientific books in the library, and the algebraic expression $x^2 + x - 6$ represents the number of literary books in the library. Write the ratio of the scientific books to the literary books in a simplest form.

19 **Geometry:** Dimensions of a rectangle are 3,5 meters. It was extended to bigger one by surrounding it by an aisle with width of x meter.Write the algebraic expression which represents the sum of the two ratios of the rectangle length before and after the extension, and the ratio of the width of the rectangle before and after the extension in a simplest form.



20 **Fireworks:** The algebraic expression $20 + 15t - 5t^2$ represents the height, in meters, of a fireworks shell which was shot from a 20 meter-high building roof, where t represents the time of reaching the shell to the target, in seconds. And the algebraic expression $4 + 19t - 5t^2$ represents the height of another shell which was shot from a roof of a 4 meter-high building.
Write the ratio of the first shell height to the height of the second shell, in simplest form.

Think

21 **Challenge:** Factoring of the following algebraic expression:

$$\frac{y^2 - 5}{2y^3 - 16} \div \frac{y - \sqrt{5}}{(2y^2 + 4y + 8)}$$

22 **Correct the mistake:** Samah simplified the algebraic expression and wrote it in a simplest form, as follow

$$\frac{z^2 - z - 30}{5 + z} \times \frac{2z + 12}{z^2 - 36} = 1$$

Discover samah's mistake and correct it.

23 **Numerical sense:** What is the result of adding the two algebraic expressions without using paper and pen ? clarify your answer.

$$\frac{5}{x^2 - 49} \times \frac{-4}{(x - 7)(x + 7)}$$

Write

The value of the algebraic expression in a simplest form:

$$\frac{z^2 + z - 6}{2z^2 + 2z - 12} \div \frac{z^2 - 16}{2z + 8}$$

Chapter Test

Find the result of multiplying an algebraic expression by another algebraic expression, each one of them consists of two terms:

1 $(x + 5)^2$

2 $(v - \sqrt{2})(v + \sqrt{2})$

3 $(2 - x)(5 - x)$

4 $(2y - 3)(y + 9)$

Find the result of multiplying a two terms algebraic expression by another algebraic expression, which consists of three terms:

5 $(x + 11)(x^2 - 11x + 121)$

6 $(\frac{1}{3} - y)(\frac{1}{9} + \frac{1}{3}y + y^2)$

7 $(y - 1)^3$

8 $(z + \frac{1}{4})^3$

Factor the expression by using the grater common factor (GCF) , then check the correction of solution:

9 $8x^2 - 12x$

10 $7y^3 + 14y^2 - 21y$

11 $\sqrt{18}\ z^3r + \sqrt{2}\ (zr^2 - zr)$

Factor the expression by using the binomial as a greatr common factor:

12 $\frac{2}{3}(y + 5) + \frac{1}{3}y(y + 5)$

13 $\sqrt{5}\ z(z^2 - 1) - \sqrt{2}\ z^2(z^2 - 1)$

Factor the expression by using the property of grouping:

14 $6x^4 - 18x^3 + 10x - 30$

15 $56 - 8y + 14y^2 - 2y^3$

Factor the expression by grouping with inverse:

16 $9x^3 - 6x^2 + 8 - 12x$

17 $\sqrt{11}\ z^3 - \sqrt{44}\ z^2 + 5(2 - z)$

Factor each of the following algebraic expressions:

18 $16 - x^2$

19 $\frac{1}{3}z^2 - \frac{1}{27}$

20 $\frac{1}{16}v - \frac{1}{2}v^4$

21 $8x^3 - \frac{1}{125}$

22 $81 - 18y + y^2$

23 $7z^3 - 36z + 5$

Determine which of the following algebraic expressions represent a perfect square, then factor it:

24 $25x^2 + 30x + 9$

25 $49 - 14y + y^2$

26 $4v^2 + 4\sqrt{5}\ v + 5$

Write the missing term in the algebraic expression ax²+bx+c to become a perfect square,then factor it:

27 $x^2 + \dots + 81$

28 $36 - 12y + \dots$

29 $7 - \dots + 4z^2$

Factor each of the following algebraic expressions:

30 $x^2 + 7x + 10$

31 $x^2 - 5\sqrt{3}\ x + 18$

32 $2v^2 + 9v + 7$

33 $32 - 16x + 2x^2$

34 $\frac{1}{4}y^2 - 2y + 3$

35 $12 - 7\sqrt{2}\ v + 2v^2$

36 $8 + 27x^3$

37 $125y^3 - 1$

38 $\frac{1}{v^3} - \frac{8}{27}$

39 $1 + 0.125y^3$

40 $z^3 - 0.027$

41 $3 - \frac{1}{9}v^3$

Write each of the following algebraic expressions in a simplest form:

42 $\frac{27 - 8z^3}{4z^2 - 9} \div \frac{9 + 6z + 4z^2}{9 + 6z}$

43 $\frac{7}{x^2 - 25} - \frac{6}{x^2 + 10x + 25}$

44 $\frac{y^2 - 1}{1 - y^3} + \frac{1 + y}{1 + 2y + y^2}$

45 $\frac{z + 3}{z + 5} - \frac{z - 5}{z - 3} + \frac{1}{z^2 + 2z - 15}$

Chapter 3

Equations

Basil and Sa’ad had travelled in tourist tours by Baghdad International airport. Basil’s group was less than sa’ad group in 22 persons. If the number of all travelers is 122 persons. We can calculate the number of each group by solving the two linear equations of first degree $x + y = 122$, $x - y = 22$, where the variable x represents the number of persons in Sa’ad's group and the variable y represents the number of persons in Basil's group.

Pretest

Find the result of multiplying an algebraic expression by another algebraic expression, each one consists of two terms:

1. $(y - 5)^2$
2. $(z + 2)(z - 2)$
3. $(x - \sqrt{5})(x + \sqrt{5})$
4. $(4 - y)(6 - y)$
5. $(3z - 2)(z + 8)$

Find the result of multiplying an algebraic expression of two terms by another algebraic expression of three terms:

6. $(x + 3)(x^2 - 3x + 9)$
7. $(\frac{1}{2} - y)(\frac{1}{4} + \frac{1}{2}y + y^2)$

Factor the expression by using the greater common factor (GCF), then check the correction of solution:

8. $5x^2 - 10x$
9. $9y^3 + 6y^2 - 3y$
10. $\sqrt{12}\,z^2 + \sqrt{3}\,z$

Factor the expression by using the binomial as a greater common factor:

11. $x(5 - x) - 3(5 - x)$
12. $\frac{1}{2}(y + 1) + \frac{1}{2}y(y + 1)$
13. $\sqrt{3}\,z(z - 1) - \sqrt{2}(z - 1)$

Factor the expression by grouping:

14. $6x^3 - 12x^2 + 5x - 10$
15. $9 - 18y + 7y^2 - 14y^3$
16. $\sqrt{2}\,z^4 - \sqrt{6}\,z^3 + z - \sqrt{3}$

Factor the expression by grouping with the inverse:

17. $4x^3 - 2x^2 + 9 - 6x$
18. $\frac{3}{4}y^3 - \frac{1}{4}y^2 + 4 - 12y$
19. $\sqrt{4}\,z^3 - \sqrt{25}\,z^2 + 3(5 - 2z)$

Factor each of the following algebraic expressions:

20. $y^2 - 25$
21. $\frac{1}{2}z^2 - \frac{1}{8}$
22. $36 - 12x + x^2$
23. $y^2 - 2y - 15$

Determine which of the following algebraic expressions represents a perfect square, then factor it:

24. $16x^2 + 40x + 25$
25. $64 - 16y + y^2$
26. $z^2 - 6z - 9$

Write the missing term in the algebraic expression $ax^2 + bx + c$ to become a perfect square, then factor it.

27. $x^2 + ... + 64$
28. $9 - 24y + ...$
29. $5 - .... + 4z^2$

Factor each of the following algebraic expressions:

30. $18 - 3y - y^2$
31. $z^2 - 2\sqrt{3}\,z + 3$
32. $4 - 21x + 5x^2$
33. $1 + 27z^3$
34. $y^3 - 125$
35. $y^3 - \frac{1}{8}$
36. $\frac{1}{x^3} - \frac{1}{64}$
37. $1 - 0.125z^3$

Lesson [3-1] Solving the System of two Linear Equations with two variables.

Idea of the lesson:
*Solve the system of two linear equations graphically and by substitution and elimination.

Vocabulary:
*Linear equation.
*Linear Equations system.
*Solving the system.

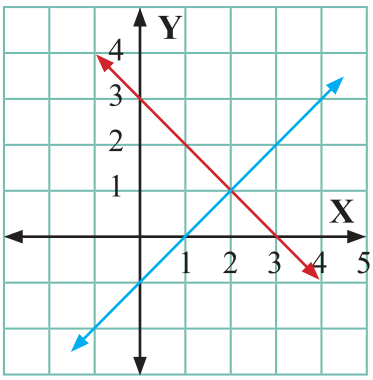
Learn

The sum of Diaa's age and Osama's age is 40 years. Five years age, Diaa's age was 4 times Osama's age . How old are each of them now?

[3-1-1] Solving a System of two Linear Equations graphically.

Assume $\overleftrightarrow{L_1} = a_1x + b_1y = c_1$, $\overleftrightarrow{L_2} = a_2x + b_2y = c_2$ ، represent two equations of first degree (linear) with two variables x,y. To solve this system graphically we follow:
1) Representing each of the two lines in the coordinate plane.
2) Finding a point of intersection the two lines by drawing two colums from the point on the two axis X - axis and Y - axis then the intersection point will represent the solution set.

Example (1) Find a solution set for the system graphically in R .



$x - y = 1$ .... (1)

$x + y = 3$ .... (2)

eq(1)

x	y= x-1
0	-1
1	0

eq(2)

x	y= 3-x
0	3
3	0

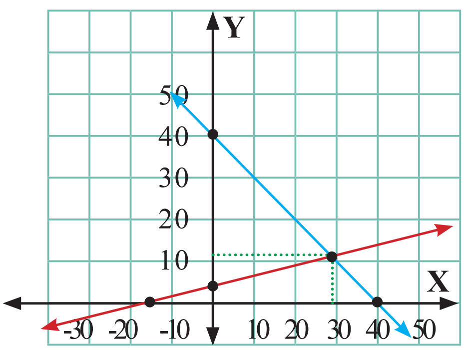
We represent the two equations graphically and determine the intersection point of the two lines (2, 1).
The solution set of the system is S = { (2,1) }
To check the correction of solution, we substitute the value of the two variables x,y in both equations to get two correct statements.

$x - y = 1 \rightarrow 2 - 1 = 1 \rightarrow 1 = 1$ *Substituting by the equations…(1).*

$x + y = 3 \rightarrow 2 + 1 = 3 \rightarrow 3 = 3$ *Substituting by the equations....(2).*

Example (2) From learn parograph and to find age of all of them.

We assume Diaa's age = x
Osama's age = y
The equation that represent the sum of ages is $x + y = 40$ ...(1)
The equation that represent the relation between two ages befor 5 years $\Rightarrow$ $(x - 5) = 4(y-5)$
$4y = x + 15$ ...(2)



We represent every equation as aline like as the solution of (ex.1) and then determine intersection point of two lines which represent the age of all of them which is (29, 11).

x	y= 40-x
0	40
40	0

x	4y= x+15
-15	0
0	$\frac{15}{4}$

[3-1-2] Solving the system of two Linear Equations by Substitution method.

We can summarize this method to solve a system of two linear with tow variables x , y by transforming one of the two equations to an equation with only one variable by finding a relation between x,y from one of the two equations, then substituting it in the other equation.

Example (3) **Find the solution set of the system by using the substitution:**

i) $y = 4x$ .... (1)
$y = x + 6$ ...... (2) $\Rightarrow 4x = x + 6$

$\Rightarrow 4x - x = 6 \Rightarrow x = 2$

$y = x + 6 \Rightarrow y = 2 + 6 \Rightarrow y = 8$

we Substitute the value y from the equation...(1) in the equation ....(2) solving the equation, then find the variable value x.
Substituting the value of (x) by the equation...(2) to find the variable value (y).

So the solution set for the system is {(2 , 8)}

ii) $x + 8y = 10$ .... (1)
$x - 4y = 2$ ...... (2) $\Rightarrow x = 2 + 4y$

$2 + 4y + 8y = 10 \Rightarrow 12y = 8 \Rightarrow y = \frac{2}{3}$

$x + 8y = 10 \Rightarrow x + 8 \times \frac{2}{3} = 10 \Rightarrow x = 10 - \frac{16}{3} \Rightarrow x = \frac{14}{3}$

We find the value of x from the equation...(2), then Substituting in the equation...(1).
Substituting the value of y by the equation...(1).

So the solution set of the system is $\{(\frac{14}{3}, \frac{2}{3})\}$

[3-1-3] Solving the system of two linear Equations by Elimiation method.

This method can be summarized to solve a system of two equations with two varibles x , y by eliminating one of the two variables through making the coefficient of one of them equaled in value and different in sign in both equations.

Example (4) **Find the solution set for the system by using the elimination method:**

i) $x + 2y = 5$ ....(1)
$3x - y = 1$ ...... (2)

$\Rightarrow \begin{cases} x + 2y = 5 & ....(1) \\ 6x - 2y = 2 & ....(2) \end{cases}$ by adding

$7x = 7 \Rightarrow x = 1$

$x + 2y = 5 \Rightarrow 1 + 2y = 5 \Rightarrow 2y = 4 \Rightarrow y = 2$

Multiplying the equation...(2) by the number 2 then adding them to the equation....(1).
Substituting the value x in one of two equations (simplest equation)
Substituting in the equation ...(1)

So the solution set of the system is {(1 , 2)}

ii) $3x + 4y = 10$ .... (1)
$2x + 3y = 7$ ...... (2)

$\Rightarrow \begin{cases} 6x + 8y = 20 & ......(2) \\ 6x \pm 9y = \pm 21 & ....(1) \end{cases}$ by subracting

$y = 1$

$2x + (3 \times 1) = 7 \Rightarrow 2x = 4 \Rightarrow x = 2$

Multiplying the equation...(1) by the number 2 and the equation...(2) by the number3, then we subtract the two equations.

We substitute the value x in the one of the two equations (before changing the sign) substitute in the equation...(2)

So the solution set of the system is {(2 , 1)}

Make sure of your understanding

Find a solution set of the system in R graphically:

1. $\left.\begin{array}{l} 3x - y = 6 \\ x - y = 3 \end{array}\right\}$

2. $\left.\begin{array}{l} y - x = 3 \\ y + x = 0 \end{array}\right\}$

3. $\left.\begin{array}{l} y = x - 2 \\ y = 3 - x \end{array}\right\}$

Questions 1-3 are similar to examples 1-2

Find the solution set of the system by using the method of substitution, for each of the following :

4. $\left.\begin{array}{l} 2x + 3y = 1 \\ 3x - 2y = 0 \end{array}\right\}$

5. $\left.\begin{array}{l} x - 2y = 11 \\ 2x - 3y = 18 \end{array}\right\}$

6. $\left.\begin{array}{l} y - 5x = 10 \\ y - 3x = 8 \end{array}\right\}$

Questions 4-6 are similar to example 3

Find the solution set of the system by using the elimination method, for each of the following :

7. $\left.\begin{array}{l} 3x - 4y = 12 \\ 5x + 2y = -6 \end{array}\right\}$

8. $\left.\begin{array}{l} x - 3y = 6 \\ 2x - 4y = 24 \end{array}\right\}$

9. $\left.\begin{array}{l} 3y - 2x - 7 = 0 \\ y + 3x + 5 = 0 \end{array}\right\}$

Questions 7-9 are similar to example 4

Find the solution set of the system, then check the correction of the solution:

10. $\left.\begin{array}{l} \frac{2x}{3} - \frac{y}{2} = 1 \\ y - \frac{x}{3} = 4 \end{array}\right\}$

11. $\left.\begin{array}{l} 0.2x - 6y = 4 \\ 0.1x - 7y = -2 \end{array}\right\}$

12. $\left.\begin{array}{l} \frac{1}{2}x + \frac{2}{3}y = 2\frac{3}{4} \\ \frac{1}{4}x - \frac{2}{3}y = 6\frac{1}{4} \end{array}\right\}$

Solve the Exercises

Find the solution set of the system graphically:

13. $\left.\begin{array}{l} x - y = -4 \\ y + x = 6 \end{array}\right\}$

14. $\left.\begin{array}{l} y = x - 4 \\ x = 2 - y \end{array}\right\}$

Find the solution set of the system by using the method of substitution, for each of the following:

15. $\left.\begin{array}{l} 3x + 2y = 2 \\ x - y = 8 \end{array}\right\}$

16. $\left.\begin{array}{l} 2x - y = -4 \\ 3x - y = 3 \end{array}\right\}$

Find the solution set of the system by using the method of elimination, for each of the following:

17. $\left.\begin{array}{l} 3x = 22 - 4y \\ 4y = 3x - 14 \end{array}\right\}$

18. $\left.\begin{array}{l} 5x - 3y = 6 \\ 2x + 5y = -10 \end{array}\right\}$

Solve the problems

19 Weather :During January, the number of days (x) in which the temperature goes down in Baghdad decreases about 10 Celsius , is nearly less in 9 days than the number of days (y) in which the temperature goes up about 10 Celsius. Write two equations represent this situation, then find their solution by using the method of elimination to find the number of days in each situation.

20 Trade :A commercial shop had sold 25 fridges and washing machines. The price of one fridge was million dinars while price of one washer was 500000 dinars. If the total cost of fridges and washers was 20millions dinars, then how many appliances did the seller sell from each type?
Write two equations represent the problem, then solve them by using the substitution method.

21 Graduation party : Sajad and Anwer held a party on the occasion of their graduation from college. The number of friends who were invited by Sajad is more in three from the friends who were invited by Anwer. The total number of friends who came to the party is 23 persons. How many persons did each of Sajad and Anwer invite? write two equations represent the problem , then solve them to find the required.

Think

22 Challenge: Find the solution set for the system:

$$\left.\begin{array}{l} \frac{2}{6}x - \frac{1}{3}y = 1 \\ \frac{1}{2}x + \frac{1}{2}y = 3 \end{array}\right\}$$

23 Correct the mistake: Ahmed said that the solution set of the system:

Is the set $\{(\frac{5}{16}, \frac{5}{9})\}$

$$\left.\begin{array}{l} 2x + 3y = 6 \\ 3x + 2y = 1 \end{array}\right\}$$

Discover Ahmed's mistake, then correct it.

Write

Asolution set for the system : $5x - 6y = 0$, $x + 2y = 4$

Lesson [3-2] Solving Quadratic Equations with one variable

Idea of the lesson:
*Solving the equation which. consists of two terms by factoring the difference between two squares.

Vocabulary:
*Equation
*Second degree
*One variable
*Difference between two squares

Learn

Zaqura is one of the Iraqi civilized landmarks. It sites in south of Iraq. Basil drew a square-shaped picture its area is 9 m^2.
Find the side length of the picture.

[3-2-1] Using difference between two squres to solve equations

The general equation of second degree with one variable $ax^2 + bx + c = 0$ where, $a \neq 0$ and $a,b,c \in R$ and solving it means finding a values set of the variable (x) which satisfies the equation , by making it correct statement.We will study in this item solving the equations consist of two terms by using the greater common factor, difference between two squares and the property of zero-product.

Example (1) Write an equation represents the area of picture, then solve it to find the side length of the picture.

Assume that the side length of picture is the variable (x) and the equation which represents the area of picture is: $x^2 = 9$

$x^2 - 9 = 0 \Rightarrow (x + 3)(x - 3) = 0$ *Factoring by using the difference between two squares*

$\Rightarrow x + 3 = 0$ or $x - 3 = 0$ *property of zero-product*

$\Rightarrow x = -3$, neglect or $x = 3$ *The wall picture length is 3 m*

Example (2) solve the following equation by using the difference between two squares, then check the correct of solution.

$16 - y^2 = 0 \Rightarrow (4 + y)(4 - y) = 0$ *Factoring by using the difference between two squares*

$4 + y = 0 \Rightarrow$ or $4 - y = 0 \Rightarrow y = -4$ or $y = 4 \Rightarrow S = \{-4 , 4\}$ *solution set.*

Check: each value in the solution set for the variable (y) must satisfies the equation

$L.S = 16 - y^2 = 16 - (-4)^2 = 16 - 16 = 0 = R.S$ *By substitution y = -4*

$L.S = 16 - y^2 = 16 - 4^2 = 16 - 16 = 0 = R.S$ *By substitution y = 4*

Example (3) **Solve the following equations by using the difference between two squares:**

i) $4x^2 - 25 = 0 \Rightarrow (2x + 5)(2x - 5) = 0 \Rightarrow 2x + 5 = 0$ or $2x - 5 = 0$

$\Rightarrow x = \frac{-5}{2}$ or $x = \frac{5}{2} \Rightarrow s = \{\frac{5}{2} , \frac{-5}{2}\}$

ii) $3z^2 - 12 = 0 \Rightarrow 3(z^2 - 4) = 0 \Rightarrow (z + 2)(z - 2) = 0$ *By dividing the two parties on 3, then by factoring.*

$\Rightarrow z + 2 = 0$ or $z - 2 = 0 \Rightarrow s = \{-2 , 2\}$

iii) $2y^2 - 6 = 0 \Rightarrow y^2 - 3 = 0 \Rightarrow (y + \sqrt{3})(y - \sqrt{3}) \Rightarrow y = -\sqrt{3}$ or $y = \sqrt{3} \Rightarrow s = \{-\sqrt{3} , \sqrt{3}\}$

iv) $x^2 - 5 = 0 \Rightarrow (x + \sqrt{5})(x - \sqrt{5}) = 0 \Rightarrow x = -\sqrt{5}$ or $x = \sqrt{5} \Rightarrow s = \{-\sqrt{5} , \sqrt{5}\}$

v) $(z + 1)^2 - 36 = 0 \Rightarrow (z + 1 + 6)(z + 1 - 6) = 0 \Rightarrow (z + 7)(z - 5) = 0 \Rightarrow s = \{-7 , 5\}$

[3-2-2] Using Square root Property to Solve the equations

You have learned in the previous item how to solve the equation of second degree with one variable by the factoring method using the difference between two squares. Now, we will find the solution set for the second –degree equation with one variable by using the method of square root property .

$\sqrt{x^2} = |x| \geq 0$

$25 = 5^2 \Rightarrow \sqrt{25} = \sqrt{5^2} = |5| = 5$

$25 = (-5)^2 \Rightarrow -\sqrt{(-5)^2} = |-5| = 5$

In general form : If a is positive real number then $x^2 = a \Rightarrow x = \pm\sqrt{a}$

Example (4) Solve the following equation by using the property of square root, then check the correction of solution:

$x^2 = 9 \Rightarrow x = \pm\sqrt{9} \Rightarrow x = \pm 3$ *By using the property of the square root*

$\Rightarrow S = \{3, -3\}$ *Solution set of the equation*

Check: Each value in the solution set of the variable (x) must satisfy the equation

$L.S = x^2 = 3^2 = 9 = R.S$ *By subtitution $x = 3$*

$L.S = x^2 = (-3)^2 = -3 \times -3 = 9 = R.S$ *By subtitution $x = -3$*

Example (5) **Solve the following equation by using the square root rule:**

i) $y^2 = 36 \Rightarrow y = \pm\sqrt{36} \Rightarrow x = \pm 6 \Rightarrow S = \{6, -6\}$

ii) $z^2 = \frac{9}{25} \Rightarrow z = \pm\sqrt{\frac{9}{25}} \Rightarrow z = \pm\frac{3}{5} \Rightarrow S = \{\frac{3}{5}, -\frac{3}{5}\}$

iii) $x^2 + 81 = 0 \Rightarrow x^2 = -81$ There is no solution for this equation in the real numbers (there is no a real number has a negative square)

iv) $3y^2 = 7 \Rightarrow y^2 = \frac{7}{3} \Rightarrow y = \pm\sqrt{\frac{7}{3}} \Rightarrow y = \pm\frac{\sqrt{7}}{\sqrt{3}} \Rightarrow S = \{\frac{\sqrt{7}}{\sqrt{3}}, -\frac{\sqrt{7}}{\sqrt{3}}\}$

v) $4x^2 - 5 = 0 \Rightarrow 4x^2 = 5 \Rightarrow x^2 = \frac{5}{4} \Rightarrow x = \pm\sqrt{\frac{5}{4}} \Rightarrow x = \pm\frac{\sqrt{5}}{2} \Rightarrow S = \{\frac{\sqrt{5}}{2}, -\frac{\sqrt{5}}{2}\}$

Notice: If the two sides of a correct equation were quadrted, then the resulted equation stills correct $(y = x \Rightarrow y^2 = x^2)$ example $\sqrt{x} = 5 \Rightarrow (\sqrt{x})^2 = 5^2 \Rightarrow x = 25$

The inverted is not correct $x^2 = y^2 \not\Rightarrow y = x$

Example (6) **Solve the following equations :**

i) $3\sqrt{x} = 18 \Rightarrow \sqrt{x} = 6 \Rightarrow (\sqrt{x})^2 = 6^2 \Rightarrow x = 36 \Rightarrow S = \{36\}$ *By squared the both sides of the cquation*

ii) $\sqrt{y+8} = 3 \Rightarrow (\sqrt{y+8})^2 = 3^2 \Rightarrow y + 8 = 9 \Rightarrow y = 9 - 8 \Rightarrow y = 1 \Rightarrow S = \{1\}$

iii) $\sqrt{5z} = 7 \Rightarrow (\sqrt{5z})^2 = 7^2 \Rightarrow 5z = 49 \Rightarrow z = \frac{49}{5} \Rightarrow S = \{\frac{49}{5}\}$

iv) $\sqrt{\frac{x}{13}} - 1 = 0 \Rightarrow \sqrt{\frac{x}{13}} = 1 \Rightarrow (\sqrt{\frac{x}{13}})^2 = 1^2 \Rightarrow \frac{x}{13} = 1 \Rightarrow x = 13 \quad S = \{13\}$

Make sure of your understanding

Solve the following equations by using the difference between two squares, then check the correction of solution:

Questions 1-3 are similar to example 2

1. $x^2 - 16 = 0$
2. $81 - y^2 = 0$
3. $2z^2 - 8 = 0$

Solve the following equations by using the difference between two squares:

Questions 4-9 are similar to example 3

4. $4x^2 - 9 = 0$
5. $5y^2 - 20 = 0$
6. $(y + 2)^2 - 49 = 0$
7. $(3 - z)^2 - 1 = 0$
8. $x^2 - 3 = 0$
9. $y^2 - \frac{1}{9} = 0$

Solve the following equations by using the rule of the square root:

Questions 10-15 are similar to example 4

10. $x^2 = 64$
11. $z^2 = 7$
12. $2y^2 = \frac{49}{8}$
13. $6z^2 - 5 = 0$
14. $4(x^2 - 12) = 33$
15. $z^2 + \frac{2}{3} = \frac{5}{6}$

Questions 16-18 are similar to example 5

Solve the following equations:

16. $3\sqrt{x} = 15$
17. $\sqrt{y - 5} = 2$
18. $\sqrt{2z} = 6$

Solve the Exercises

Solve the following equations, then check the correction of solution:

19. $x^2 = 49$
20. $5y^2 - 10 = 0$

Solve the following equations in R by using the difference between two squares:

21. $9x^2 - 36 = 0$
22. $9(x^2 - 1) - 7 = 0$
23. $y^2 - \frac{1}{36} = 0$

Solve the following equations by using the rule of the square root:

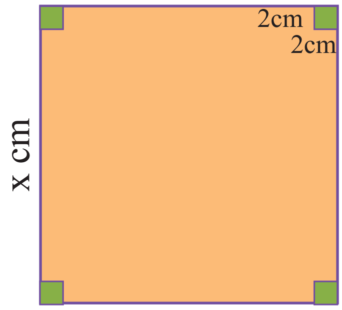
24. $x^2 = 121$
25. $50 - 2y^2 = 0$
26. $x^2 = \frac{1}{64}$
27. $7(x^2 - 2) = 50$

Solve the following equations:

28. $6\sqrt{x} = 30$
29. $\sqrt{4z} = 8$

Solve the problems

30 Carpets: A rectangular- shaped carpet, its length is 12 m and width 3m. It was divided into parts to cover the floor of a square- shaped room. write the equation which represent the problem and find the side length of the room?



31 Geometry: A piece of cardboord which was square-shaped, its side length is x cm. four equaled equaled pieces were cut fromits four corners.The side length of each square is 2cm. It was folded to form a box without cover. which was a rectangular parallele surfaces-shaped box, its volme 32 cm^3 write the equation which represent the problem and. Find the side length of the origin cardboord.

32 Fountain: A square- shaped swimming pool was designed in the center of a square-shaped garden. It's side length is 3m.The remained area of the garden which was surrounded the pool was $40m^2$. write the equation which represent the problem and the side length of the garden?

Think

33 Challenge: **Solve the following equations:**

i) $9(x^2 + 1) = 34$ ii) $4x^2 - 3 = 0$

34 **Does the given set represent the solution set for the equation or not?**

i) $(2y + 1)^2 = 16$, $\{\frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}}\}$ ii) $3x^2 - 7 = 0$, $\{\frac{7}{\sqrt{3}}, -\frac{7}{\sqrt{3}}\}$

35 Correct the mistake: Salah said that the set ($\{\frac{4}{\sqrt{5}}, -\frac{4}{\sqrt{5}}\}$) represents the solution set for the equation $5x^2 = 4$ Discover Salah's mistake and correct it.

36 Numerical sense: A positive integer consists of one digit, If one was subtracted from it's square, the result would be a number from the multiplying of ten.What is the number?

Write

The solution set for the equation:

$(8 - 3y)^2 - 1 = 0$

Lesson [3-3] Using Probe and Error to Solve the Quadratic Equations(Experiment)

Idea of the lesson:
*Solving the equations of the second degree which consist of three terms by factoring in experiment.

Vocabulary:
*Quadratic equation
*Experiment

Learn

If the length of the basketball court increases in about 2m more than the double of its width, and its area is 480m^2.
Find the two dimensions of the court.

[3-3-1] Solving the equation $x^2 + bx + c = 0$.

You have previously learned how to find the factoring of an algebraic expression which consists of three terms by experiment. Now, you will use the factoring to solve the equations of second degree which consist of three terms $x^2 + bx + c = 0$ where b , c are real numbers (factoring the expression to two brackets with two different signs or two similar signs according to the sign of the absolute term and the middle term).

Example (1) Finding the two dimensions of a basketball court.

Assume that the width of the court is the variable x, so the length of the court will be 2x+2.

Court area = length × width

$x(2x + 2) = 480 \Rightarrow x^2 + 2x - 480 = 0 \Rightarrow x^2 + x - 240 = 0$

$\Rightarrow (x + 16)(x - 15) = 0$

$\Rightarrow x + 16 = 0 \Rightarrow x = -16$

or $x - 15 = 0 \Rightarrow x = 15$

The middle term $-15x + 16x = x$.
We neglect it because there is no negative length.

So the width of the court is 15m and the length is $2 \times 15 + 2 = 32$m

Example (2) **Solve the following equations by factoring in experiment:**

i) $x^2 - 7x + 12 = 0 \Rightarrow (x - 3)(x - 4) = 0 \Rightarrow x = 3$ or $x = 4 \Rightarrow S = \{3 , 4\}$

ii) $y^2 + 8y + 15 = 0 \Rightarrow (y + 3)(y + 5) = 0 \Rightarrow y = -3$ or $y = -5 \Rightarrow S = \{-3 , -5\}$

iii) $z^2 + z - 30 = 0 \Rightarrow (z + 6)(z - 5) = 0 \Rightarrow z = -6$ or $z = 5 \Rightarrow S = \{-6 , 5\}$

iv) $x^2 - 2x - 63 = 0 \Rightarrow (x - 9)(x + 7) = 0 \Rightarrow x = 9$ or $x = -7 \Rightarrow S = \{9 , -7\}$

v) What is the number which its square increases in 12?

Assume that the number is x, then the square of the number will be x^2, and the numerical sentence which represents the problem is

$x^2 - x = 12 \Rightarrow x^2 - x - 12 = 0 \Rightarrow (x - 4)(x + 3) = 0 \Rightarrow x = 4$ or $x = -3$

So, the number is either 4 or -3

[3 -3-2] Solving the equation $ax^2 + bx + c = 0$, $a \neq 0$

You have previously learned how to solve an equation by experiment method and the variable x^2 without cofficients. Now, you will learn how to solve the same equation but with existence of cofficients for the variable x^2.

Example (3) **A swimming pool which its length is less in three times of its width in 1 m. If the area of the swimming pool is 140m2, find its dimensions.**

Assume that the width of the swimming pool is the variable x so the length of the pool is $3x - 1$

The equation which represents the problem is

$x(3x-1) = 140$:

$x(3x-1) = 140 \Rightarrow 3x^2 - x - 140 = 0$

$\Rightarrow (3x + 20)(x - 7) = 0$ *The middle term* $-21x + 20x = -x$

$\Rightarrow 3x + 20 = 0 \Rightarrow x = -\frac{20}{3}$ We neglect it because there is no negative length

or $x - 7 = 0 \Rightarrow x = 7$

$\therefore$ The length $3(7) - 1 = 20$ The width of swimming pool .

Example (4) **Solve the following equations by factoring in experiment:**

i) $4y^2 - 14y + 6 = 0 \Rightarrow (4y - 2)(y - 3) = 0$ $-12y - 2y = -14y$ *The middle term*

$\Rightarrow \begin{cases} 4y - 2 = 0 \Rightarrow y = \frac{1}{2} \\ \text{or } y - 3 = 0 \Rightarrow y = 3 \Rightarrow S = \{\frac{1}{2}, 3\} \end{cases}$

ii) $3x^2 - x - 21 = 0 \Rightarrow (3x - 3)(x + 7) = 0$ $21x - 3x = 18x$ *The middle term*

$\Rightarrow \begin{cases} 3x - 3 = 0 \Rightarrow x = 1 \\ \text{or } x + 7 = 0 \Rightarrow x = -7 \Rightarrow S = \{1, -7\} \end{cases}$

iii) $20 + 13z + 2z^2 = 0 \Rightarrow (4 + z)(5 + 2z) = 0$ $8z + 5z = 13z$ *The middle term*

$\Rightarrow \begin{cases} 4 + z = 0 \Rightarrow z = -4 \\ \text{or } 5 + 2z = 0 \Rightarrow z = -\frac{5}{2} \Rightarrow S = \{-4, -\frac{5}{2}\} \end{cases}$

iv) $9x^2 - 69x - 24 = 0 \Rightarrow 3(3x^2 - 23x - 21) = 0 \Rightarrow 3x^2 - 23x - 21 = 0$

$\Rightarrow (3x + 1)(x - 8) = 0$ $-24x - x = -23x$ *The middle term*

$\Rightarrow \begin{cases} 3x + 1 = 0 \Rightarrow x = -\frac{1}{3} \\ \text{or } x - 8 = 0 \Rightarrow x = 8 \Rightarrow S = \{-\frac{1}{3}, 8\} \end{cases}$

Make sure of your understanding

Solve the following equations by factoring in experiment:

Questions 1-6 are similar to example 2

1. $x^2 - 9x + 18 = 0$
2. $x^2 - 4x - 32 = 0$
3. $y^2 + 48y - 49 = 0$
4. $y^2 + 9y - 36 = 0$
5. $x^2 - 3x + 2 = 0$
6. $y^2 - 8y - 33 = 0$

Questions 7-9 are similar to example 1

7. What is the number which its square is more greater than its double in 35?
8. What is the number that if we add its four times to it's square, the result will be 45?
9. A carpet ,its length is more than its width in about 2m and its area is $48m^2$, what are the dimensions of the carpet?

Questions 10-14 are similar to example 4

10. $15x^2 - 11x - 14 = 0$
11. $6 + 7x - 5x^2 = 0$
12. $42 + 64y + 24y^2 = 0$
13. $36 - 75x + 6x^2 = 0$
14. $70 - 33y + 2y^2 = 0$

Questions 15 is similar to example 3

15. A rectangular- shaped land, its length is more than its width in 4m, what are the two dimensions of the land if its area is $60m^2$?

Solve the Exercises

Solve the following equations by factoring in experiment:

16. $x^2 - 15x + 56 = 0$
17. $y^2 + 16y + 63 = 0$
18. $x^2 + 15x - 16 = 0$
19. $y^2 - y - 42 = 0$
20. A rectangular- shaped metal, its width decreases in 2m than its length. What are the two dimensions of the piece of metal? If its area is $24m^2$?
21. A dining hall, its length less than the twice of its width in 3m and its area is$54m^2$, what are the dimensions of the hall?

Find the solution set for the following equations, then check the correction of solution:

22. $x^2 - 4x + 3 = 0$
23. $y^2 - 9y - 36 = 0$
24. $4 - 26x + 12x^2 = 0$

Solve the problems

25 Sport: If the length of a picture of football stadium increases in 4m more than the twice of its width, its area was $160m^2$.
What are the two dimensions of the picture?

26 Field of ostriches: If the length of a field for breeding ostriches decreases in 4m than the twice of its width. If its area was $96m^2$, will a 44m length fence be enough to surround the field?

27 Picture frame: Samir bought a picture frame, its length is twice of its width.
Samir needs to shorten the frame in 2cm from both length and width to make it suitable for picture. What are the dimensions of the frame which Samir bought if the picture area is 40 cm^2?

Think

28 Challenge: **Solve the following equations by factoring in experiment.**

i) $(x - 3)(x + 2) = 14$ ii) $3y^2 - 11y + 10 = 80$

29 Clarify: **Does the given set represents a solution set for the equation or not?**

i) $4x^2 + 2x = 30$, $\{\frac{-2}{5}, 3\}$ ii) $42 - 33y + 6y^2 = 0$, $\{2, \frac{7}{2}\}$

30 Correct the mistake: Rana said that the solution set for the equation $2x^2 - 34x + 60 = 0$ is $\{3, 15\}$.

Determine Rana's mistake, then correct it.

Write

An equation represents the following problem, then find its solution:

What is the integer number which its square is less than its twice in 35?

Lesson [3-4] Solving the Quadratic Equations by the Perfect square.

Learn

Idea of the lesson:
*Solving the quadratic equations by method of perfect square.

Vocabulary:
*First term
*Last term
*Perfect square
*Completing the square

Jaguar (Panthera onca) is a kind of tigers.
The equation $x^2 - 20x + 100 = 0$ represents a square region area in square meters which is allocated to the tiger inside a zoo.
What is the expression which represented the side length of the squared area ?

[3-4-1] Solving the equations by the Perfect square.

You have previaosly learned how to factor the algebraic expression which is as a perfect square. Now, we will use the factoring in solving equations by factoring the complete square to find the solution set of the equation.

Example (1) What is the expression which represented by the side length of the square area?

$x^2 - 20x + 100 = 0$ — To factor the left side of the equation, we should be sure that the expression represents a perfect square.

$2 (x \times 10) = 20x$ — A perfect square because: the middle term $=2\times$ (the first term root $\times$ the last term root).

$x^2 - 20x + 100 = 0 \Rightarrow (x - 10)^2 = 0 \Rightarrow (x - 10)(x - 10) = 0$ — Factoring of the expression

$\Rightarrow x - 10 = 0 \Rightarrow x = 10$

or $x - 10 = 0 \Rightarrow x = 10$

So the side length of the square region area which is allocated to tiger is 10m .

Example (2) **Solving the following equations by the complete square:**

i) $4x^2 + 20x + 25 = 0$ — The middle term $2 \times (2x \times 5) = 20x$

$\Rightarrow (2x + 5)^2 = 0 \Rightarrow 2x + 5 = 0 \Rightarrow 2x = -5 \Rightarrow x = -\frac{5}{2}$ — We take one of the repeated factors

ii) $y^2 - y + \frac{1}{4} = 0$ — The middle term $2 \times (y x \times \frac{1}{2}) = y$

$\Rightarrow (y - \frac{1}{2})^2 = 0 \Rightarrow y - \frac{1}{2} = 0 \Rightarrow y = \frac{1}{2}$ — We take one of the repeated factors

iii) $3 - 6\sqrt{3}z + 9z^2 = 0$ — The middle term $2 \times (\sqrt{3} \times 3z) = 6\sqrt{3}z$

$\Rightarrow (\sqrt{3} - 3z)^2 = 0 \Rightarrow \sqrt{3} - 3z = 0 \Rightarrow 3z = \sqrt{3}$ — We take one of the repeated factors

$z = \frac{1}{\sqrt{3}}$

[3-4-2] Solving Quadratic Equations by Completing the square

Now, you will learn how to solve an equation of second degree by completing the square:

1- We put the quadratic equation as follow: $ax^2 + bx = -c$ ، where $a \neq 0$

2- If $1 \neq a$, the equation will be divided by a.

3- We add the expression (quadrate of the half factor x) to the two sides of the equation.

4- We factor the left side which becomes a perfect square after step 3 , then we simplify the right part.

5- We take the square root for the two sides, then we find the values of x.

Example (3) **Solve the following equations by completing the square:**

i) $x^2 - 4x - 12 = 0 \Rightarrow x^2 - 4x = 12$ — we write the equation as in the first step

$\Rightarrow x^2 - 4x + 4 = 12 + 4$ — Adding the expression $(\frac{1}{2} \times -4) = 4$ to the two sides of the equation.

$\Rightarrow x^2 - 4x + 4 = 16 \Rightarrow (x - 2)^2 = 16$

$\Rightarrow x - 2 = \pm 4 \Rightarrow \begin{cases} x - 2 = 4 \Rightarrow x = 6 \\ \text{or } x - 2 = -4 \Rightarrow x = -2 \end{cases} \Rightarrow S = \{6, -2\}$ — We take the square root of the two sides of the equation

ii) $2y^2 - 3 = 3y \Rightarrow 2y^2 - 3y = 3$ — We write the equation as in the first step

$\Rightarrow y^2 - \frac{3}{2} y = \frac{3}{2}$ — By dividing the two sides of the equation by 2

$\Rightarrow y^2 - \frac{3}{2} y + \frac{9}{16} = 3 + \frac{9}{16}$ — Adding the expression$(\frac{1}{2} \times -\frac{3}{2})^2 = \frac{9}{16}$ to the two sides of the equation by factoring the left side and simplify the right side of the equation.

$\Rightarrow (y - \frac{3}{4})^2 = \frac{33}{16}$

$\Rightarrow y - \frac{3}{4} = \pm \frac{\sqrt{33}}{4}$ — We take the square root of the two sides of the equation.

$\Rightarrow \begin{cases} y - \frac{3}{4} = \frac{\sqrt{33}}{4} \Rightarrow y = \frac{3 + \sqrt{33}}{4} \\ \text{or} \\ y - \frac{3}{4} = -\frac{\sqrt{33}}{4} \Rightarrow y = \frac{3 - \sqrt{33}}{4} \end{cases} \Rightarrow S = \left\{ \frac{3 - \sqrt{33}}{4}, \frac{3 + \sqrt{33}}{4} \right\}$

Example (4) The length of a rectangle is greater than its width in 2cm estimate, the length and width of the rectangle by nearing to the nearest integer n if its area was $36cm^2$.

Assume that the rectangular width is the variable x, then the rectangle length is x+2.The equation which represent the problem:

$x(x + 2) = 36$

$\Rightarrow x^2 + 2x = 36$ — We solve the equation by completing the square

$\Rightarrow x^2 + 2x + 1 = 36 + 1$ — We add $(\frac{1}{2} \times 2)^2 = 1$ to the two sides of the equation . neglect $x \approx -1$

$\Rightarrow (x + 1)^2 = 37 \Rightarrow x + 1 = \pm\sqrt{37} \Rightarrow x + 1 \approx \pm 6 \Rightarrow x + 1 \approx 6 \Rightarrow x \approx 5$

Note : $\sqrt{37} \approx \sqrt{36} = 6$ — $\Rightarrow x + 1 \approx -6 \Rightarrow$ neglect $x \approx -7$

So the approximate width of the rectangle is 5 cm and its length is 7 cm.

Make sure of your understanding

Solve the following equations by the perfect square:

Questions 1- 6 are similar to example 2

1. $x^2 + 12x + 36 = 0$
2. $y^2 - 10y + 25 = 0$
3. $4x^2 - 4x + 1 = 0$
4. $y^2 + 2\sqrt{7}\, y + 7 = 0$
5. $x^2 + 16x = - 64$
6. $\frac{1}{16} - \frac{1}{2} x + x^2 = 0$

Solve the following equations by complete square :

Questions 7-12 are similar to example 3

7. $x^2 - 10x - 24 = 0$
8. $y^2 - 3 = 2y$
9. $4x^2 - 3x - 16 = 0$
10. $3y^2 + 2y = 1$
11. $x^2 - \frac{6}{5} x = \frac{1}{5}$
12. $5y^2 + 15 y - 30 = 0$

Solve the Exercises

Solve the following equations by the perfect square:

13. $x^2 + 24x + 144 = 0$
14. $y^2 + 4\sqrt{2}\, y + 8 = 0$
15. $3y^2 + 36 - 12\sqrt{3}\, y = 0$

Solve the following equations by complete square

16. $y^2 + 2\sqrt{3}\, y = 3$
17. $x^2 - 2x = 0$
18. $x^2 - \frac{2}{3} x = 4$

Solve the following equations by complete square, then find the result by rounding to the nearest integer:

19. $x^2 - 6x = 15$
20. $y (2y + 28) = 28$

Solve the problems

21 Babylon city: It is Babylon in Latin. It is an Iraqi city which sites nearby the river of Euphrates. It was the capital of Babylonians during the reign of Hamoraby (1750-1792) BC. Find the from the equation $x^2 - 28x + 196 = 0$ which represents the lenght of the side of one of the square – shaped hall find the value of x.

22 Panda bear: The area which was allocated to the Panda bear in a zoo is a rectangular- shaped area. which is 126 m^2 . its width is less than the length in 8 m.find the dimensions of the allocated area for panda by nearing to the nearest integer .

23 Whales: Some whales are swimming in groups towards the beach and no one know why because there is no scientific illustration to this phenomena. Those who interested in protecting the environment try to return them to the sea. Solve the equation $x^2 + 20x = 525$ by the method of completing the square to find the value of x which represents the number of whales which swam toward the beach in Australia.

Think

24 Challenge:**Solve the following equations by completing the square, then find the result by rounding to the nearest integer :**

i) $4x(x - 6) = 27$ ii) $6y^2 - 48y = 6$

25 Correct the mistake: Sawsen solved the equation $4x^2 - 4\sqrt{3}x + 3 = 0$ by the method of completing the square, then wrote the solution set for the equation as follows $S = \{\frac{\sqrt{3}}{4}, -\frac{\sqrt{3}}{4}\}$.

Discover Sawsen's mistake and correct it.

26 Numerical sense:Does the solution set of the equation $y^2 - 4y + 4 = 0$ contains two equaled values in the expression which one of them is positive and the other is negative? Clarify your answer.

Write

The solution set for the equation:

$\frac{1}{81} - \frac{2}{9}z + z^2 = 0$

Lesson [3-5] Using Quadratic formula to Solve the Quadratic Equations.

Idea of the lesson:

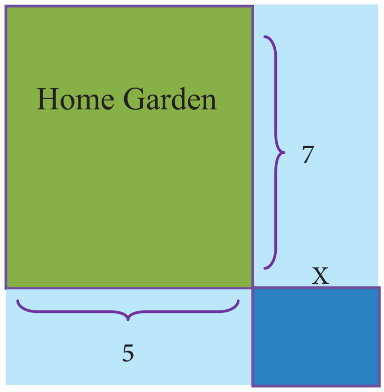
* Solving equtions from the second degree by using the quadratic formala.

Vocabulary:

* Coefficient
* Absolute term
* General law

Learn

An aisle was required to be paved on the two sides of a home garden. It was paved with ceramics. The length of the garden is 7m and its width is 5m. And the area of paving is 45 m^2 find the width of the aisle which was required to be paved with ceramics.



[3-5-1] Solve the Equation by Using the Law $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and $a \neq 0$.

You have previously learned how to solve an equation of second degree by many methods, but there are equations which cannot be solved by the previous methods, so we will solve them by the quodratic formula (constitution) by finding the real roots for the quadratic equation, as follow:

1) We put the quadratic equation in the general form (standard) $ax^2 + bx + c = 0$.

2) We write the values of coefficients : a coefficient x^2, b coefficient x with its sign, c represents the absolute term with its sign.

Supstitutiong by the quadratic formula to find the two values of the variable.

Example (1) From learn paragraph , what is the width of the aisle which needs to be paved on the two sides of the garden?

Assume that the width of the aisle is x, then the area of the right part of the aisle equals 7x, the area of the front part =5x , the area of the aisle angle = x^2 and the sum of the two area of the paving is $45m^2$.

$x^2 + 7x + 5x = 45 \Rightarrow x^2 + 12x = 45$ The equation which represents the problem

$x^2 + 12x - 45 = 0$ Put the equation in the general form

$a = 1$, $b = 12$, $c = -45$ Determine the coefficients and substituting, in the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-12 \pm \sqrt{144 - 4\times1\times(-45)}}{2\times1} \Rightarrow x = \frac{-12 \pm \sqrt{144 + 180}}{2}$$

$$\Rightarrow x = \frac{-12 \pm \sqrt{324}}{2} \Rightarrow x = \frac{-12 \pm 18}{2} \Rightarrow \begin{cases} x = \frac{-12 + 18}{2} \Rightarrow x = 3 \\ \text{or } x = \frac{-12 - 18}{2} \Rightarrow x = -15 \end{cases}$$

$x = 3$ The width of the aisle is3m

$x = -15$ Neglect because it is impossible

Example (2) Find the solution set for the following equations by using the quadratic formula:

$x^2 - 3x - 5 = 0$, $a = 1$, $b = -3$, $c = -5$

$$x=\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x=\frac{3 \pm \sqrt{9 + 20}}{2} \Rightarrow x=\frac{3 \pm \sqrt{29}}{2} \Rightarrow \begin{cases} x = \frac{3 + \sqrt{29}}{2} \\ \text{or } x= \frac{3 - \sqrt{29}}{2} \end{cases} \Rightarrow S=\{\frac{3 + \sqrt{29}}{2}, \frac{3 - \sqrt{29}}{2}\}$$

[3-5-2] The Discriminant ($\Delta = b^2 - 4ac$)

In the first part of this lesson, you have learned how to solve the equation by the quadratic formula to find the real roots of the equation. Now, we will talk about the discriminate of the quadratic equation $ax^2 + bx + c = 0$ which is $\Delta = b^2 - 4ac$

and the type of the two roots of the equation determines as follow:

Roots type :	$\Delta = b^2 - 4ac$.
1- Two rational real roots.	1)positive and perfect square (prameter rational number
2- Two irrational real roots.	2)Positive and not a complete square
3- Two equaled real roots ($\frac{-b}{2a}$).	3)Zero
4-Two irrational real roots. (the solution set in R = Ø)	4)Negative

Example (3) **Determine the equation roots, firstly, then find the solution set if it is possible:**

i) $2x^2 + 3x - 2 = 0$, $a = 2$, $b = 3$, $c = -2$

$\Delta = b^2 - 4ac \Rightarrow \Delta = 9 - 4 \times 2 \times (-2) = 25$

The discriminant expression is a perfect square that means the equation has two rational roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-3 \pm \sqrt{9 + 16}}{4} \Rightarrow x = \frac{-3 \pm \sqrt{25}}{4} \Rightarrow x = \frac{-3 + 5}{4} = \frac{1}{2} \text{ or } x = \frac{-3 - 5}{4} = -2$$

ii) $y^2 - 4y - 9 = 0$, $a = 1$, $b = -4$, $c = -9$

$\Delta = b^2 - 4ac \Rightarrow \Delta = 16 - 4 \times 1 \times (-9) = 52$

The discriminant expression is not a perfect square, so the equation has two irrational roots.

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{4 \pm \sqrt{16 + 36}}{2} \Rightarrow x = \frac{4 \pm \sqrt{52}}{2} \Rightarrow x = 2 + \sqrt{13} \text{ or } x = 2 - \sqrt{13}$$

iii) $z^2 + 8z = -16 \Rightarrow z^2 + 8z + 16 = 0$, $a = 1$, $b = 8$, $c = 16$

$\Delta = b^2 - 4ac \Rightarrow \Delta = 64 - 4 \times 1 \times 16 = 0$

The discriminative expression is zero , that means the equation has two equaled real roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-8 \pm \sqrt{64 - 64}}{2} = -4$$

iv) $x^2 - 2x + 10 = 0$, $a = 1$, $b = -2$, $c = 10$

$\Delta = b^2 - 4ac \Rightarrow \Delta = 4 - 4 \times 1 \times 10 = -36$

The discriminant expression is negative, therefore the equation doesn't have a solution in R.

Example (4) What is the value of the constant (K) which makes the two roots of the equation $x^2 - (k+1)x + 4 = 0$ equaled? Check your answer.

The two roots of the equation will be equaled when the value of the discriminant expression

Δ equals zero.

$a = 1$, $b = -(k+1)$, $c = 4$ Determine the values of the coefficients

$\Delta = b^2 - 4ac \Rightarrow \Delta = (k+1)^2 - 4 \times 1 \times 4 \Rightarrow \Delta = (k+1)^2 - 16$

$\Delta = 0 \Rightarrow (k+1)^2 - 16 = 0$

$\Rightarrow (k+1)^2 - 16 = 0 \Rightarrow (k+1)^2 = 16$

We substitute the value of the discriminant by zero because the two roots of the equation are equal to the root of the two sides of the equation.

$$\Rightarrow k + 1 = \pm 4 \Rightarrow \begin{cases} k + 1 = +4 \Rightarrow k = 3 \\ \text{or } k + 1 = -4 \Rightarrow k = -5 \end{cases}$$

Check:

We substitute by the value K= 3 in the original equation, then we find the roots of the equation:

$x^2 - (k+1)x + 4 = 0 \Rightarrow x^2 - 4x + 4 = 0 \Rightarrow (x - 2)^2 = 0 \Rightarrow x = 2$

We substitute by the value K=-5 in the original equation, then we find the equation roots..

$x^2 - (k+1)x + 4 = 0 \Rightarrow x^2 + 4x + 4 = 0 \Rightarrow (x + 2)^2 = 0 \Rightarrow x = -2$

Make sure of your understanding

Find the solution set for the following equations by using the quodratic formula:

Questions 1-6 are similar to examples 1-2

1. $x^2 - 4x - 5 = 0$
2. $y^2 + 5y - 1 = 0$
3. $3x^2 - 9x = - 2$
4. $4y^2 + 8y = 6$
5. $4x^2 - 12x + 9 = 0$
6. $2y^2 -3 = -5y$

Determine the roots of equation at firstly, then find the solution set if it is possible:

Questions 7-10 are similar to example 3

7. $2x^2 + 3x = 5$
9. $y^2 - 2y + 1 = 0$
8. $3x^2 - 7x + 6 = 0$
10. $y^2 + 12 = - 9y$

Questions 11-14 are similar to examples 3-4

11. What is the value of the constant (k) which makes the two roots of the equation $x^2 - (k + 2)x + 36 = 0$ equaled? Check your answer.
12. What is the value of the constant (K)) which makes the two roots of the equation $4y^2 + 25 = (k - 5)y$ equaled? Check your answer.
13. What is the value of the constant (K)) which makes the two roots of the equation $z^2 + 16 = (k + 4)z$ equaled? Check your answer.
14. Show that the equation $z^2 - 6z + 28 = 0$ doesn't have a solution set in real numer.

Solve the Exercises

Find the solution set for the following equations by using the quadratic formula:

15. $x^2 - 7x - 14= 0$
16. $y^2 + 3y - 9 = 0$
17. $2x^2 - 8 (3x + 2) = 0$
18. $2y^2 - 2 = -10y$

Determine the equation roots at firstly, then find the solution set in R if it is possible:

19. $y^2 - 2y - 10 = 0$
20. $y^2 - 14y + 49 = 0$
21. What is the value of the constant (K) which makes the two roots of the equation $x^2 - (k + 6)x + 49 = 0$ equaled? Check your answer.
22. Show that the equation $2z^2 - 3z + 10 = 0$ doesn't have a solution set in real numer.

Solve the problems

23 Fireworks: In one of the occasions, a group of fireworks was shot vertically, they reached acertain height of 140m. Calculate the time (t) second in which the fireworks reached up to that height, if the following equation $5t^2 + 60t = 140$.

24 Trade: Samir calculates the cost of one men's suit, then he adds amount of profit and sell it in 120,000 dinars. If (P) in the equation $p^2 - 30p + 225 = 0$ represents the amount of Samir's profit in one suit which is in thousands of dinars. What is the cost of one suit?

Think

25 Challenge: **Determine the roots of the equation at firstly, then find the solution set if it is possible:**

i) $x^2 + 8x = 10$ ii) $3y^2 - 6y - 42 = 0$

26 Correct the mistake:Sa'ad said that the equation $2x^2 - 3x - 9 = 0$ doesn't have a solution in the set of the real numbers.
Discover sa,ad,s mistake and correct it.

27 Numerical sense:Marwa used the discriminant expression for writing the two roots of the equation $z^2 - 8z + 16 = 0$ without factoring. illustrate how Marrwa was able to write the two roots of the equation.

Write

The type of the two roots of the equation $x^2 + 100 = 20x$ by using the discriminant expression without solving it.

Lesson [3-6] Solving the Fractional Equations

Idea of the lesson:
*Solving the fractional equations of second degree.

Vocabulary:
* Numerator
*Denominator
*Fractional equation

Learn

If the price of a masterwork is $2x + 3$ thousands dinars, and the price of buying 3 pieces of masterworks is $x^2 + 3x - 1$ thousands dinars.So if the ratio of one masterwork price to the price of three masterworks is $\frac{1}{3}$, what is the price of buying one masterwork?.

You have previously learned how to simplify the fractional algebraic expressions by dividing both the numerator and denominator by a common factor. Now you will use the factoring of algebraic expressions to solve fractional equations which have a variable in its denominator by get rid of fractions, then solve them by using one of the methods that you have previously learned.

Example (1) Write the price of buying one masterwork.

$$\frac{\text{one master work price}}{\text{three master work price}} = \frac{1}{3} \Rightarrow \frac{2x+3}{(x^2+3x-1)} = \frac{1}{3}$$

Simplify the fraction by multiplying the two sides by the two middles.

$$\Rightarrow x^2 + 3x - 1 = 6x + 9 \Rightarrow x^2 - 3x - 10 = 0$$

Simplify the equation for factoring.

$$\Rightarrow (x-5)(x+2) = 0 \Rightarrow \begin{cases} x - 5 = 0 \Rightarrow x = 5 \\ \text{or } x + 2 = 0 \Rightarrow x = -2 \end{cases}$$

Neglect because there is no price in negative

Then the price of buying one masterwork is$(2x + 3 = 13)$ 13000 dinars.

Example (2) Find the solution set for the following equation, then check the correction of the solution.

$$5x + \frac{x-2}{3x} = \frac{2}{3}$$

We multiply the two sides of the equation by the least common multiple (LCM) to get rid of fractions

$$3x(5x) + 3x\left(\frac{x-2}{3x}\right) = 3x\left(\frac{2}{3}\right) \Rightarrow 15x^2 + x - 2 = 2x \Rightarrow 15x^2 - x - 2 = 0$$

Simplify the equation

$$\Rightarrow (3x+1)(5x-2) = 0$$

Factoring by experiment

$$\Rightarrow \left\{ \begin{matrix} 3x + 1 = 0 \Rightarrow x = \frac{-1}{3} \\ \text{or } 5x - 2 = 0 \Rightarrow x = \frac{2}{5} \end{matrix} \right\} \Rightarrow S = \left\{\frac{-1}{3}, \frac{2}{5}\right\}$$

Solution set

Check:Substituting by the original equation when $x = \frac{-1}{3}$

$$LHS = 5\left(\frac{-1}{3}\right) + \frac{\frac{-1}{3} - 2}{3 \times \frac{-1}{3}} = \frac{-5}{3} + \frac{-1}{-3} + 2 = \frac{-5}{3} + \frac{1}{3} + 2 = \frac{-5+1+6}{3} = \frac{2}{3} = RHS$$

it is also easy to check when $x = \frac{2}{5}$ (leave it to students)

You have previously learned how to simplify the adding of the (fractional) relative algebraic expressions and subtract them by factoring each of the numerator and denominator of the fraction to simplest form, then doing the operation of adding and subtracting the fractional expressions by using the least common multiple and simplify the expressions to the simplest form. Now, you will use that to solve the fractional equations to find the solutions set of the fractional equation.

Example (3) Find the solution set for the equation:

$$\frac{x}{x-3} + \frac{4x}{x+3} = \frac{18}{x^2-9}$$

Factoring the denominators to the possible

$$\Rightarrow \frac{x}{x-3} + \frac{4x}{x+3} = \frac{18}{(x-3)(x+3)}$$

simplest form by multiplying the two sides of the equation by LCM (x-3) (x+3)

$$\Rightarrow x(x+3) + 4x(x-3) = 18$$

Simplifying the equation and solve it to find the values of the variable.

$$\Rightarrow x^2 + 3x + 4x^2 - 12x - 18 = 0 \Rightarrow 5x^2 - 9x - 18 = 0$$

$$\Rightarrow (5x+6)(x-3) = 0 \Rightarrow x = -\frac{6}{5} \text{ or } x = 3$$

Note:
We have to exclude the values which make the denominator of any fractional term from the original equation terms, zero because it leads to divide by zero and that considers impossible.

So we exclude x=3 from the solution because ($\frac{x}{x-3} = \frac{3}{0}$), and the solution will be only $x = -\frac{6}{5}$

Check:Substituting by the original equation $x = -\frac{6}{5}$ to see if the two sides of the equation are equaled or not ?

$$LHS = \frac{x}{x-3} + \frac{4x}{x+3} = \frac{-\frac{6}{5}}{-\frac{6}{5}-3} + \frac{4 \times \frac{-6}{5}}{-\frac{6}{5}+3} = \frac{6}{21} - \frac{24}{9} = -\frac{50}{21}$$

$$RHS = \frac{18}{x^2-9} = \frac{18}{(-\frac{6}{5})^2 - 9} = \frac{18}{(\frac{36}{25}) - 9} = -\frac{450}{189} = -\frac{50}{21}$$

LHS = RHSF

So the value $x = -\frac{6}{5}$ satisfy the equation

Example (4) Find the solution set for the equation:

$$\frac{2}{x+2} - \frac{x}{2-x} = \frac{x^2+4}{x^2-4}$$

$$\Rightarrow \frac{2}{x+2} + \frac{x}{x-2} = \frac{x^2+4}{(x+2)(x-2)}$$

Before multiplying the two sides of the equation by LCM for the denominators we try to factor the denominator of the fraction for the right side and change 2-x = - (x-2) by using the information a-b= - (b-a)

$$\Rightarrow 2(x-2) + x(x+2) = x^2 + 4$$

By multiplying the two sides of the equation by LCM(x + 2) (x - 2)

$$\Rightarrow 2x - 4 + x^2 + 2x - x^2 - 4 = 0 \Rightarrow 4x - 8 = 0 \Rightarrow x = 2$$

When we substitute x=2 in the original equation , we get a process of dividing by zero and it is impossible ($\frac{x}{2-x} = \frac{2}{0}$), so the equation doesn't have a solution in the real number set (R), that means the solution set in R is an empty set (Ø).

Make sure of your understanding

Find the solution set for each of the following equations and check the correct of solution:

Questions 1-6 are similar to examples 1-2

1. $\frac{1}{x} + \frac{1}{2} = \frac{6}{4x^2}$

2. $\frac{y}{2} - \frac{7}{5} = \frac{3}{10y}$

3. $\frac{x+4}{2} = \frac{-3}{2x}$

4. $\frac{y+1}{y^2} = \frac{3}{4}$

5. $\frac{9x-14}{x-5} = \frac{x^2}{x-5}$

6. $\frac{1}{y^2-6} = \frac{2}{y+3}$

Find the solution set for each of the following equations:

Questions 7-10 are similar to examples 3-4

7. $\frac{y-4}{y+2} - \frac{2}{y-2} = \frac{17}{y^2-4}$

8. $\frac{9}{x^2-x-6} - \frac{5}{x-3} = 1$

9. $\frac{12}{y^2-16} + \frac{6}{y+4} = 2$

10. $\frac{2x}{x+1} + \frac{3x}{x-1} = \frac{8+7x+3x^2}{x^2-1}$

Solve the Exercises

Find the solution set for each of the following equations and check the correct of solution:

11. $\frac{4}{6x^2} + \frac{1}{3} = \frac{1}{x}$

12. $\frac{3y}{4} - \frac{6}{12y} + \frac{1}{4} = 0$

13. $\frac{9x+22}{x^2} = 1$

13. $\frac{9}{(y+2)^2} = \frac{3y}{y+2}$

Find the solution set for each of the following equations:

15. $\frac{3}{x-4} - \frac{2}{x-3} = 1$

16. $\frac{y-5}{y+5} - \frac{y+5}{y-5} = \frac{4y^2-24}{y^2-25}$

17. $\frac{6-x}{x^2+x-12} - \frac{2}{x+4} = 1$

18. $\frac{4+8y}{y^2-9} + \frac{6}{y-3} = 3$

Solve the problems

19 **Sports:** If a cyclist wanted to cut a distance of 60 km between the two cities A and B in certain speed. If his speed increases in about 10 km/h , then he will be able to cut this distance in an hour less than the first time. Find his speed at first.

20 **passengers Transporting:** One of the Iraqi airlines planes cuts a distance of 350 km from Baghdad to Erbil in a certain speed. If the speed of the plane increases in 100 km/h, then the plane will be able to cut the distance in time which will be less in 12 minutes from the first time. Find approximate speed of the plane at first.

21 **Racing:** Nawfel participated in triple race which includes swimming, riding bicycles and running, and he took two hours to finish the race, as shown in the nearby table, considering x represents Nawfel's speed average in swimming. Find the average of his approximate speed in swimming racing .

	Distance km	Speed km/h	Time
Swimming	$d_s = 1$	x	t_s
Riding bicycles	$d_b = 20$	5x	t_b
running	$d_r = 4$	x + 4	t_r

Note: use the equation of total time which Nawfel spent in the race, in term of his speed in swimming is $T(x) = t_s + t_b + t_r$

Think

22 **Challenge:** Find the solution set for each of the following equations:

$$\frac{3}{x+5} + \frac{4}{5-x} = \frac{x^2 - 15x + 14}{x^2 - 25}$$

23 **Correct the mistake:** Nammeer used the discriminant expression to show the roots of equation,

$$\frac{2}{x-7} \times \frac{1}{x-1} = 1$$

he said that the equation has two relative real roots. Discover Nammeer's mistake and correct it.

Write

The solution set in real number .

$$\frac{1}{x+6} - \frac{5}{x-6} = 2$$

Lesson [3-7] problem solving plan (Writing Equation)

Learn

Idea of the lesson:
*Using the strategy of writing an equation to solve problem.

A ship sailing a distance of 240 km between the port A and the port B in certain speed. If its speed increases in 10 km/h, It will be able to sailing the distance in time which is less in 2 hours from the first time.
Find the speed of ship at first.

UNDERSTAND

What is the data in the problem: A ship sailing a distance of 240 km between the two cities A and B in certain speed. It cuts the same distance in time which is less in two hours from the first time if the speed of the ship increases in 10 km/h.

What is wanted in the problem: Finding the speed of ship at first.

PLAN

How can you solve the problem? Write an equation represents the problem, then solve it to find the speed of liner at first.

SOLVE

Assume that the first speed of ship = v , The first time = $\frac{240}{v}$

So its second speed = v + 10 , the second time = $\frac{240}{v + 10}$

$\frac{240}{v} - \frac{240}{v+10} = 2$

The first time – the second time equals 2

By multiplying the two sides of the equation by LCM v(v + 10)

$240v + 2400 - 240v = 2v(v + 10)$

$2400 = 2v^2 + 20v$

$v^2 + 10v - 1200 = 0 \Rightarrow (v + 40)(v - 30) = 0$

$\Rightarrow \begin{cases} v + 40 = 0 \Rightarrow v = -40 & \text{Neglect} \\ \text{or } v - 30 = 0 \Rightarrow v = 30 \text{ km/h} & \text{The speed of ship at first} \end{cases}$

CHECK

The first time of the ship = $\frac{240}{v} = \frac{240}{30} = 8$ h

The second time of the ship = $\frac{240}{v + 10} = \frac{240}{40} = 6$ h

The second time of ship is less than its first time in about two hours (8-6=2h), so the solution is correct.

Problems

Solve the following problems by the strategy of (writing an equation)

1 Fountain: Asquare–shaped area was planted with flowers in the middle of a square- shaped garden of a hotel, its side length is 4m. The remaining area which surrounds it is 84 m^2. What is the side length of the garden?

2 Babylon lion: It is a statue which was found in Babylonian archeological city in Iraq in 1776.It was made of the solid black basalt stone. It locates on a base in the middle of a rectangular- shaped area which its length is greater than its width in 2m and its area is 15 m^2. What are its dimensions?

3 Lion: Lion is one of the strongest animals on earth. It is the king of forest according to its strength. If the equation $x^2 - 30x$ represents the area which is under the control of the lion.What is the side length of the area? which represnted by x , if the area 175 km^2 ?

4 Fireworks: In one occasion, a group of fireworks was shot vertically. They reached up to 200m height. Calculate the time in which the fireworks reached up to that height if the following equation $2t^2 + 30t = h$ represents the relation between height, in meters (h), in which the fireworks reach after t second.

Chapter Test

Find the solution set for the two equations graphically:

1 $\begin{cases} y = 1 + x \\ y = 2 - x \end{cases}$

2 $\begin{cases} y + x = 0 \\ y - x = 0 \end{cases}$

3 $\begin{cases} y - x - 5 = 0 \\ y + x - 1 = 0 \end{cases}$

Find the solution set for the two equations by using the substitution or elimination for each of the following:

4 $\begin{cases} 2x + y = 1 \\ x - y = 8 \end{cases}$

5 $\begin{cases} 4x - 2y = -4 \\ x + y = 6 \end{cases}$

6 $\begin{cases} \frac{x}{3} + \frac{y}{2} = 1 \\ x + y = 2 \end{cases}$

Solve the following equations by using the greater common factor and the difference between two squares:

7 $9x^2 - 25 = 0$

8 $3y^2 - 12 = 0$

9 $(7 - z)^2 - 1 = 0$

Solve the following equations by using the rule of square root:

10 $x^2 = 49$

11 $81 - y^2 = 0$

12 $z^2 = \frac{36}{9}$

Solve the following equation by factoring in experiment:

13 $x^2 + 9x + 18 = 0$

14 $z^2 - 2z - 48 = 0$

15 $3x^2 - x - 10 = 0$

16 $7z^2 - 18z - 9 = 0$

17 What is the number which its square decreases from its four times in 3 ?

18 Aswimming pool , its length is gerter than the twice of its width in 4 m and its area is $48m^2$.
What are the dimensions of the pool?

Solve the following equations by the perfect square:

19 $x^2 - 16x + 64 = 0$

20 $\frac{1}{9} - \frac{1}{3}z + \frac{1}{4}z^2 = 0$

Solve the following equations by completing the square:

21 $x^2 - 14x = 32$

22 $4y^2 + 20y - 11 = 0$

23 $z^2 - \frac{2}{3}z = 1$

Find the solution set for the following equations by using the quadratic formula:

24 $x^2 - 3x - 7 = 0$

25 $3y^2 - 12y = -3$

26 $5z^2 + 6z = 9$

Determine the equation roots then find the solution set if it is possible:

27 $2x^2 + 8x + 8 = 0$

28 $y^2 - 6y - 9 = 0$

29 $4z^2 - 3z + 7 = 0$

30 What is the value of constant (K) which makes the two roots of the equation $x^2 - (k + 6)x + 9 = 0$
Equaled? Check your answer.

Find the solution set for each of the following equations and check the correction of solution:

31 $\frac{6x}{5} = \frac{5}{6x}$

32 $\frac{1}{6y^2} + \frac{1}{2} = \frac{1}{y}$

33 $\frac{z + 4}{z^2} = \frac{1}{2}$

Find the solution set for each of the following equations :

34 $\frac{4}{x - 5} - \frac{3}{x - 2} = 1$

35 $\frac{2y}{y + 2} + \frac{y}{2 - y} = \frac{7}{y^2 - 4}$

Chapter 4

Coordinate Geometry

Skiing is one of the enjoyable sports in the world, where the mountain declines represent a good example for slope. Higher slope requires greater skills of skiers

Pretest

Determine the points on the coordinate plane and then determine its location in the quadrants or in axes for each of the following :-

1 A (3,6)

2 B (-3,-5)

3 C (0,2)

4 D (-3,0)

5 E (-4,2)

6 F (3,-2)

Determine the points on the coordinate plane , then named the resulting shap for each of the following:-

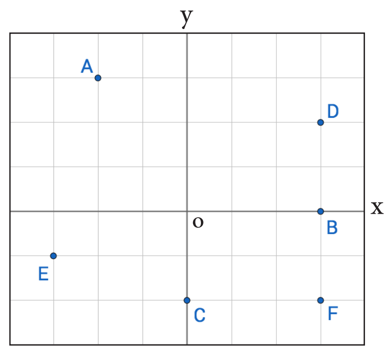
7 A (0,3),B(3,0) C(-3,0).

8 A (1,4),B(2,4) C(4,4)D(6,4).

9 A (-2,4),B(-2,-3) C(1,4),D(1,-3).

10 A (0,3),B(3,0) C(0,-3),D(-3,0).

11 Write the coordinate of the points which are indicated in the nearby coordinate plane :



Represent the following tables in the coordinate plane :

12

x	y
1	3
2	4
5	7

13

x	y
5	2
-2	-5
0	3

Find the value of (y) in each of the following:

14 $y = 2x - 5,\ x = 0$

15 $y = -x + 7,\ x = -1$

16 $y = x^2 + x + 2,\ x = 1$

17 $3y - x^2 = 9,\ x = -2$

If $A(x_1, y_1)$, $B(x_2, y_2)$ **find the numerical value of the expression** $\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$ **for each of the following .**

18 A(3,-5), B(-2,1)

19 A(-1,5), B(4,5)

Lesson [4-1] Graphical Representation of The Equations In The Coordinate Plane.

Idea of the lesson
- -Representing the linear equation in the Coordinate plane.
- -Representing the quadratic equation in the Coordinate plane

Vocabulary:
- -Ordered Pair
- -Coordinate plane
- -Linear equation
- -Quadratic equation

Learn

In a study to determine the quantity of milk which the new born of Anteater needs in liters for some days, the researcher reached to the following equation:
$2y-x=0$ where x represents number of days ,y represent the quantity of milk in liters.
How I can represent the relation in the coordinate plane.

[4- 1- 1] Graphical representation of the linear equation in the coordinate plane

Linear Equation : The general formula of linear equation is $ax+by+c = 0, a,b,c \in R$
Where a , b are not equal zero together . Its representation in the coordinate plane is line .

Example (1) To represent the equation $2y-x=0$ in the coordinate plane we following :-

First step :- We make the equation in the form $y = f(x)$ (y denoted by x)

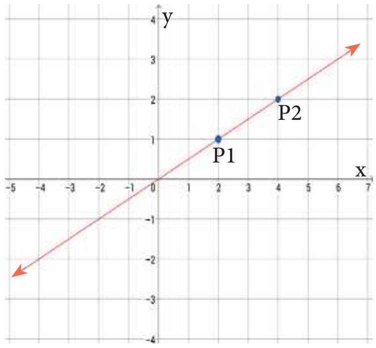
$$2y - x = 0 \Rightarrow 2y = x \Rightarrow y = \frac{1}{2}x$$

Second step :- Choose ,at least ,two values to the variable x , let x=4,x=2, then we substitute them in the equation to get ordered pairs .

$$x=2 \Rightarrow y = \frac{1}{2}(2) \Rightarrow y=1 \Rightarrow P_1(2,1)$$

$$x=4 \Rightarrow y = \frac{1}{2}(4) \Rightarrow y=2 \Rightarrow P_2(4,2)$$

Third step:- We make a table with the resulting values and represent the ordered pairs in the coordinate plane and connect the two points ,the figure that we get represents a line.



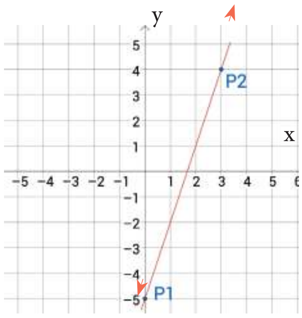
x	y	(x,y)
2	1	$P_1(2,1)$
4	2	$P_2(4,2)$

Note:- The equation of the line which passes through the origion is without the absolute term.

Example (2): **Represent the following equations in the coordinate plane. What do you notice?**

i) y-3x+5=0 ii) y = 4 iii) x = -3

i) y-3x+5=0 ⟶ y=3x-5

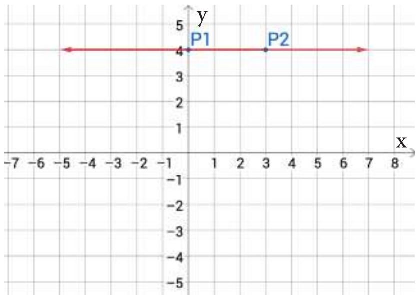


x	y=3x-5	(x,y)
0	3(0)-5=-5	$P_1(0,-5)$
3	3(3)-5=4	$P_2(3,4)$

The line is intersect X- axis , and Y- axis and it does not through the origion.

ii) y = 4

x	y=4	(x,y)
0	4	$P_1(0,4)$
3	4	$P_2(3,4)$

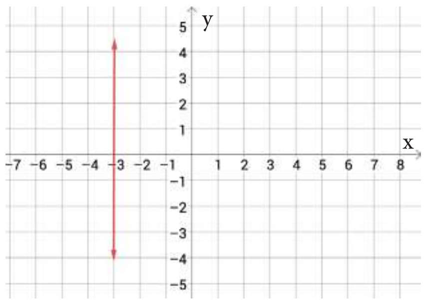


The line y= 4 parallel to the X- axis

and perpendicular to
Y_axis at the point (0,4)

iii) x = -3

The line x=-3 parallel to the Y –axis and perpendicular to X_axis at (-3,0)



It can put mentioned above in the following table:

Equation	the relation with the two axes
ax+by+c = 0	The line intersects the two axes and it does not passes through the origion
ax+by = 0	The line intersects the axes and passes through the origion
$y = k, k \in R$	The line parallel to the X-axis and perpendicularly to the Y- axis at the point (0,k)
$x = h, h \in R$	The line parallel to the Y-axis and perpendicularly to the X-axis at the point (h,0)

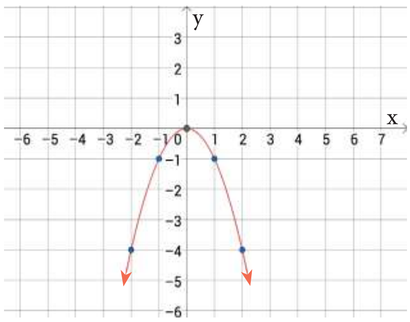
[4 - 1 -2] The Graphical representation of the quadratic equation in the coordinate plane .

The general formula of the quadratic equation is : $y = ax^2 + bx + c$, $a \neq 0, a, b, c \in R$

- In this item we will deal with the quadratic equation in the formula of $y = ax^2 + c$ where $a \neq 0,\ a, c \in R$ and the way its representation .
-To represent the equation $y = ax^2 + c$ we do the nearby table and the graphical representation of the equation will be $\cup$ or $\cap$

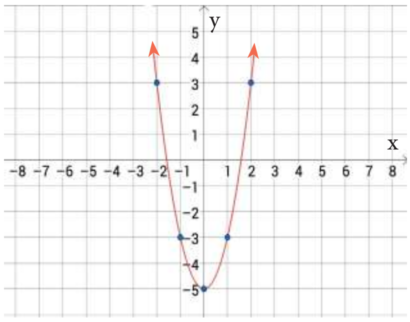
x	$y = ax^2 + c$	y	(x,y)
Supposed Values	**Substitute the value of X**	**The Result**	**Ordered Pairs**
-2			
-1			
0			
1			
2			

Example (3): Represent the equation $y = -x^2$



x	$y = -x^2$	y	(x,y)
-2	$-(-2)^2$	-4	(-2,-4)
-1	$-(-1)^2$	-1	(-1,-1)
0	$-(0)^2$	0	(0,0)
1	$-(1)^2$	-1	(1,-1)
2	$-(2)^2$	-4	(2,-4)

Example (4): Represent the equation $y = 2x^2 - 5$



x	$y = 2x^2-5$	y	(x,y)
-2	$2(-2)^2-5$	3	(-2,3)
-1	$2(-1)^2-5$	-3	(-1,-3)
0	$2(0)^2-5$	-5	(0,-5)
1	$2(1)^2-5$	-3	(1,-3)
2	$2(2)^2-5$	3	(2,3)

Make sure of your understanding

Represent the following linear equations in the coordinate plane, then explain their relation with the two axes:

Questions 1 -6 are similar to examples (1, 2)

1. $y = 3x+1$
2. $y = -4x$
3. $y + 3x - 2 = 0$
4. $y = 1 - 3x$
5. $y+5=0$
6. $x-5 = 0$

Represent the following quadratic equations in the coordinate plane:

Questions 7 -9 are similar to examples (3,4)

7. $y=x^2 + 4$
8. $y= x^2$
9. $y= 1- 3x^2$

Solve the Exercises

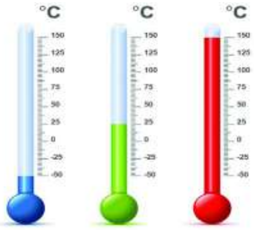
Represent the following linear equations in the coordinate plane then explain their relation with the two axes:

10. $y=-x + 4$
11. $y=x$
12. $x=-\frac{5}{2}$
13. $y= 0$

Represent the following quadratic equations in the coordinate plane:

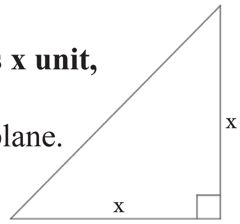
14. $y=x^2 - 1$
15. $y=2x^2 + 3$
16. $y=- 3x^2$
17. $y= 2 x^2$
18. $4y=x^2$

Solve the problems



19 **Temperatures:** The equation $F^0 = \frac{9}{5}C^0 + 32$ shows the relation between the Celsius temperatures and Fahrenheit temperatures .Represent the equation graphically.

20 **Geometry: A right – angled and isosceles triangle, its length right side is x unit, f (x) represents its area.** i) Write the relation f (x) in terms of x.
ii) Represent the relation f(x) in the coordinate plane.

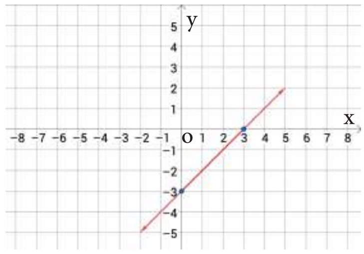


21 **Physics :** The law $F=9.8m$ represents the power of the earth gravity on a body, where F is the power in Newton, m is the body mass in Kg. Represent the law in the coordinate plane.

22 **Works :** A company for construction equipment takes 10,000 dinars as insurance, as well 5,000 dinars for every hour. Write the equation that expresses the problem, and then represent it graphically in the coordinate plane.

Think

23 **Discover the mistake:** Mohammed represented the following linear equation (y=-3x+9) in the nearby graphic figure. Discover Mohammed's mistake, then correct it.



24 **Open problem: Give an example of linear equation as ax+by+c=0 for each case :**
i) a= 0 ii) b= 0 iii) c = 0

25 **Challenge :** The following ordered pairs (0,4) ,(1,6),(-1,2) had formed a line, what is the intersection point of this line with the X – axis ?

26 **Justification:** Show if the following ordered pairs (2,4),(1,1),(0,0),(-1,1),(-2,4) represent a linear function or quadratic one .

27 **Numerical sense :** $y = x^2 + 1$, $y = x+1$. Which one represents a quadratic function ? clarify that.

Write

The steps which show that y=4x+3 is a linear equation?

Lesson [4-2] Slope of the line

Idea of the lesson:
- Finding the slope of the line.
- Finding the Y- Intercept
- Finding the X- Intercept

Vocabulary:
- The vertical change
- The horizontal change
- X-Intercept
- Y-Intercept
- Slope

Learn

The mountain declines are good example for slope, whenever the higher the mount hight, the slope will be increased. How can we determine the slope of the declines?

[4- 2- 1] Finding the slope of the line

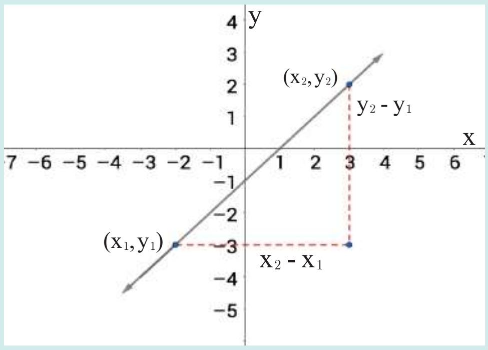
The slope: The slope of the non- vertical line can be defined as the ratio between the vertical change to horizontal change.

Vertical change : Is the Y- change which equals $y_2 - y_1$

Horizontal change : Is the X- change which equals $x_2 - x_1$

$$\text{The slope} = \frac{\text{vertical change}}{\text{horizontal change}}$$

i.e. $m = \frac{y_2 - y_1}{x_2 - x_1}$ where $x_2 - x_1 \neq 0$



m: is the slope of the line which passes through the two points $(x_1, y_1), (x_2, y_2)$

The slope of the line can be either positive or negative if it is not horizontal or vertical It also may be zero (horizontal) or undefined (vertical) .

Example(1) **Find the slope of the line which passes through two points in each of the following:**

i) A(5,7), B(-2,1)

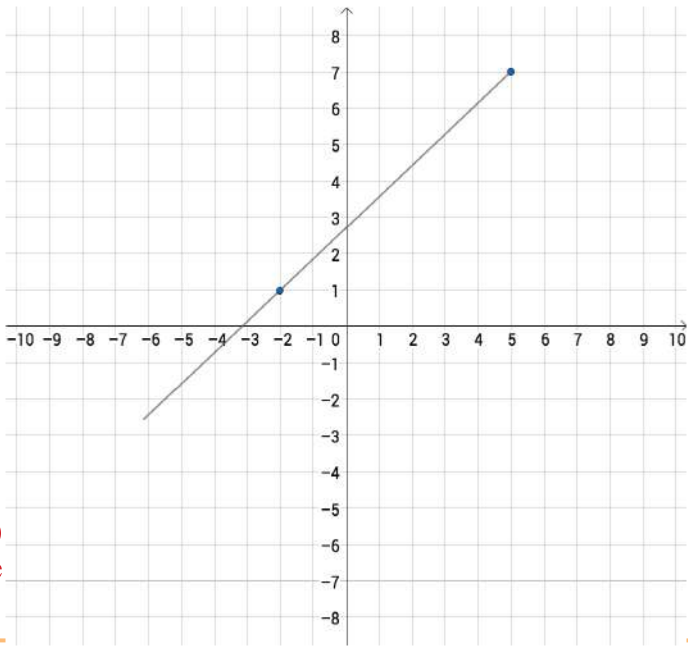
$m = \frac{y_2 - y_1}{x_2 - x_1}$ The slope of the line which passes through two points

$m = \frac{1 - 7}{-2 - 5}$ Substitute by the two points

$m = \frac{-6}{-7}$ By simplify

$m = \frac{6}{7}$ So the slope of $\overleftrightarrow{AB}$ is $(\frac{6}{7})$ (positive).

The slope is positive (the line is to upward) when we move from left to right , values of y are increasing.

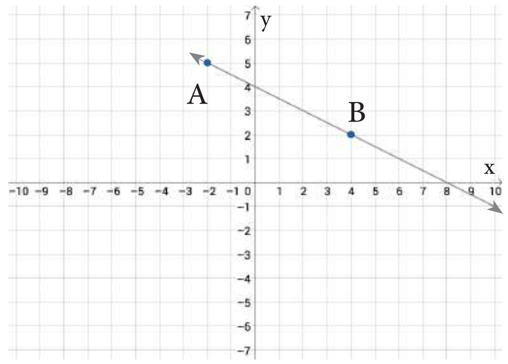


ii) A (-1,5), B (4,2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ The slope of the line which passes through two points

$$= \frac{2 - 5}{4 - (-1)}$$ Substitute by the two points.

$$= \frac{-3}{5}$$ So the slope of ($\overleftrightarrow{AB}$) is ($\frac{-3}{5}$) negative



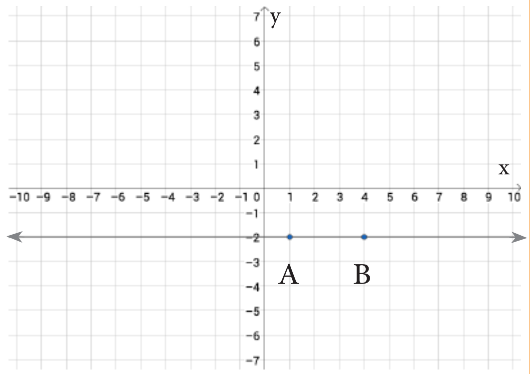
The slope is negative (the line is to down ward) when we move from left to right, values of y are decreasing.

iii) A (1, - 2), B (4, - 2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ The slop of the line which passes through two points

$$= \frac{-2 - (-2)}{4 - 1}$$ Substitute by the two points.

$$= \frac{0}{3} = 0$$ So the slope of ($\overleftrightarrow{AB}$) is (0)



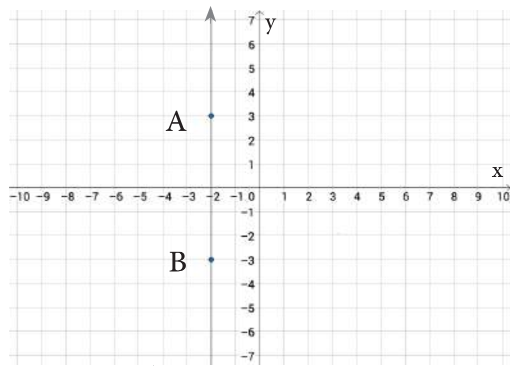
The slope is zero (the line is horizontal), it parallel to the X-axis ,the values of y are constant.

iv) A (-2,3), B (-2, - 3)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ The slope of the line which passes through two points

$$= \frac{-3 - 3}{-(-2) - (-2)}$$ Substitute by the two points

$$= \frac{-6}{0}$$ It is impossible to divide by 0,so the slope of ($\overleftrightarrow{AB}$) is undefined



The slope is undefined (the line is vertical),it parallel to the Y-axis, the values of x are constant.

Example (2) The nearby table represents the change of temperatures by time (hours).Find the slope of line and explain what it means?

Time (hours)	Temperatures
1	-2
2	1
3	4
5	10

$(x_1, y_1) = (1, -2)$
$(x_2, y_2) = (3, 4)$ Choose any two points from the table, let it be

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ The slope of the line which passes through two points.

$$= \frac{4 + 2}{3 - 1} = \frac{6}{2} = 3$$ The substitution and simplification

Since the slope of the line is 3 , then the temperatures are increasing 3 Celsius degrees every hour

[4-2-2] The Intersection of the line with axes in Coordinate plane

You can easily represent the equation of the line by finding the two points of line with the two axes.
X – intercept: is the value of x from the intersection of the line with the X – axis by substituting y= 0. The interection point is (x,0).

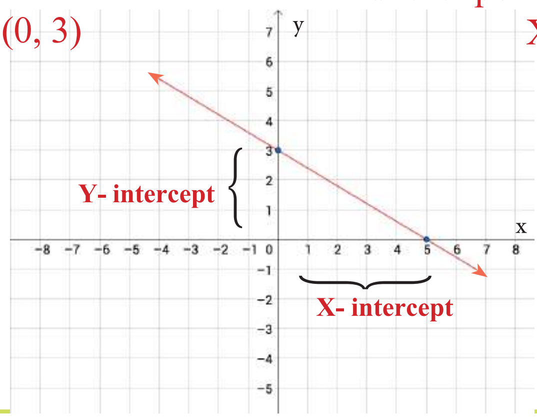
Y – intercept : is the value of y from the interection of the line with the Y– axis by substituting x= 0 and the interection point is (0,y).

Example (3) Find the X-intercept and Y- intercept for the line 3x+ 5y =15

Y- Intercept		X-Intercept	
$3x + 5y = 15$	The equation	$3x + 5y = 15$	The equation
$3(0) + 5y = 15$	Substituting x= 0	$3x + 5(0) = 15$	Substituting y=0
$5y = 15$	Simplify	$3x = 15$	Simplify
$y = \frac{15}{5}$	By dividing the two sides of equation by5	$x = \frac{15}{3}$	By dividing the two sides of equation by3
$y = 3$	So the Y-intercept is 3	$x = 5$	So the X-intercept is 5

And the point of intersection with the Y- axis is (0, 3)

And the point of intersection with the X-axis is (5, 0)



Example 4 **Find the X- intercept and Y- intercept if it is for each of the following:**

i) x = -2, ii) y = 4

x=-2, represents X- intercept and the point of intersection is (-2,0) The line // Y-axis

y = 4, represents Y- intercept and the point of intersection is (0,4) The line // X-axis

Make sure of your understanding

Find the slope of the line which through the two points.Is the slope positive or negative or zero or undefined ?Then determinethe direction of its movement for each of the following:

Questions 1- 6 are Similar to Examples (1, 2)

1 (-2, - 2), (-4, 1)
2 (0, 0), (3, 2)
3 (-4, 4), (2, - 5)
4 (5, 0), (0, 2)
5 (4, 3), (4, - 3)
6 (-6, - 1), (-2, - 1)

Find the X-intercept and the Y- intercept for each the following:

Questions 7- 18 are Similar to Examples (3, 4)

7 $3x + 6y = 18$
8 $y + 2 = 5x - 4$
9 $y = -4x$
10 $y = -x + 8$
11 $5x = y - 8$
12 $y = -\frac{3}{4}x - 5$
13 $2x + 6y = 12$
14 $y + 4 = 2x - 4$
15 $y = -5x$
16 $x = 4$
17 $3y = -6$
18 $y = -\frac{1}{2}x + 4$

Solve the Exercises

Find the slope of the line which passes through the two points .Is the slope positive or negative or zero or undefined, then determine the direction of its movement for each of the following:

19 $(6,2),(0,2)$

20 $(-2,-3),(2,4)$

21 $(3,-5),(0,0)$

22 $(\frac{3}{2},\frac{1}{4}),(\frac{3}{2},\frac{3}{4})$

Find the X-intercept and the Y-intercept for the each of the following:

23 $3y - 7x = 9$

24 $y = -\frac{3}{2}x$

25 $x = -4$

26 $0 = y+3$

Solve the Problems

27 **Physics:** The nearby table represents the amount of liquid which flows from a basin during certain time, find the slope of the line which represented by the table, then illustrate what it means.

The amount of leaky liquid	
Time second	Volume of liquid m^3
10	40
13	52
16	64
19	76

28 **Plants: If the length of a plant is 30 cm, during each two months, it grows in constant distance which is 4cm.**

i) Complete the table.

ii) What is the slope of the line which represented by the relation between the length of the plant and the time.

iii) Write the linear function which represented by the table.

iv) Represent the function in the coordinate plane.

Time	0	2	4
length of plant			

Think

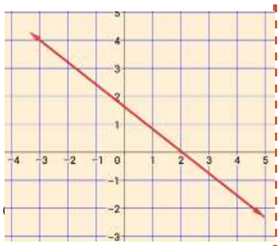
29 **Challenge:** Find the value of (a) which makes the slope of the line which passes through the two points $(1,6),(-5,a)$ equals $(\frac{1}{2})$.

30 **Critical thinking :-** Can you determine a slope of a line passes through the two points (7,3),(7,-3)?

31 **Discover the mistake:** The slope of the line which passes through the two points $(0,3),(3,-1)$ is $(\frac{3-0}{3-(-1)} = \frac{3}{4})$,Discover the mistake and correct it.

32 **Open problem:** Mention two points on a line which its slope $= -\frac{1}{3}$

33 **Critical thinking:** From the nearby graphical figure, determine the direction



write

In your style the meaning of the slope equals zero and the slope is undefined.

Lesson [4-3]

The Equations of The line.

Idea of the lesson:
-Finding equation of a line Which its following data are given:
-Two points .
-Slope - point .
-Slope - intercept .

Vocabulary:
-The slope.
-The intercept.

Learn

A cyclist round off a distance 20 Km in two hours and 50 Km in five hours, what is the linear equation which connects between the distance and the time?

[4- 3- 1] Writing an Equation of line with two points of it

The equation which passes through the two points $B(x_2,y_2), A(x_1,y_1)$

You have previously learned finding a slope of a line passes through the two points A,B

where $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Assuming that the point C (x,y) lie on the line , then the slope of the line which pasçs through the tço points A,C is $m = \dfrac{y - y_1}{x - x_1}$

It is known that the slope of the line is constant in all its points , so : $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$

This equation represents the equation of the line AB

Example (1) We find the linear equation in the paragraph of (Learn)

Assume that $C(x,y) \in \overleftrightarrow{AB}$, $B(5,50)$, $A(2,20)$

$\Downarrow$ $x_2 = 5, y_2 = 50$ $\quad$ $\Downarrow$ $x_1 = 2, y_1 = 20$

$\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$ Writing the equation of the line which passes through two points.

$\dfrac{y - 20}{x - 2} = \dfrac{50 - 20}{5 - 2}$ Substituting by $(x_2,y_2),(x_1,y_1)$

$\dfrac{y - 20}{x - 2} = \dfrac{30}{3}$ By simplify

$y - 20 = 10x - 20$ Commutative multiplication

Then the line equation is $y - 10x = 0$

[4- 3- 2] Writing Equation of line with the slope and one point of it

The equation of the line which its slope m and passes through the point (x_1,y_1)

You have previously learned an equation of a line which passes through two points which is

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

You have also learned that the slope of a line passes through the two points $(x_2,y_2),(x_1,y_1)$

is $m = \dfrac{y_2 - y_1}{x_2 - x}$

So the above equation can be written as $\dfrac{y - y_1}{x - x_1} = m$

And by commutative multiplication we get the required equation $y - y_1 = m(x - x_1)$

Example (2)

Use the equation of the slope and the point for each line to determine its slope and the point in which it passes.

i) $y - 3 = -5(x - 2)$

$$y - 3 = -5(x - 2)$$
$$y - y_1 = m(x - x_1)$$

Equation of the slope –the point By comparison

$m = -5, (x_1, y_1) = (2, 3)$

ii) $y + 7 = \frac{2}{5}x$

$$y - (-7) = \frac{2}{5}(x - 0)$$
$$y - y_1 = m(x - x_1)$$

Equation of the slope –the point By comparison

$m = \frac{2}{5}, (x_1, y_1) = (0, -7)$

Example (3) Find the equation of the line which its slope is $\frac{1}{2}$ and its X-intercept equals -1

$y - y_1 = m(x - x_1)$ — **Equation of the slope- the point**

$m = \frac{1}{2}, \quad x_1 = -1, \quad y_1 = 0 \Rightarrow p(-1, 0)$ — **The slope, the point**

$y - 0 = \frac{1}{2}(x - (-1))$ — **By substituting the slope and the point**

$y = \frac{1}{2}(x + 1)$ — **Multiplying the two sides of equation by two**

$2y = x + 1$

The equation of the required line. $2y - x = 1$

[4 -3- 3] Writing Equation of the line with the slope of it and one intercept with axes

The equation of the line according to its slope m and its Y-intercept K is $y = mx + k$

Example (4)

Use the equation of the slope and the intercept for each line to determine its slope and intercept:

i) $2x + 3y = 6$ ii) $5x = 7y + 8$ iii) $y = x$ iv) $y = 1$ v) $y = 0$ vi) $y + x = 5$

i) $2x + 3y = 6 \Rightarrow 3y = -2x + 6$ — By dividing the two sides of the equation by 3

$$y = \frac{-2}{3}x + 2$$
$$y = mx + k$$

The comparison with the equation of the slope-intercept

$\therefore m = \frac{-2}{3}, \quad k = 2$

iii) $y = x \Rightarrow y = 1x + 0$

$$y = 1x + 0$$
$$y = mx + k$$

The comparison with the equation of the slope-intercept

$\therefore m = 1 , \ k = 0$

ii) $5x = 7y + 8 \Rightarrow 7y = 5x - 8$ — By dividing the equation by 7

$$y = \frac{5}{7}x - \frac{8}{7}$$
$$y = mx + k$$

The comparison with equation of the slope-intercept section

$\therefore m = \frac{5}{7}, \quad k = \frac{-8}{7}$

iv) $y = 0x + 1$

$$y = mx + k$$

The comparison with equation of the slope-intercept

$\therefore m = 0, \quad k = 1$

v) $y = 0x + 0$

$$y = mx + k$$

The comparison with the equation of the slope intercept

$\therefore m = 0, k = 0$

vi) $y = -1x + 5$

$$y = mx + k$$

The comparison with equation of the slope-intercept

$\therefore m = -1, \quad k = 5$

Example (5)

A line passes through the point $(5, -1)$ and its slope $\frac{-2}{5}$, find its intercept and equation.

The first method		The second method	
$y = mx + k$	Equation of slope-intercept	$y - y_1 = m(x - x_1)$	Equation slope - point
$m = \frac{-2}{5}$	Given	$m = \frac{-2}{5}, p(5, -1)$	Given
$y = \frac{-2}{5}x + k$	By substituting the slope	$y - (-1) = \frac{-2}{5}(x - 5)$	By substituting the point and the slope
$-1 = \frac{-2}{5}(5) + k$	By simplifying	$5y + 5 = -2x + 10$	By Multipling the two sides by 5 by dividing sides by 5 after simplifying
$-1 = -2 + k$		$5y + 2x = 5$	
$k = 1$			
$y = \frac{-2}{5}x + 1$	the equation of the line	$y = \frac{-2}{5}(x) + 1$	The equation of the line

$k = 1$ its intercept

Make sure of your understanding

Find the line equation which each one of them passes through two points in each of the following:

1. $(-3, 1), (2, -1)$
2. $(0, 2), (2, -4)$

Questions 1 -2 are Similar to example 1

Use the equation of slope and point for each line to determine its slope and the point which it passes through.

3. $y - 1 = 2(x - 3)$
4. $y + 1 = -x + 4$

Questions 3 -4 are Similar to example 2

Find the line equation for each of the following , then find its intercept:

5. $(4, 6), \frac{-2}{5}$
6. $(-1, -3), \frac{1}{3}$

Questions 5 -6 are Similar to examples (3,5)

Use the equation of the slope and point for each line to determine its slope and intercept:

7. $5y = -2x - 1$
8. $-y = 7x$

Questions 7 -8 are Similar to example 4

Solve the Exercises

Find the line equation which each one of them passes through two points in each of the following:

9. $(0, 0), (-3, 7)$
10. $(0, 7), (-5, 0)$

Use the equation of the slope and intercept for each to detrmine its slope and intesrcept:

11. $y + \frac{3}{2} = -5(x - 8)$
12. $y - x = 8$

Find the line equation for each of the following then find its intercept:

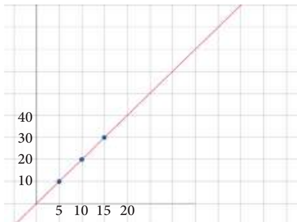
13. $(-3, 7)$ -3 **The slope**
14. $(1, -4)$ $\frac{-1}{2}$ **The slope**

Use the equation of the slope and intercept for each line to determine its slope and intercept:

15. $y + 7 = 3x + 5$
16. $\frac{1}{3}y = -5x - 1$

Solve the problems

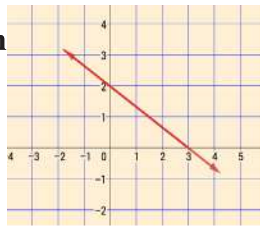
17 Biology: Elephant's ivory is continuously grows at a rate of 1cm monthly. Assuming that you start monitoring an elephant which its ivory length was 100cm.Write in the form of slope-point, an equation represents the growing of the elephant ivory after (n) months of the monitoring.



18 Physics: The nearby graphic representation represents the amount of leaky water from a tank during certain time. Write in the form of two points, an equation represents the leakage of the water after (n) seconds.

19 Money: A person wants to pay 30 million dinars as monthly installments which will not exceed 1.5 million dinars, the following linear equation $y = -1.5x + 30$, where y represents the remaining value from the money, x represents the number of months. Use the equation of slope and intercept to determine its slope and intercept.

20 Health: In a new study, a man loses two hours of his age when he smoke a pack of cigarettes. Write the equation which represents that and represent it graphically.



21 Geometry: Use the data in the nearby figure and then find the line equation in the following cases:

i)Two points ii) Slope-point iii) Slope-Y-intercept

Think

22 Critical thinking: Is it possible to find a line which its slope is 4 and passes through the two points $(5,7),(8,-2)$?If it is possible to find a line like this, then write an equation, otherwise illustrate your answer.

23 Challenge: A line, its horizontal intercept represents the additive inverse of its vertical intercept. It passes through the point $(2,3)$, write the equation of the slope-point for this line.

24 Which one is correct : An equation of line ,its slope is $\frac{3}{5}$ and passes through the point $(-1,7)$

Ahmed wrote the equation in the form $y - 7 = \frac{5}{3}(x+1)$

Mohammed wrote the equation in the form $y - 7 = \frac{3}{5}(x+1)$, which answer is correct?

Write

A problem from life that can be represented by the equation of the line.

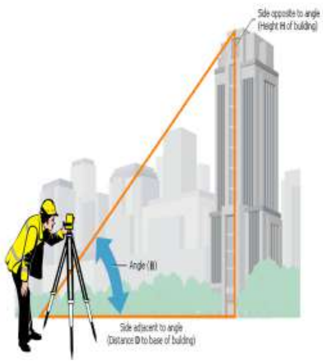
Lesson [4-4] Trigonometric Ratios

Idea of the lesson:
- -Identifing the fundamental trigonometric ratios
- -The trigonometric ratios for special angles
- -Finding the values of statements including special angles

Vocabulary:
- - Trigonometric ratios
- -sin,cos tan,sec,csc,cot
- Special angles: $60°, 45°, 30°, 90°, 0°$

Learn

A surveyor stood a distance d meter from a building by using his appliance, he looking at the upper level of the building with a certain angle. How can the trigonometric ratios help him in finding the height of the building?



[4- 6- 1] Trigonometric ratios (sinθ ,cosθ ,tanθ)

You have previously learned the elements of the triangle ,where it consists of three angles and three sides. According to its angles, we call the triangle (acute angles ,optuse angle, right angle)or according to its sides,we call it (equilateral,isosceles,scalene)

-Caculating the triangles :Is the study of the relation between the angles and sides of the triangle.

-Trigonometric ratio : Is the ratio which compares between the length of two sides of a right –angled triangle

-The fundamental ratio: Is the sine (sin) and the cosine (cos)and the tangent (tan).

-The sine of $\angle\theta$ (we refer to by sinθ): Is the ratio between the side which opposite to the angle (θ)and the hypotenuse $\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$

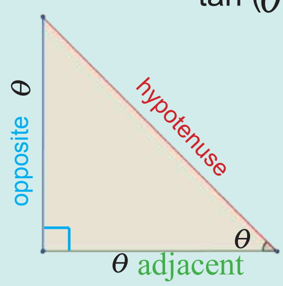
-The cosine of $\angle\theta$(we refer to by cosθ) : Is the ratio between the side which is adjacent to the angle (θ) and the hypotenuse. $\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$

-The tangent of $\angle\theta$ (we refer to by tanθ): Is the ratio between the side which is opposite to the angle (θ) and the adjacent.

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

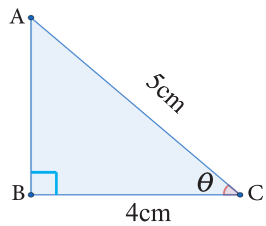
-To find the trigonometric ratios (sin,cos,tan) we follow:-

1) Drawing a diagram of a right –angled triangle and we fix the data on it.
2) Using Pythagorean theorem to find the missing side.
3) Using the trigonometric ratios to find the rquired.



Example(1)

From the nearby figure, find the three values of the trigonometric ratios of the angle θ .
Use Pythagorean theorem to find the length of the side AB (opposite)



$(AC)^2 = (AB)^2 + (BC)^2$ Pythagorean theorem
$(5)^2 = (AB)^2 + (4)^2$ By substituting and simplifying
$(AB)^2 = 25 - 16 = 9$ Take the square root of both sides
$AB = 3$ (positive sign because it is length)

-use the trigonometric ratios ,then the substitution

$$\sin\theta = \frac{\text{opposite of } \angle\theta}{\text{hypotenuse}} = \frac{3}{5} \qquad \cos\theta = \frac{\text{adjacent of } \angle\theta}{\text{hypotenuse}} = \frac{4}{5} \qquad \tan\theta = \frac{\text{opposite of } \angle\theta}{\text{adjacent of } \angle\theta} = \frac{3}{4}$$

Example (2)

The triangle ABC has a right –angle in B ,if $\tan A = \frac{15}{8}$ **, find:**

i) sin A ii) cos A

$\tan A = \frac{15k}{8k}$, $\tan A = \frac{BC}{BA}$ — By multiplying the numerator and the denominator by the constant K, where K is greater than 0. -Tangent formula

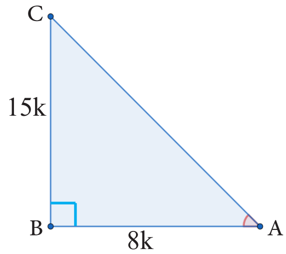
$\therefore BC = 15k, AB = 8k$ — By comparison

$(AC)^2 = (AB)^2 + (BC)^2$ — Pythagoras theorem

$= (8k)^2 + (15k)^2$ — By substituting

$= 64k^2 + 225k^2$ — Simplification

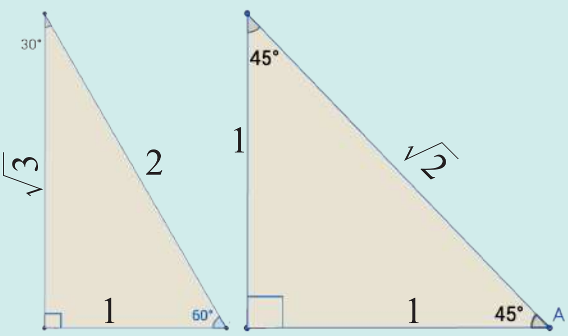
$(AC)^2 = 289k^2 \Rightarrow \therefore AC = 17k$ — Take the square root of both sides



i) $\sin A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$ ii) $\cos A = \frac{AB}{AC} = \frac{8k}{17k} = \frac{8}{17}$

[4 -6 -2]The Trigonometric Ratios for Special Angles.

The nearby table shows the values of the trigonometric for special angles



Trigonometric ratios	30°	60°	45°	90°	0°
Sine (sin)	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	1	0
Cosine (cos)	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	0	1
Tangent (tan)	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1	undifiend	0

Example (3) Prove that $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ = \sin 90^\circ$

$\sin 60^\circ = \frac{\sqrt{3}}{2}, \cos 30^\circ = \frac{\sqrt{3}}{2}, \cos 60^\circ = \frac{1}{2}, \sin 30^\circ = \frac{1}{2}, \sin 90^\circ = 1$ — From the table, we find

R.H.S: $\sin 90^\circ = 1$

L.H.S: $(\frac{\sqrt{3}}{2})(\frac{\sqrt{3}}{2}) + (\frac{1}{2})(\frac{1}{2})$ — By substituting in the right side and the left side

$\frac{3}{4} + \frac{1}{4} = 1$

$\therefore$ L.H.S = R.H.S

Example(4) A man stood in front of a building in a distance which is 12 m from its base, he looked at the top of the building in a angle which is (30°). Find the height of the building.

The trigonometric ratio which connects between the height of the building h and the distance of the man from its base ,is the tangent ratio.

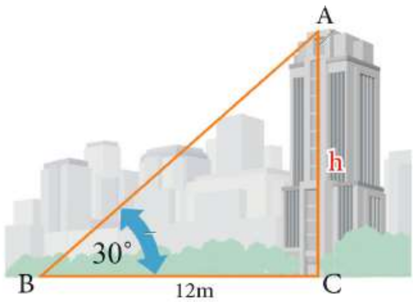
$\tan 30^\circ = \frac{h}{12}$ — Tangent formula

$\frac{1}{\sqrt{3}} = \frac{h}{12}$ — Substituting

$\sqrt{3}\,h = 12$ — Commutative Multiplication

$h = \frac{12}{\sqrt{3}} = 4\sqrt{3}$ m — Simplifying

The height of the building is $4\sqrt{3}$ m



[4-6-3] Relations of Trigonometric Ratios

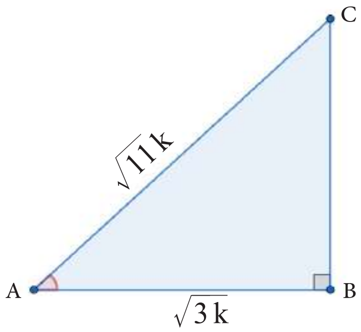
In this item, we will focus on the inversion of the trigonometric ratios sin,cos,tan,and as it shown in the following table:

Trigonometric ratio	$\sin\theta$	$\cos\theta$	$\tan\theta$
Its inversion	$\csc\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{1}{\tan\theta}$

Example(5)

A triangle has a right angle in B, if $\cos A = \frac{\sqrt{3}}{11}$ **find** i) $\sec A$ ii) $\csc A$ iii) $\cot A$

$\cos A = \frac{\sqrt{3}\,k}{\sqrt{11}\,k} = \frac{AB}{AC} \Rightarrow AB = \sqrt{3}\,k, AC = \sqrt{11}\,k$



$(AC)^2 = (AB)^2 + (BC)^2$ Pythagorean theorem

$(\sqrt{11}\,k)^2 = (\sqrt{3}\,k)^2 + (BC)$ By substituting

$11k^2 = 3k^2 + (BC)^2$ Simplifying

$(BC)^2 = 8k^2$ Take the square root of both sides

$\therefore BC = \sqrt{8}\,k$

i) $\cos A = \frac{\sqrt{3}}{\sqrt{11}} \Rightarrow \sec A = \frac{1}{\cos A} = \frac{\sqrt{11}}{\sqrt{3}}$ ii) $\sin A = \frac{\sqrt{8}}{\sqrt{11}} \Rightarrow \csc A = \frac{1}{\sin A} = \frac{\sqrt{11}}{\sqrt{8}}$

iii) $\tan A = \frac{\sqrt{8}}{\sqrt{3}} \Rightarrow \cot A = \frac{1}{\tan A} = \frac{\sqrt{3}}{\sqrt{8}}$ Inversion of fundamental trigonometric ratios

Example(6)

Find the numerical value for the expression: $(\sin 45^\circ)(\sec 45^\circ) - (\tan 60^\circ)(\cot 30^\circ) + 2\csc 90^\circ$

$\sin 45^\circ = \frac{1}{\sqrt{2}}, \sec 45^\circ = \frac{1}{\cos 45^\circ} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

$\tan 60^\circ = \sqrt{3}, \cot 30^\circ = \frac{1}{\tan 30^\circ} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$

$\csc 90^\circ = \frac{1}{\sin 90^\circ} = \frac{1}{1} = 1$

From the table, we find the values of the special trigonometric ratios and the inversions of the fundamental trigonometric ratios.

$(\sin 45^\circ)(\sec 45^\circ) - (\tan 60^\circ)(\cot 30^\circ) + 2\csc 90^\circ$ The given expression

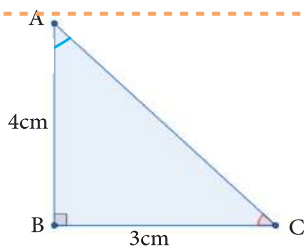
$(\frac{1}{\sqrt{2}})(\sqrt{2}) - (\sqrt{3})(\sqrt{3}) + 2(1) \Rightarrow 1 - 3 + 2 = 0$ by substituting and simplifying

$\therefore$ The numerical result of the expression equals 0

Make sure of your understanding

1 **From the nearby figure, find the following trigonometric ratios**

i) $\sin A$ ii) $\cos C$ iii) $\cot C$ iv) $\sec A$



Question 1 is similar to examples 1,2,5

2 The triangle ABC has a right angle in B, if $\cot A = \sqrt{3}$, find :

i) $\tan A$ ii) $\sin A$ iii) $\csc A$ iv) $\sec A$ v) $\cos A$

Question 2 is similar to examples 2,5

3 **Prove the following:**

i) $(\cos 30^\circ - \csc 45^\circ)(\sin 60^\circ + \sec 45^\circ) = \frac{-5}{4}$, ii) $2\sin 30^\circ \sec 30^\circ = \csc 60^\circ$

iii) $(\cos 45^\circ - \csc 45^\circ)(\tan 45^\circ)(\csc 90^\circ) = -\cos 45^0$, iv) $\sqrt{\frac{1 - \cos 60^\circ}{2}} = \sin 30^0$

Question 3 is similar to examples 6,3

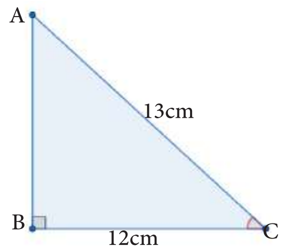
4 A Kite had raised ($3\sqrt{3}$ m) from the surface of the land ,if the string which was connected with it formed an angle of 60° with the surface of the land . find the length of the string

Question 4 is similar to example 4

Solve the Exercises

5 **From the nearby figure,find the following trigonometric ratios:**

i) $\cot A$ ii) $\cot C$ iii) $\sec C$ iv) $\csc A$



6 **The triangle ABC has a right angle in B, if $\sec A = \sqrt{2}$, find :**

i) $\sin A$ ii) $\cot C$ iii) $\csc A$ iv) $\cos C$

7 **Prove the following :**

i) $\cos 60^\circ \csc 60^\circ + \sin 60^\circ \sec 60^\circ = \frac{4}{\sqrt{3}}$

ii) $\sin 45^0 \sec 45^0 + \csc 45^0 \cos 45^0 = 2$,

Solve the problems

8 **Tower :** A wire used install acommcunication tower forms an angle of slop 30° withe the ground if the hight of the tower is 12 m what is the horizantal distance between end the wire on the ground and the base of the tower?

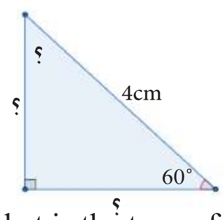
9 **Skiing :** In a position for skiing , the height of the main hill is 500m and the angle of its inclination from the surface of the land is 60°.How long is the skiing surface?

10 **A ladder of fire fighting :**The long of a ladder of fire fighting is 20m and the measurement of the angle which it creates with the land is (45°). Find the height of long of ladder on the building.

11 **Garden :** Benan stood in a distance of 25m far from base of a tree which its height is 25m. What is the measurement of the angle which she forms with the top of the tree?

Think

12 **Challenge:** In the nearby figure, find the indicated values (?) by using the trigonometric ratios.



13 **Open problem :** The triangle ABC has a right angle is B, $\sin A = \frac{\sqrt{3}}{2}$ how can you find the value of the angle C?

14 **Justification :** If the sine and cosine of a right –angled triangle were equaled , what is the type of the triangle in term of its sides length?

Write

A problem in which you can use the sine ratio to find the length of an unknown side in a right – angled triangle, then solve it.

Chapter Test

1 **Represent the following equations in the coordinate plane:**

i) $2x-4y=8$

ii) $y=2$

iii) $x=2$

iv) $y=x^2-1$

2 Find the equation of the line which passes through the two ponits A(-2,-3), B(2,3).

3 Find the X-intercept and Y-intercept for the equation $y-x=4$.

4 **Find the equation of the line for each the following:**

i) passes through the two points (3,-2), (1,5).

ii) Slope $\frac{3}{2}$ and Y-intercept equals -5.

iii) Slope $-\frac{1}{5}$ and X-intercept equals 3.

5 Use slope equation and point to determine the slope and one of it's points:

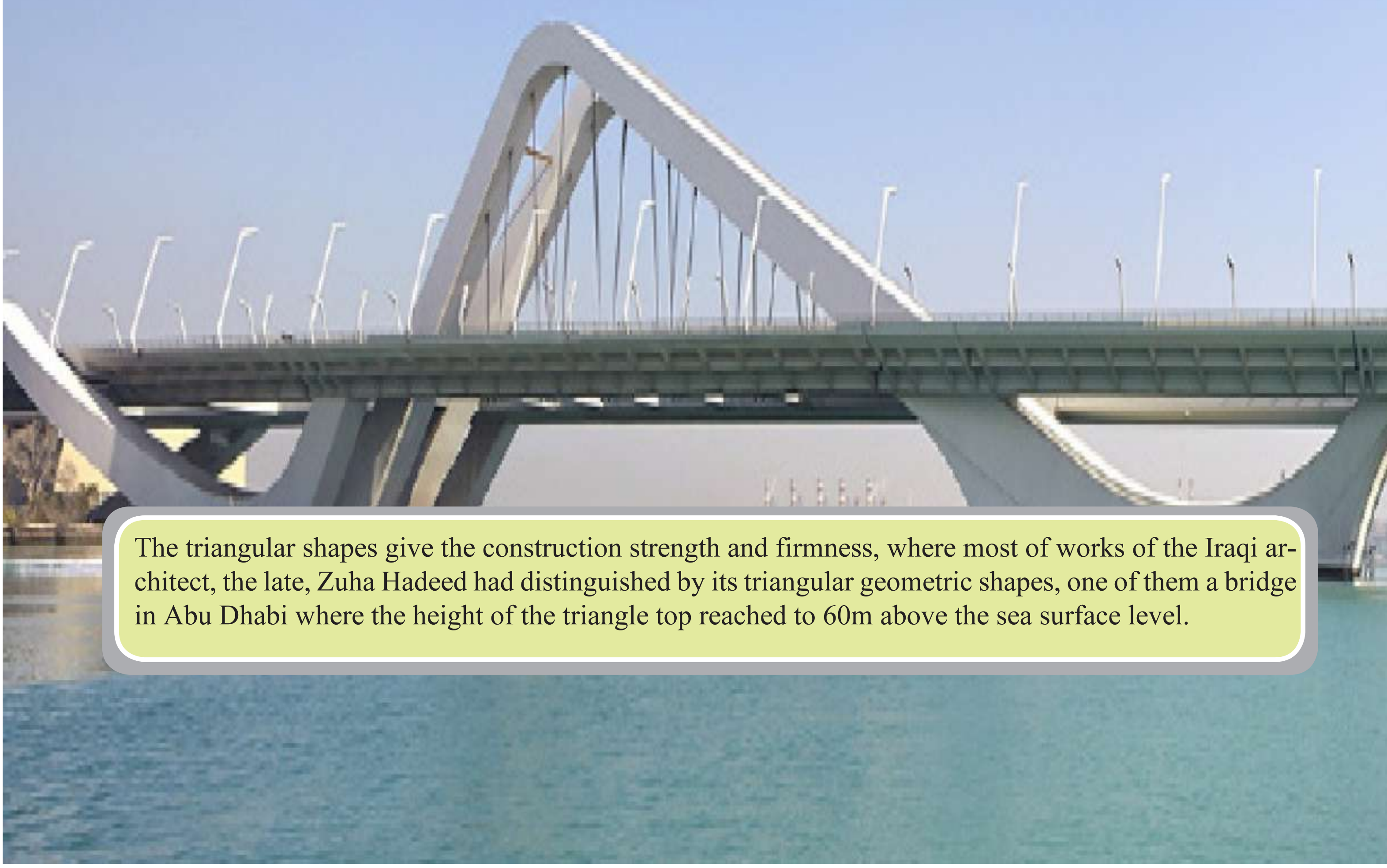
$2y-3x=8$.

6 The triangle ABC has a right- angle in B, if $\sin A = \frac{1}{2}$, find the:

i) cos A ii) tan A iii) cot C iv) sec A

Chapter 5

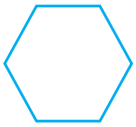
Geometry and Measurement

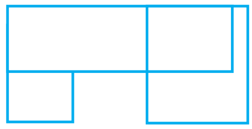


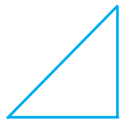
The triangular shapes give the construction strength and firmness, where most of works of the Iraqi architect, the late, Zuha Hadeed had distinguished by its triangular geometric shapes, one of them a bridge in Abu Dhabi where the height of the triangle top reached to 60m above the sea surface level.

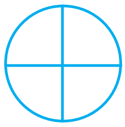
Pretest

Determine if the figure is polygon or not, and if it is a polygon, then, is it a regular polygon or irregular one?

1 

2 

3 

4 

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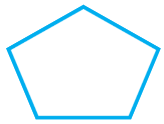
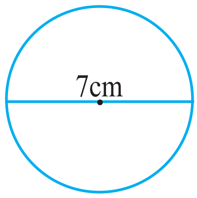
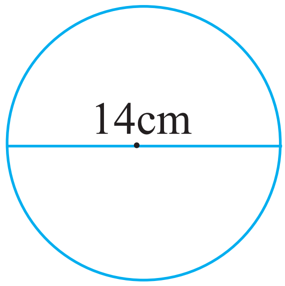
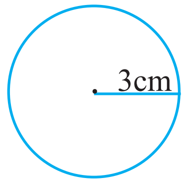
6 

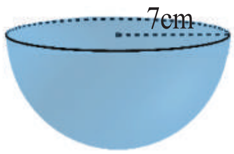
Find the area and the perimeter of each of the following circles:

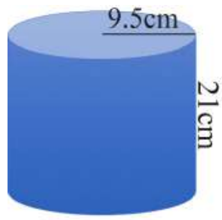
7 

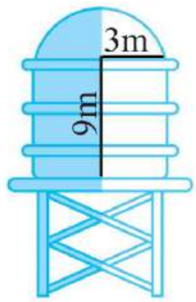
8 

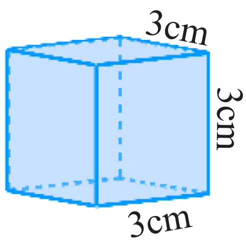
9 

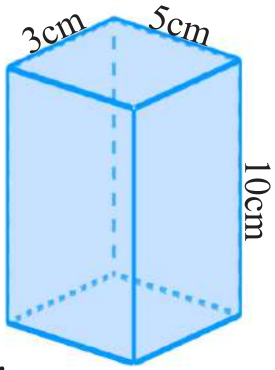
Find the surface area and the volume of each of the following:

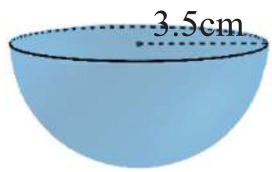
10 

11 

12 

13 

14 

15 

Find the value of x in each of the following:

16 $\frac{7}{6} = \frac{x - 3}{2}$

17 $\frac{7}{x} = \frac{1}{2}$

18 $\frac{3}{16} = \frac{x}{4}$

Find the measure of the central angle and the sum of the interior and exterior angles for each of the following:

19 regular pentagonal

20 regular octagonal

21 regular hexagonal

22 A commercial company includes 20 employees, the ratio of males to females is $\frac{3}{2}$, how many female employees are there? And, how many male employees are there?

23 An equilateral triangle, each side of its three sides equals $(2x - 1)$ cm, and its perimeter is 57 cm, find the value of x and find the length of each side.

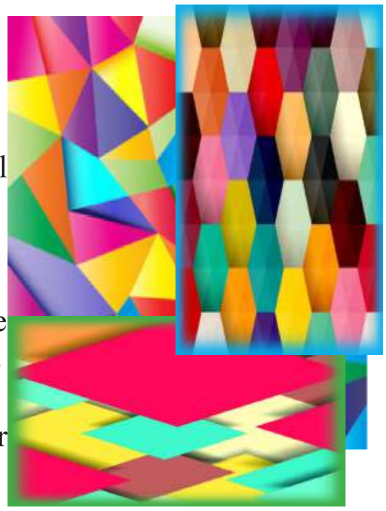
Lesson [5-1] Polygons and Polyhedrons (Pyramid and cone)

Idea of the lesson:
- Finding the perimeter and the area of the regular polygons.
- Finding the volume and the surface area for each of pyramid and cone.

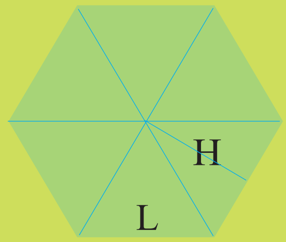
Vocabulary
- Apothem (H)
- Lateral height
- Cone
- Pyramid

Lesson

You have previously learned the regular and irregular polygons and how to find the internal and external of the regular polygon. You have also learned how to find the central angle of the polygon. You have been able to distinguish between the convex and concave polygon.in this lesson you will be able to find the area and the perimeter of the regular polygons.



[5- 1- 1] Regular Polygons

The regular polygon perimeter = the number of sides multiplying by the side length	$P = n \times L$	
The regular polygon area = the triangle area which its vertex is the center of the polygon and its base is the side of the polygon $\times$ the number of its sides.	$A = \frac{1}{2} L \times H \times n$	

If you know that the side length is L and the apothem is H (it is the descending column from the polygon center on one of the polygon sides).
We can calculate the triangle area as follow: The triangle area $= \frac{1}{2} \times$ Base $\times$ height (Apothem),

$$A = \frac{1}{2} L \times H$$

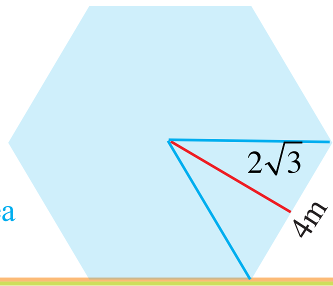
Example (1): Find the perimeter and the area of the regular hexagonal, its side length is 4m and the length of the apothem is $2\sqrt{3}$ m.

$P = n \times L$ — By using the formula of the polygon perimeter

$P = 6 \times 4 = 24m$

$A = \frac{1}{2} L \times H \times n$ — The polygon area — By using the formula of the polygon area

$= \frac{1}{2} \times 4 \times 2\sqrt{3} \times 6 = 24\sqrt{3}$ m^2 — By substituting and simplifying



Example (2): Find the square area which its apothem length is 4 cm

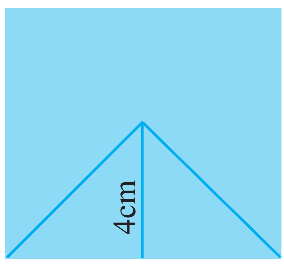
$A = \frac{1}{2} L \times H \times n$ — Method (1) by using the formula of the regular polygon area.

$L = 4 \times 2 = 8cm$ — The length of the square side

$A = \frac{1}{2} \times 8 \times 4 \times 4 = 64cm^2$

$A = L \times L$ — Method (2) by using the formula of the square area (the side length x itself)

$A = 8 \times 8 = 64cm^2$



[5-1-2] pyramid and Cone

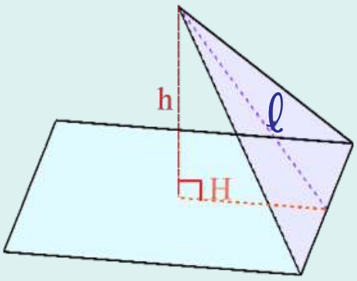
Pyramid: Is a polyhedron which has, at least , three triangular faces and one base represents a polygon figure (the shape of the base determines the name of the pyramid)

h = height

H = Apothem

ℓ = lateral height (slant height)

$\ell^2 = h^2 + H^2$



Cone: Is the polyhedron which has one base represents a circle and has one vertex.

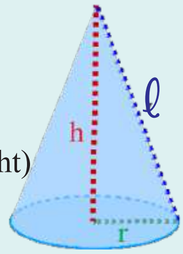
ℓ = the lateral height (slant height)

h = height

r = radius

$\ell^2 = h^2 + r^2$

The total area = the lateral area + the base area



The area formula of the regular pyramid and the right circular cone			The volume of pyramid and cone	
	The regular pyramid	The Right cone	The pyramid volume	$V = \frac{1}{3} b \times h$
Lateral area	$LA = \frac{1}{2} p \times \ell$ The base perimeter p	$LA = \pi r \times \ell$		
Total area	$TA = \frac{1}{2} p \times \ell + b$ The base area b	$TA = \pi r \times \ell + \pi r^2$	The cone volume	$V = \frac{1}{3} \pi r^2 \times h$

Example (3): Find the lateral area and the total area for a regular pyramid which its lateral height is 8 cm and the side length of its square base is 3cm.

$LA = \frac{1}{2} p \times \ell$ **Lateral area**

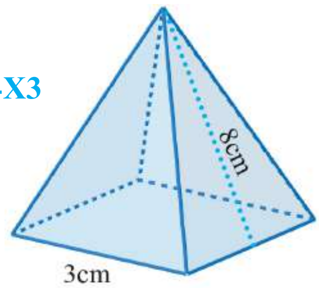
$LA = \frac{1}{2} \times 12 \times 8$ **The base perimeter = the square perimeter = 4X3**

$LA = 48cm^2$ **The total area**

$TA = \frac{1}{2} p \times \ell + b$ **The base area= the square area = 3X3**

$TA = 48 + 9 = 57cm^2$ **The total area**

$TA = 57cm^2$



Example (4): Use the nearby figure to find **i)**The lateral area **ii)**The total area **iii)**The volume

i) $LA = \pi r \times \ell$ **The lateral area of the cone**

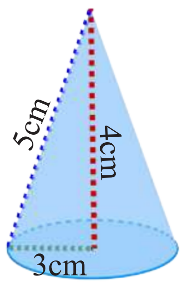
$= \pi \times 3 \times 5 = 15\pi \ cm^2$ **By substituting and simplifying**

ii) $TA = \pi r \times \ell + \pi r^2$ **The total area of the cone**

$= 15\pi + 9\pi = 24\pi cm^2$ **By substituting and simplifying the cone volume**

iii) $V = \frac{1}{3} \pi r^2 \times h$

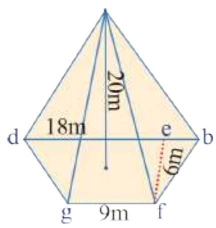
$= \frac{1}{3} \times \pi \times 9 \times 4 = 12\pi \ cm^3$ **By substituting and simplifying**



Example (5): Find the volume of the nearby pyramid

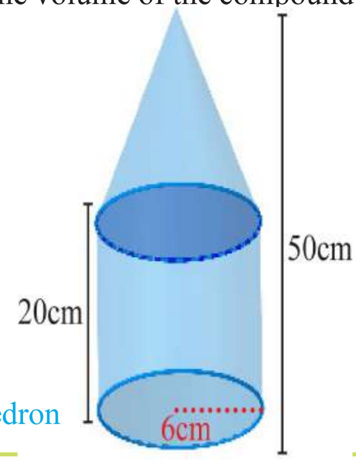
$b = \frac{1}{2}(gf + bd) \times fe = \frac{1}{2}(9 + 18) \times 6 = 81m^2$ **The area of the trapezoid**

$V = \frac{1}{3} b \times h = \frac{1}{3} \times 81 \times 20 = 540m^3$ **The pyramid volume**



Example (6): Find the volume of the nearby compound polyhedron.

To find the volume of the compound polyhedron, we firstly find the volume of the cylinder and the cone, after that, we add the volumes to find the volume of the compound polyhedron.



$V_1 = \pi r^2 h \Rightarrow V_1 = 36\pi \times 20$ The formula of the cylinder volume

$V_1 = 720\pi\ cm^3$ By substituting and simplifying

$V_2 = \frac{1}{3} r^2 \pi \times h$ The formula of the cone volume

$V_2 = \frac{1}{3} \times 36\pi \times 30 = 360\pi\ cm^3$ By substituting and simplifying

$V = V_1 + V_2$

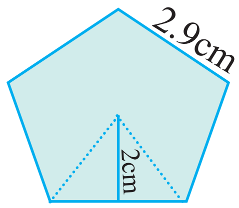
$V = 720\pi + 360\pi = 1080\pi\ cm^3$ The volume of the compound polyhedron

Make sure of your understanding

Find the perimeter and the area for each regular polygon:

Questions 1- 2 are similar to example 1 .

1



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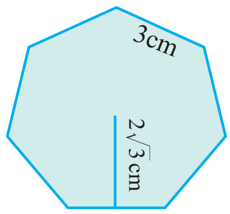


Questions 3 - 4 are similar to examples 6

3 **Find the volume , the lateral area and the total area for each of the following:**

i) Right circule cone: its base area is $225\pi cm^2$, its base perimeter is $30\pi cm$,its height is 20 cm and its lateral height is 25 cm.

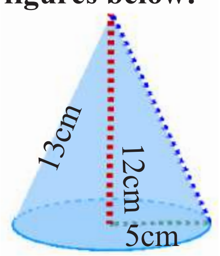
ii) A pyramid; its base area is $54\sqrt{3}\ cm^2$,its perimeter is 36cm , its height is $3\sqrt{6}\ cm$ and its lateral height is 9 cm.

4 **Find the volume and the lateral area and the total area for each of the following:**

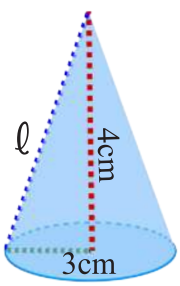
i) A pyramid has a square base, the length of its side is 12 cm, its height is 8 cm and its lateral height is 10cm.

5 **Find the volume and the lateral area and the total area for each of the following by using the figures below:**

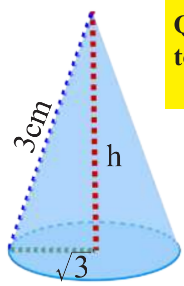
i)



ii)

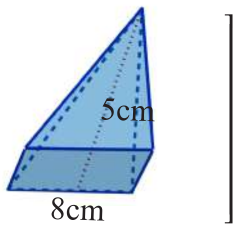


iii)



Questions 6 is similar to example 5.

6 **Find the volume and the lateral area and the total area for each of the following by using the figures below:**



Its base is a square

Solve the Exercises

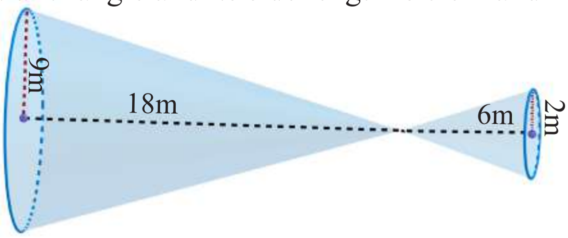
7 Find the lateral area of the pyramid which has a square- shaped base, its side length is 8 cm and its lateral height is 7.2 cm.

8 Find the lateral area of the pyramid which has a base of a regular octahedral polygon, its side length is 1.16 cm and its lateral height is 2 cm.

9 Find the lateral area and the total area of a right circle cone which its diameter is 35m and its lateral height is 20m, and write the answer by π .

10 Find the volume of a pyramid which its base is a regular triangle and its side length is 6 cm and its height is 13cm

11 Find the volume of the nearby figure Compsite form.

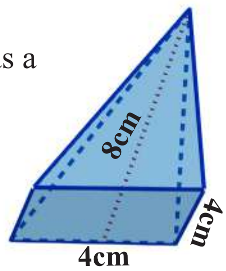


Solve the problems

12 Science: A volcanic model in a shape of a right circle cone,Its radius length is 3 cm, if the model volume is about 203 cm^3, what is its height?

13 Construction: The height of the Arab tower is 321m and it represents an arched pyramid, calculate the approximate area of its base if the pyramid volume which represents it is $1904000m^3$.

14 Geometry: Find the lateral area of the pyramid which has a square-shaped base, as it is shown in the nearby figure.



Think

15 Challenge: A cone and a cylinder have the same base and volume, the cylinder diameter is 40 cm and its height is 7 cm, what is the lateral area of the cone?

16 Discover the mistake: Which of the two solutions is wrong? Clarify your answer.

The first solution

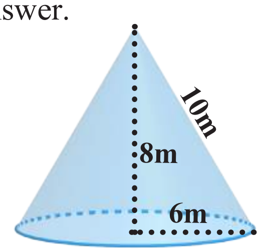
$V = \frac{1}{3} \times b \times h$

$V = \frac{1}{3} \times 36\pi \times 10 = 120\pi m^3$

The second solution

$V = \frac{1}{3} \times b \times h$

$V = \frac{1}{3} \times 6 \times 6 \times \pi \times 8 = 96\pi m^3$



Write

A problem about a regular polygon which its data allows to find the perimeter and the area of the polygon.

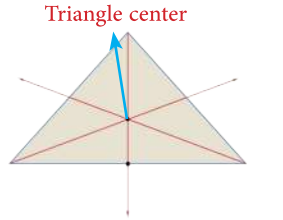
Lesson [5-2] Triangles

Idea of the lesson:
-Identifying the triangles bisections of the angles, and median of a triangle, the similarity of two triangles and using the similarity in solve problems.

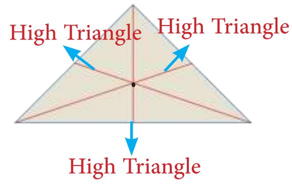
Vocabulary:
•The two similar triangles
•The similarity ratio

Learn

You have previously learned the properties of the triangle. In this lesson, we will learn the median of a triangle: it is a segment, its two ends represent one of the triangle's vertices and the midpoint of the side which is opposite of that vertices. Each triangle has three median intersect in one point which is called the meeting point of the median of the triangle (the triangle center).



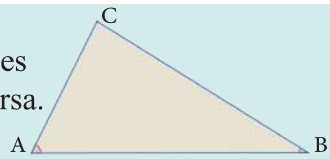
The triangle height: is the vertices descending column from one vertex of the triangles on the line which contains the side which is opposite of that vertice, each triangle has three heights which are intersected in one point called (the meeting point of the heights)



[5-2-1] Sides and Angles in the triangle

(theorems without reasoning)
A theorem **:** If two sides of a triangle were variated, then the two opposite angles will be variated too, the great one will be opposite of the great side and vice versa.

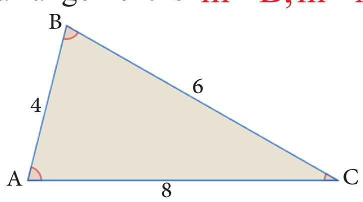
$$BC > AC \Leftrightarrow m\angle A > m\angle B$$



Example(1)

i) In the triangle below, arrange the angles from the smallest to the greatest.

The shortest side AB, then the small angle is $\angle C$.
The longest side AC, then the great angle is $\angle B$.
The arrangement is $m\angle B, m\angle A, m\angle C$



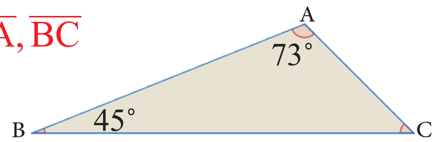
ii) In the triangle below, arrange the sides from the shortest to the longest and calculate the measure of $\angle C$.

$m\angle A + m\angle B + m\angle C = 180^\circ$

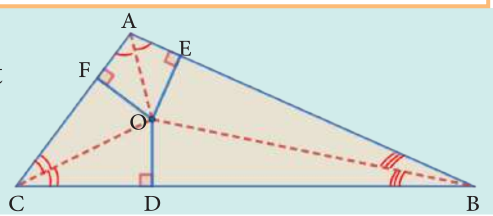
$m\angle C = 180^\circ - (73^\circ + 45^\circ) = 62^\circ$

$\therefore m\angle B < m\angle C < m\angle A$

$\overline{AC}, \overline{BA}, \overline{BC}$

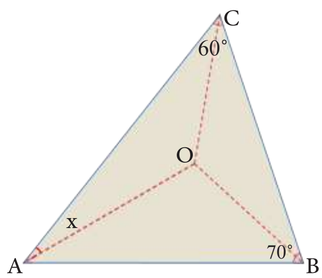


(theorems without reasoning) in each triangle:
A theorem : The bisectors of the triangles angles meet in one point which is in equidistant from its sides.
If $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, are bisectors of the angles A ,B,C respectively, they meet in the point O , then OD= OE = OF



Example(2)

In the nearby triangle, find the value of x.
$\overline{BO}$ bisectors $\angle B$, $\overline{CO}$ bisects $\angle C$, then O is the meeting point of the bisectors of the angles of the triangle ABC
($\overline{AO}$ bisects $\angle A$) $x = \frac{1}{2} m\angle A$ theorem
The sum of the triangle angles $m\angle A + m\angle B + m\angle C = 180^\circ$
$m\angle A = 180^\circ - (70^\circ + 60^\circ) = 50^\circ \Rightarrow$ $\therefore x = 25^\circ$

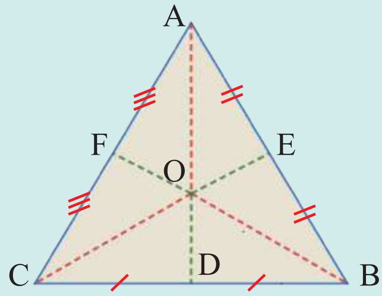


(theorems without reasoning) in each triangle:

A theorem: The medians of the triangle are meet in one point called the center of triangle gravity. It divides each of them in a ration of $\frac{2}{3}$ from the vertices.to tenmid point of the side which is opposite of the vertices.

$$AO = \frac{2}{3}AD, BO = \frac{2}{3}BF, CO = \frac{2}{3}CE$$

$$OD = \frac{1}{3}AD, OF = \frac{1}{3}BF, OE = \frac{1}{3}CE$$



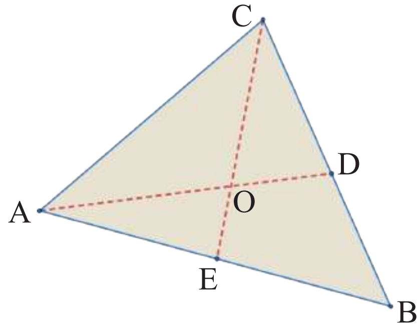
Example(3) The triangle ABC in which the two medians $\overline{AD}, \overline{CE}$ meet in the point O, CE=9 cm ,AD=6 cm, find the length of $\overline{AO}, \overline{OE}$

$OE = \frac{1}{3}CE$

$\therefore OE = \frac{1}{3} \times 9 = 3cm$ A median of $\overline{CE}$

$\because OA = \frac{2}{3}AD$ also a median of $\overline{AD}$

$\therefore OA = \frac{2}{3} \times 6 = 4cm$

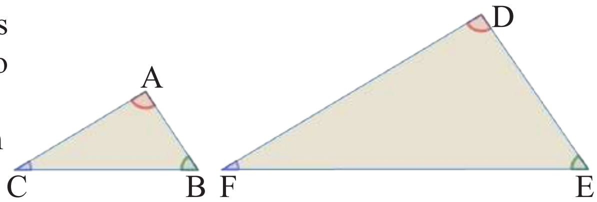


[5-2-2] Similarity of Triangles

The two similar triangles: are triangles which their angles are congruent, and their sides are proportioned we refer to the similarity by $(\sim)$ theorems without reasoning.

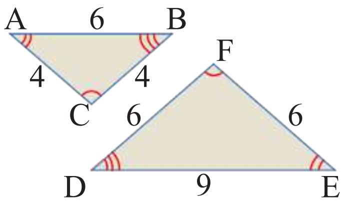
A theorem: If two angles in a triangle are congruent with two angles in another triangle, then the two triangles are similar. $m\angle A = m\angle D, m\angle C = m\angle F, \Rightarrow \triangle ABC \sim \triangle DEF$

A theorem : If three sides of a triangle are proportioned with three sides of another triangle, then the two triangles are similar.



Example(4) **Show if the two triangles in the nearby figure are similar, and write the ratio of the similarity.**

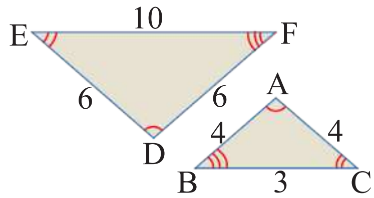
i)



$$\frac{AB}{DE} = \frac{6}{9} = \frac{2}{3}$$
$$\frac{AC}{EF} = \frac{4}{6} = \frac{2}{3}$$
$$\frac{BC}{FD} = \frac{4}{6} = \frac{2}{3}$$

Then the two triangles are similar

ii)



$$\frac{BC}{EF} = \frac{3}{10}$$
$$\frac{AB}{DF} = \frac{4}{6} = \frac{2}{3}$$
$$\frac{BC}{EF} \neq \frac{AB}{DF}$$

Then the two triangles are not similar

A theorem: If two sides of atringle the angle were proportioned with the two sides of another triangle and the angle between then is congruent with the angle of the other triangle and the sides, then the two triangles are similar.

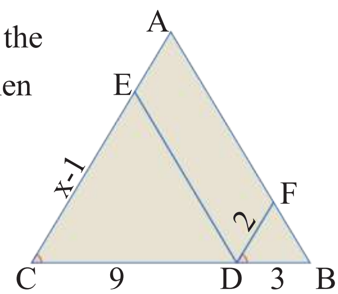
Example(5)

In the nearby figure: if ($m < C = m < FDB, \frac{EC}{FD} = \frac{CD}{DB}$) find the value of x. Since the two triangles BFD,DEC are similar, then their symmetric sides are proportion

$\frac{x - 1}{2} = \frac{9}{3}$ Proportion

$3x - 3 = 18$ Commutative multiplication

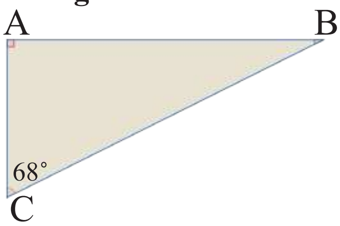
$3x = 21 \Rightarrow x = 7$ Simplifying



Make sure of your understanding

Questions 1- 4 are similar to example1

1 **Arrange the sides from the shortest to the longest:**



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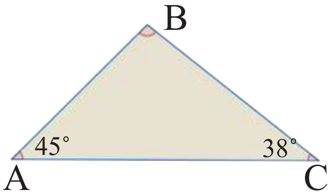
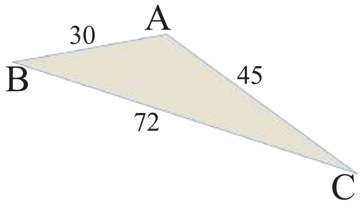


3 **Arrange the angles from the smallest to the greatest:**



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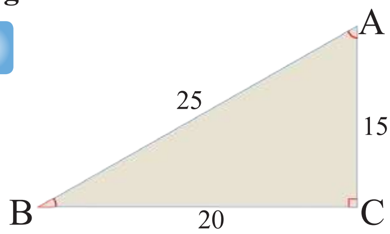
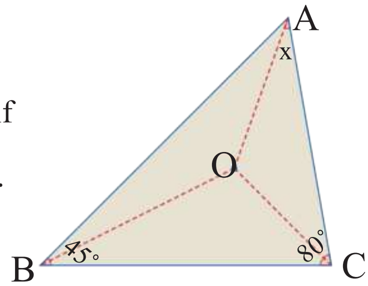


Question 5 is similar to example 2

5 In the nearby triangle, if $\overline{AO},\overline{BO},\overline{CO}$ are bisectors of the angles A,B,C find $m\angle X$.



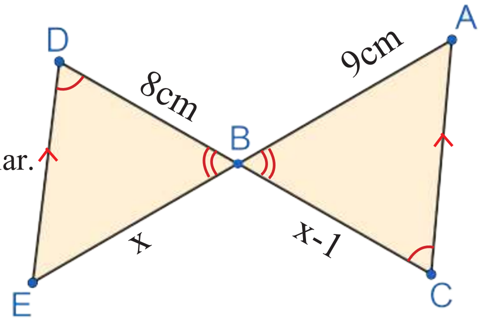
6 ABC is a triangle, O represent a point of intersection of its median, if BO =12cm,
Find the length of the median which one of its to ends is the point B.

7 In the triangle ABC, O is the point of meeting the medians, find the length of $\overline{AD}$ if your knew that

$.m\angle COB = 90^\circ, \overline{AO} \cap \overline{BC} = \{D\}, BC = 6cm$

Questions 6 - 7 are similar to example 3

8 **In the nearby figure:**
Show that:
i) The two triangles ABC,BDE are similar.
ii) Find the value of x.
iii)Find the ratio of the similarity.

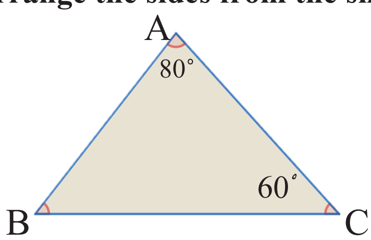


Question 8 is similar to examples 4,5

Solve the Exercises

Arrange the sides from the shortest to the longest:

9



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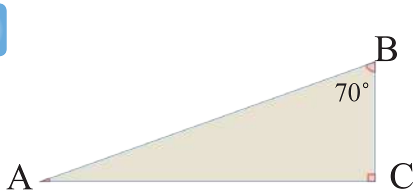
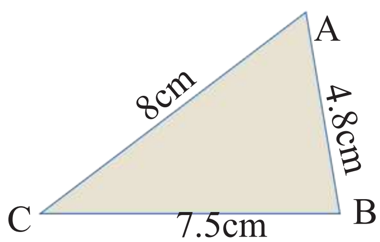


Arrange the angles from the smallest to the greatest:

11



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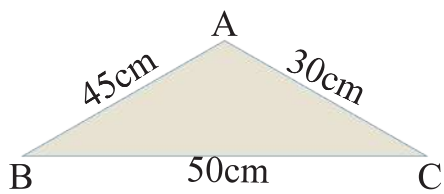
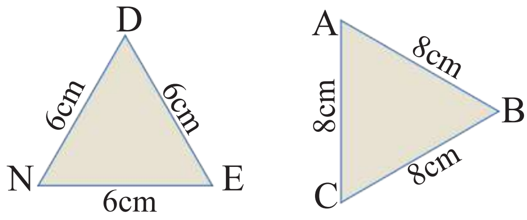
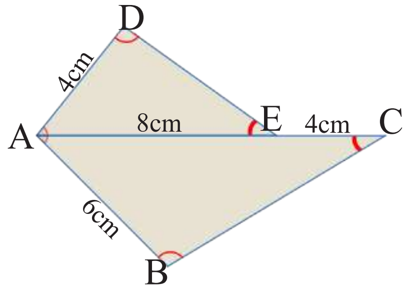


13 Show if the two triangle ABC, DEN, in the nearby figure, are similar, and write the ratio of similarity.

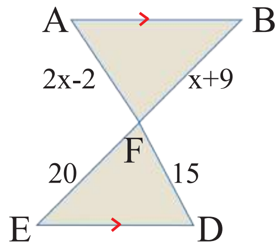


14 Show if the two triangles ABC , ADE, in the nearby figure, are similar, and write the ratio of the similarity .



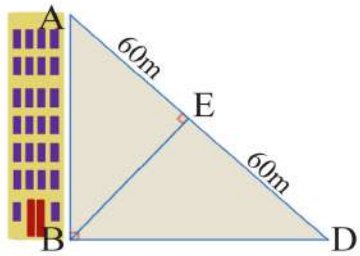
Solve the problems

15 Geometry: If you knew that Δ ABF ~ Δ DEF , $\overline{AB} // \overline{ED}$ use the information in the nearby figure to find the value of x.

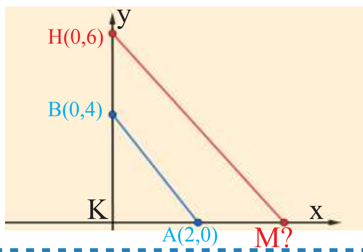


16 Building: A building which its height represented by a side right_ angled triangle as it is shown BE is the hight of tringle ABD in the nearby figure, prove that : i) ∠ EBA ≅ ∠ D

ii) ΔABE ~ Δ DBE

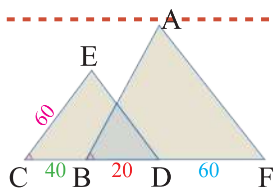


17 In the nearby figure, the two triangle KAB, KMH are similar, find the two coordinates of M and the ratio of the similarity.



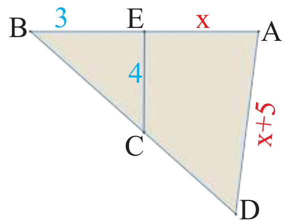
Think

18 Discover: What is the length of $\overline{AB}$ in the nearby figure? If you knew that ΔECD ~ ΔABF .



19 Challenge: (x,15,6) and (10,5,2) are the lengths of the corresponding sides in two similar triangles, what is the value of x?

20 Numerical sense: Find the value of x in the nearby figure.If the two tringle ABD , EBC are similar EC // ED



21 Opened problem: Illustrate why we need the measurements of the angles to be sure of the similarity of the triangles, give an example.

write

A problem about two isosceles triangles in which the two angles of the vertices are congruent, and find the ratio of the similarity.

Lesson [5-3] Proportion and Measure in Triangles

Idea of the lesson:
- Using the proportioned parts in the triangles to prove the parallel of two or more of lines.
- Using the proportion to find unknown measures
- Using the geometric proportion in the coordinate plane

Vocabulary:
Geometric proportion

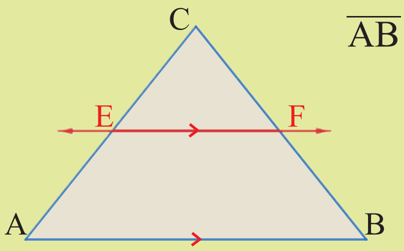
Learn

The planning of cities and streets in the mapp of the electronic appliance map include parallel and perpendicular lines, the lateral planning represents part of Baghdad city, we see that the streets are paralled and perpendicular.

[5-3-1] Proportion in Triangles

You have previously learned the similar triangles and some of the theorems of the similarity in the triangles, you will learn, in this term, the proportion in the triangle using the previous theorems.

The triangular proportion theorem

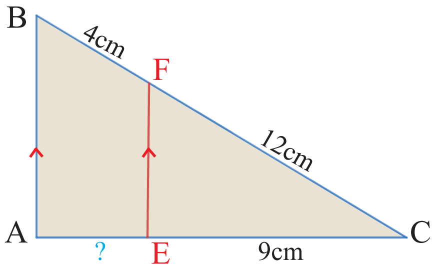
The theorem	Given	The result
If a line parallels a side of a triangle and intersects the other two sides in two different point.it divides the two sides into segments which are proportional in (without reasoning)	 $\overline{AB} \parallel \overline{EF}$	$\frac{CE}{EA} = \frac{CF}{FB}$

Example(1) Find the length of the segment AE, if you knew that $\overline{AB} \parallel \overline{EF}$ in the nearby figure.

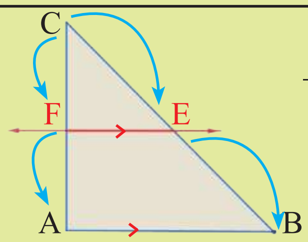
$$\frac{CE}{EA} = \frac{CF}{FB}$$

$$\frac{9}{EA} = \frac{12}{4} \Rightarrow EA = \frac{4 \times 9}{12} = \frac{36}{12} = 3cm$$

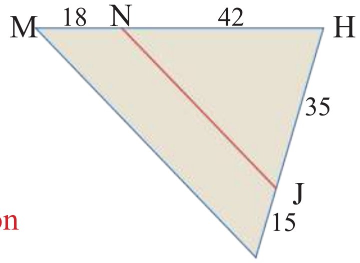
A theorem of the triangular proportion the substitution and simplifying



Converse the theorem of the triangular proportion

The theorem	Given	The result
If a line divided two sides of a triangle into proportioned segments, then it will be parallel to the third side (without reasoning)	 $\frac{CE}{EB} = \frac{CF}{FA}$	$\overline{EF} \parallel \overline{AB}$

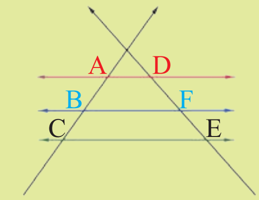
Example(2) In the nearby figure, prove $\overline{MK} \parallel \overline{NJ}$.
we find the ratio of the proportionl parts



$$\frac{HJ}{JK} = \frac{35}{15} = \frac{7}{3}, \frac{HN}{NM} = \frac{42}{18} = \frac{7}{3}$$

$$\because \frac{HJ}{JK} = \frac{HN}{NM} = \frac{7}{3}$$

$\therefore \overline{MK} \parallel \overline{NJ}$ converse the theorem of the triangular proportion

Thales theorem		
The theorem	Given	The result
If three or more parallel lines were intersected by two lines, then the segments which are intercepted by the parallel lines will be proportional	 $\overleftrightarrow{AD} \parallel \overleftrightarrow{BF} \parallel \overleftrightarrow{CE}$	$\frac{AB}{BC} = \frac{DF}{FE}$

Example (3):

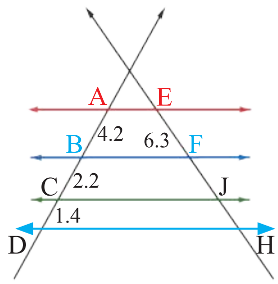
An architect had used the perspective (which is the drawing of the remote bodies where they seem small, and the near bodies where they seem great, and maintaining their structure and the proportion of their measures to seem three-dimensions) to draw initial lines to help him in drawing parallel columns of communication, he checked his drawing in measuring the distance between the columns, how long FH?

$\overleftrightarrow{AE} \parallel \overleftrightarrow{BF} \parallel \overleftrightarrow{CJ} \parallel \overline{DH}$ Thale's theorem

$$\frac{AB}{BD} = \frac{EF}{FH}$$

$BD = BC + CD = 2.2 + 1.4 = 3.6m$ By substituting and simplifying

$$\frac{4.2}{3.6} = \frac{6.3}{FH} \Rightarrow FH = \frac{6.3 \times 3.6}{4.2} = 5.4m$$



[5-3-2] Proportion and Measures

To find the ratio of the two perimeters and the ratio of the two areas of two similar triangles, I can use the following theorem (without reasoning)

A theorem: if two triangles are similar in ratio of $\frac{a}{b}$, then the ratio of the two perimeters of the two triangles equals $\frac{a}{b}$ and the ratio of the two triangle area equals $\frac{a^2}{b^2}$.

If the two triangles are similar, then the ratio between their two perimeters equals The ratio between the length of the corresponding.

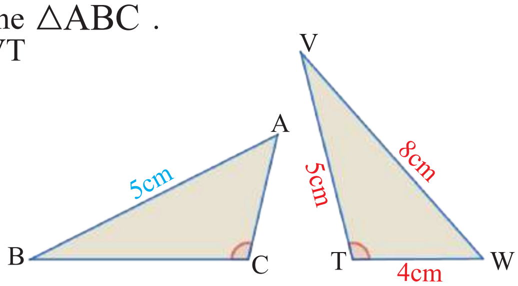
Example(4) If $\triangle WVT \sim \triangle ABC$, find the perimeter of the $\triangle ABC$.
Assume P_1 is the perimeter of the triangle WVT

$P_1 = 8 + 5 + 4 = 17cm$

Use the proportion to find the perimeter of the triangle ABC assume P_2 is the perimeter of The triangle ABC

$$\frac{P_2}{P_1} = \frac{AB}{WV} \Rightarrow \frac{P_2}{17} = \frac{5}{8}$$

$\therefore P_2 = 10.625cm$

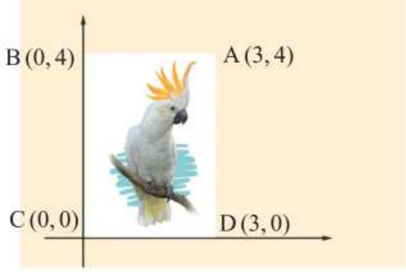


You have previously learned three geometric transformations: the reraetiax with drawaill, the reflection and rotation, these transportations maintain the structure and the measures. In this lesson, you will learn new transformation maintain the structure without the measures, it is the dilation.

[5-3-3] Dilation in the Coordinate Plane

Dilation: is a transformation which change the measures of the geometric shapes without changing their structure. In the dilation, the preimage and its image will always be similar, the proportion center is the origin.
In this lesson, we will only study the dilation in the coordinate plane, if you deal with a dilation with sacale factor M, you will be able to find the image of the point by multiplying its coordinates by M
$(x, y) \rightarrow (Mx, My)$

Example(5) the nearby figure shows a position of a picture in the internet, draw the limits of the picture after transform it in a dilation its ratio is $\frac{5}{3}$



The first step:
Multiplying the factor of the dilation by the coordinates of the vertices.

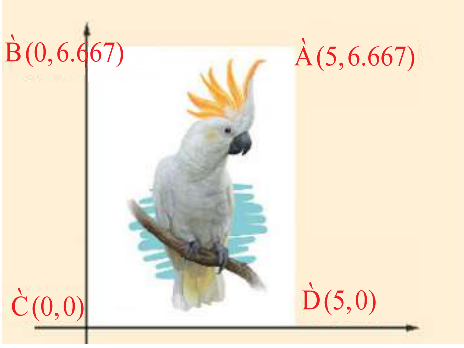
$A(3,4) \rightarrow (\frac{5}{3} \times 3, \frac{5}{3} \times 4) \longrightarrow \grave{A}(5, 6.667)$

$B(0,4) \rightarrow (\frac{5}{3} \times 0, \frac{5}{3} \times 4) \longrightarrow \grave{B}(0, 6.667)$

$C(0,0) \rightarrow (\frac{5}{3} \times 0, \frac{5}{3} \times 0) \longrightarrow \grave{C}(0, 0)$

$D(3,0) \rightarrow (\frac{5}{3} \times 3, \frac{5}{3} \times 0) \longrightarrow \grave{D}(5, 0)$

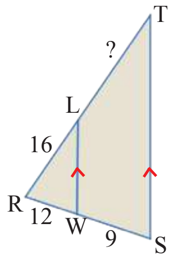
The second step: put the points $\grave{A}, \grave{B}, \grave{C}, \grave{D}$ on the coordinate plane, and then connecting them to get the rectangle $\grave{A}\ \grave{B}\ \grave{C}\ \grave{D}$



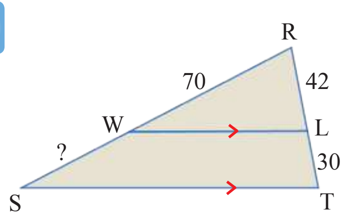
Make sure of your understanding

Find the length of the unknown segment in the following figures:

1



2

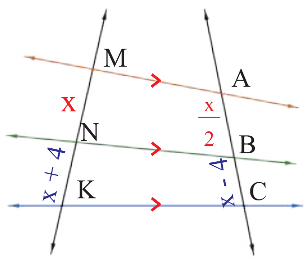


Questions 1 - 2 are similar to examples 1- 3

Questions 3 - 4 are similar to example 2

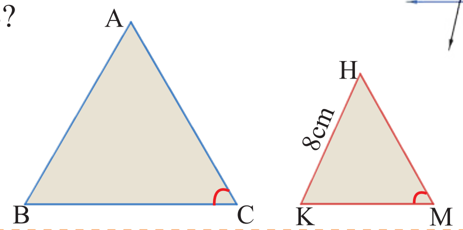
3 In the triangle $MQP, MQ = 12.5, MR = 4.5, MP = 25, MN = 9$, does $\overline{\mathbf{RN}} // \overline{\mathbf{QP}}$ or not? Explain your answer. $N \in \overline{MP}$, $R \in \overline{MQ}$

4 In the nearby figure, find the length of $\overline{KN}, \overline{MN}$.



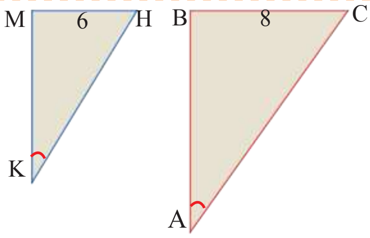
Question 4 is similar to example 3

5 The two triangles ABC,HKM are similar, the area of $\triangle ABC$ is the double of the area of $\triangle HKM$, what is the length of $\overline{AB}$?



Questions 5 , 6 are similar to examples 4,5

6 The two triangles ABC, KMH are similar, find the area and the perimeter of the triangle ABC, it should be taken in consideration that the perimeter of the triangle KMH equals18 cm and its area 15 cm^2.

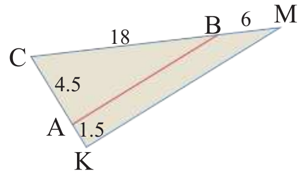


7 ABC is a triangle,where $A(6,0), B(-3,\frac{3}{2}), C(3,-6)$, find its image after minimizing it in $\frac{1}{3}$,it is worth to be mentioned that the proportion center is the origion.

Question 6 is similar to example 4

Question 7 is similar to example 5

Solve the Exercises



8 In the triangle ACD, $\overline{BE} \parallel \overline{CD}$, find the value of x and $\overline{ED}$ if $ED = 3x - 3, BC = 8, AE = 3, AB = 2$.

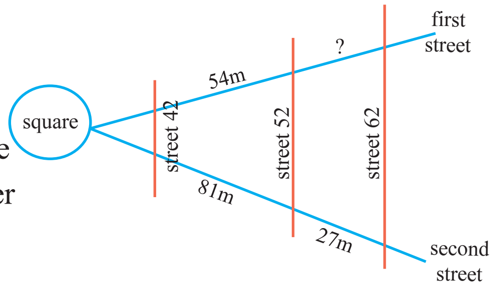
9 Determine if $\overline{AB} \parallel \overline{MK}$ in the nearby figure.

10 The area ratio of the triangle ABC to the area ratio of the triangle KMH equals $\frac{16}{25}$, what is the ratio of the similarity between the two triangles, and what is the ratio of the similarity between their two perimeters.

11 Find the image of the triangle ABC, where $A(-1,-1), B(1,-2), C(1,2)$ under the effect of the proportion of its factor 2.

Solve the problems

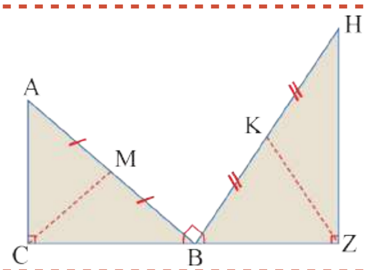
12 **Roads:** The nearby map represents some of the parallel streets and two ways crossing them, what is the length of the first way between the street number 62 and the street number 52?



13 **Geometry:** Find the image of the quadrangle, where $A(2,6), B(-4,0), C(-4,-8), D(-2,-12)$ under the effect of the proportion of its factor $\frac{1}{4}$.

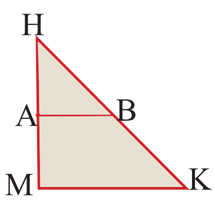
Think

14 **Challenge:** In the nearby figure, M is the midpoint of $\overline{AB}$, and K is the midpoint of $\overline{HB}$, the angles $\angle Z, \angle ABH, \angle C$ are right angles, prove that $\left(\frac{KZ}{CM}\right)^2 = \frac{(BZ)^2 + (ZH)^2}{(BC)^2 + (CA)^2}$



Write

As you can of the dilations, if you knew that $\overline{MK} \parallel \overline{AB}$ in the near by figure.



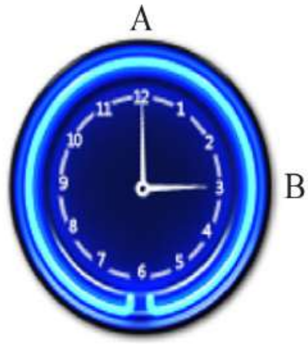
Lesson [5-4] The Circle

idea of the lesson:
- Finding the measure of the arcs and the centeral angles of the circles.
- Indentifying the tangent and the common tangent

Vocabulary:
- Arc chord
- Tangent, common tangent
- Central angles

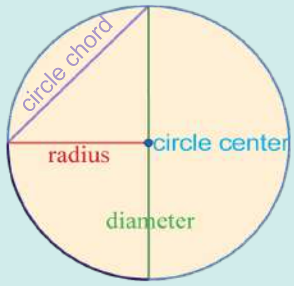
Learn

Every angle between clock hand is a central angle. The central angle is the angle which intersects the circle in two points and its vertex is the center of the circle, and every central angle in a circle, an arch would be in front of it, it is called the angle's arc. What is the measure of $\overset{\frown}{AB}$ which is in front of $\angle AOB$? Are there many types of arcs?

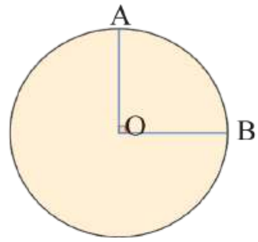


[5-4-1] Arch and chord

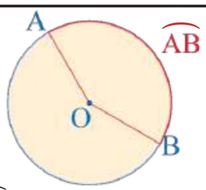
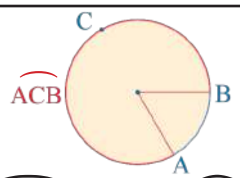
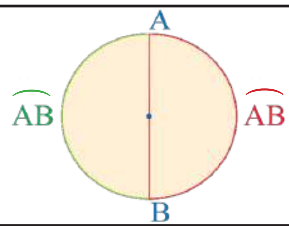
You have previously learned the concept of the circle: it is a set of the connected points in the plane which have the same distance from a fixed point called the circle center, and the radius r : is a segment connects between the circle center and a point on the circle, and the circle chord: is a segment which its two ends are on the circle, and the diameter : is a chord passes through the circle center.
In this lesson, you will increase your information by identifying the arc and its measure by using the central angle which is in front of it.



Example(1) How can I find the measure of the arc $\overset{\frown}{AB}$ by using the central angle which is in front of it?



The measure of the central angle is equivalent to the arc measure which is in front of it, and we refer to the arc $\overset{\frown}{AB}$
The angle AOB is a right angle $m\angle AOB = 90^\circ$
Then the measure of the arc which is in front of the angle AOB equals $\overset{\frown}{AB} = 90^\circ$
There are three types of the arcs in the circle:

The smallest arc (less than 180)	The greatest arc (greater than 180)	Measure of the half of the circle (equals 180)
 $m\ \overset{\frown}{ACB} = m\angle AOB \angle 180$	 $m\overset{\frown}{ACB} = 360 - m\overset{\frown}{AB} > 180$	 $m\overset{\frown}{AB} = 180$

Example(2) **Find the measure of the angles and the unknown arcs in the nearby figure:**

i) $\overset{\frown}{BC}: m\angle BOC = 30^\circ \Rightarrow m\overset{\frown}{BC} = 30$

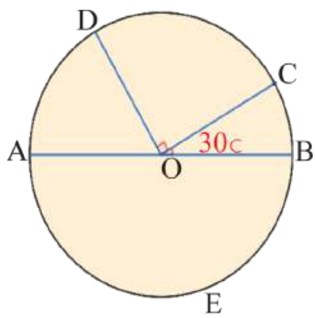
ii) $\overset{\frown}{DC}: m\angle COD = 90^\circ \Rightarrow m\overset{\frown}{DC} = 90$

iii) $\overset{\frown}{BCD}: m\angle BOC + m\angle COD = 30^\circ + 90^\circ = 120^\circ$

$m\overset{\frown}{BCD} = 120$

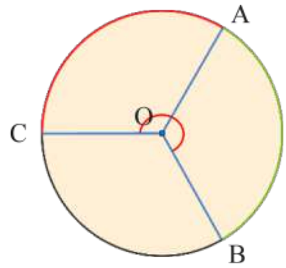
iv) $\overset{\frown}{BEA}: m\angle BOA = 180^\circ \Rightarrow m\overset{\frown}{BEA} = 180$

v) $\overset{\frown}{AD}: m\angle AOD = 180^\circ - 120^\circ = 60^\circ \Rightarrow m\overset{\frown}{AD} = 60$



Example (3): **The nearby circle is divided into three congruent parts, find the measure of the following arcs:**

There are three congruent centeral angles, its sum is 360^0:

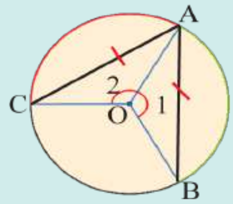


i) $\widehat{AB}$: $m\angle AOB = \frac{360^\circ}{3} = 120^\circ \Longrightarrow \widehat{AB} = 120$

ii) $\widehat{ABC}$: $m\angle ABC = 120^\circ + 120^\circ = 240^\circ \Rightarrow \widehat{ABC} = 240$

or other method $\widehat{ABC} = 360 - 120 = 240 \Rightarrow \widehat{ABC} = 240$

Notice the two triangles and the two central angles 1,2 and the two arcs $\widehat{AB}, \widehat{CA}$ and the two chords $\overline{AB}, \overline{CA}$,if the two angles are congruent, then the two arcs will be congruent and the two triangles will be congruent, so the two chords $\overline{AB}, \overline{CA}$ will be congruent. You can use a method like this to get the following theorem. (without reasoning).



The theorem of arcs, chords and the central angles in each circle or in two congruent circles.	
If two central angles are congruent, then their two chords will be congruent and vice versa.	$\angle 1 \cong \angle 2 \Leftrightarrow \overline{AB} \cong \overline{AC}$
If two central angles are congruent, then their two arcs will be congruent and vice versa.	$\angle 1 \cong \angle 2 \Leftrightarrow \widehat{AB} \cong \widehat{AC}$
If two arcs are congruent, then their two chords will be congruent and vice versa.	$\overline{AB} \cong \overline{AC} \Leftrightarrow \widehat{AB} \cong \widehat{AC}$

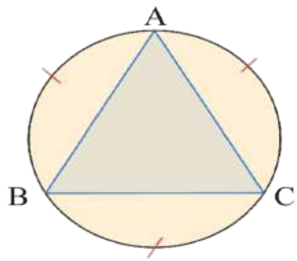
Example (4): Use the theorem of the arcs and the chords to prove that the triangle ABC is an equilateral triangle in the nearby circle, it is worth to be mentioned that:

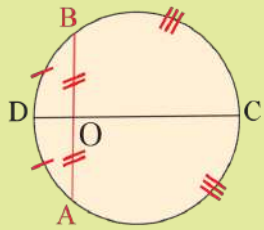
$\widehat{AB} \cong \widehat{AC} \cong \widehat{CB}$

Given from the question $\quad \because \widehat{AB} \cong \widehat{AC} \cong \widehat{CB}$

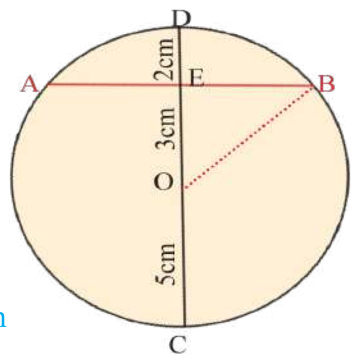
The theorem of the arcs and the chords $\quad \therefore \overline{AB} \cong \overline{AC} \cong \overline{CB}$

So, the triangle ABC is an equilateral triangle.



A theorem of the perpendicular diameter in each circle	
A theorem: The perpendicular diameter to a chord in a circle bisects the chord and the two arcs. $\overline{CD} \perp \overline{AB} \Rightarrow AO = BO, \widehat{AD} \cong \widehat{DB}, \widehat{BC} \cong \widehat{AC}$	

Example (5): Use the theorem of the perpendicular diameter to find the chord AB,if you knew that the radius OD equals 5 cm and that DE= 2cm



$OC = OD = 5cm, DE = 2cm$ — The step (1): draw the radius $\overline{OC}$

$OE = 5 - 2 = 3cm$ — The step (2):

$(EB)^2 + (EO)^2 = (OB)^2$ — Pythagorean theorem by substitution

$25 - 9 = (EB)^2$ — By simplifying

$(EB)^2 = 16 \Rightarrow EB = 4cm$ — E is the midpoint of $\overline{AB}$, the theorem of the perpendicular diameter

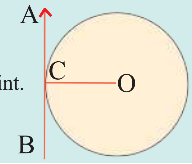
$\therefore AB = 2 \times EB = 2 \times 4 = 8cm$ — The diameter $\overline{DC}$ is perpendicular on the chord $\overline{AB}$ and bisects it.

[5-4-2] Tangent

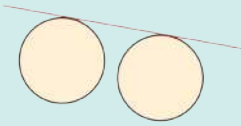
The circle tangent: is the line which meets the circle in one point and it is vertically to the radius in the tangency point.

The tangency theorem

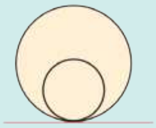
tangency point.



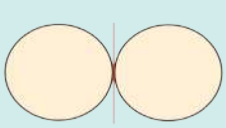
The common tangent of two circles :is tangent line for the two circles.



common tangent



Internal Common Tangent



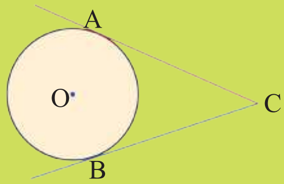
External Common Tangent

The theorem of the two tangents

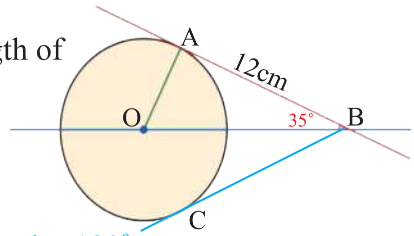
A theorem: The two segments of the tengents which are drawn to a circle from outside point are identical.

$\overline{CB}, \overline{CA}$ are two tengents to the circle from the point C.

$\therefore \overline{CB} \cong \overline{CA}$



Example (6): A circle, in the nearby figure, its center is O, and $\overline{AB}$ is a tangent of the circle at A and the measure of the angle ABO equals 35^0, find the measure of the angle AOB, then find the length of the segment BC.



$\overline{AB} \perp \overline{AO}$, $m\angle OAB = 90°$ — $\overline{BA}$ is the circle tangent at the point A

The tangent theorem

$\because m\angle OBA = 35°$ — Given

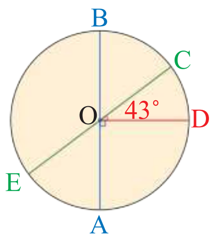
$\therefore m\angle AOB = 180° - (90° + 35°) = 55°$ — The sum of the triangle angles 180^0

BC=12cm — The theorem of the two tengents

Make sure of your understanding

In the circle below, find the measure of the angles and the arcs in each of the following:

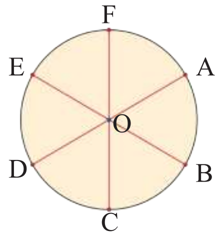
1. $\angle AOD$
2. $\angle COB$
3. $\overset{\frown}{DBE}$
4. $\overset{\frown}{DAB}$



Questions 1- 4 are similar to examples 1- 2

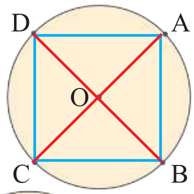
A circle is divided into 6 identical parts, find the measure each arc of the following:

5. $\overset{\frown}{AB}$
6. $\overset{\frown}{ABC}$
7. $\overset{\frown}{ABD}$

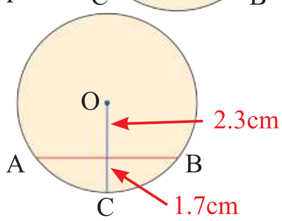


Questions 5- 7 are similar to example 3

Questio 8 is similar to example 4

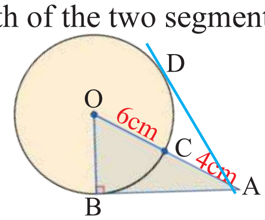


8. The nearby circle is divided into 4 identical parts, prove that the figure ABCD is a square .
9. In the nearby figure, use the theorem of the perpendicular diameter, and find the length of the segment AB in the nearby circle. Approximate the result to the nearest tenth.



10. Use the tangent theorem to find the length of the two segments AB, AD in the nearby figure.

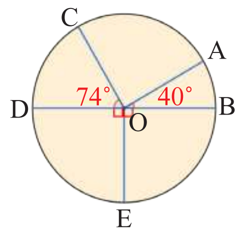
Question 9 is similar to example 5



Question 10 is similar to example 6

Solve the Exercises

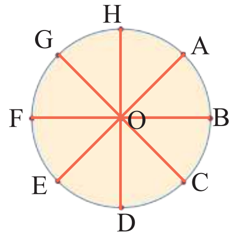
Find the measure of the angles and the arcs in the following:



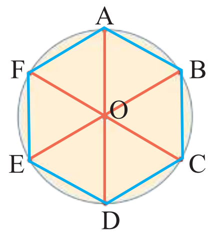
11 $\angle COA$ 12 $\overset{\frown}{DBE}$

13 $\overset{\frown}{BAC}$ 14 $\overset{\frown}{DCA}$

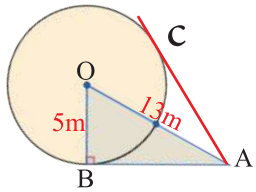
The circle is divided into 8 identical parts, find the measure of each arc in the following:



15 $\overset{\frown}{AB}$ 16 $\overset{\frown}{ABC}$ 17 $\overset{\frown}{GDB}$



18 The nearby circle is divided into 6 identical parts, prove that the figure ABCDEF is a regular hexagonal.



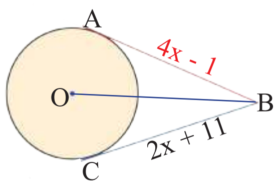
19 Use the theorem of the tangent to find the length of the two line segments AB,AC in the nearby circle.

Solve the problems

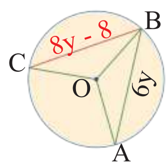
20 Geography (volcanos): The vent of the Hulaii volcano rises from the level of the see surface in 2.52 km, calculate the distance between the top of the volcano and the level of the horizon, if you knew that the radius of the land is 6437 km, approximate the result to the nearest kilometer.

21 Space station: Russian station, mir rises from the level of the see surface in about 390 km, what is the distance between the station and the horizon, approximate the result, if you know that the radius of the land is 6437 km to the nearest kilometer.

Think

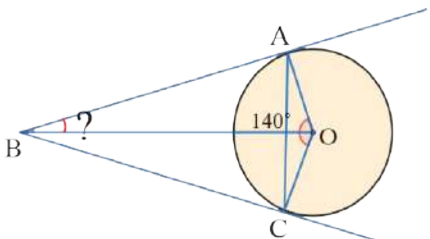


22 Challenge: Use the theorem of the two tangents to find the length of $\overline{AB}$ in the nearby circle.



23 Numerical sense: If the two angles COB,AOB are congruent, find the length of $\overline{CB}$ in the nearby circle.

Write



The required steps to find the measure of the angle ABC in the nearby figure, if you knew that $\overline{BO}$ bisects the angle AOC which its measure equals 140^0

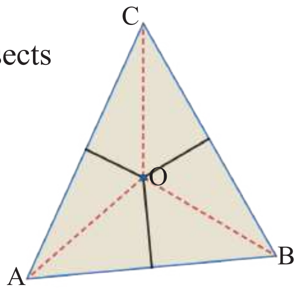
Lesson [5-5] Triangle and Circle, Line Segments and Circle

Idea of the lesson:
-Using the properties of the axes and the bisectors of the angles to draw the circumscribed circle and the inscribed circle in a triangle
- Finding the lengths of the segments which determine by two lines on a circle.

Vocabulary:
- Circumscribed circle
- Inscribed circle

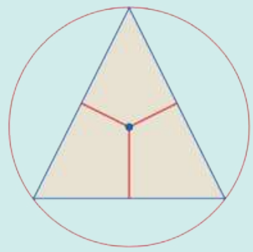
Learn

In the nearby $\triangle ABC$, the axis of BC intersects the axis of AB in O
OC=OB beause O in the axis of BC
OA = OC
So O located on the axis of AC, that is the axis of AC passes through O
$\because$ OA = OB = OC
We can draw a circle, its center is O and passes through the vertices of the triangle ABC



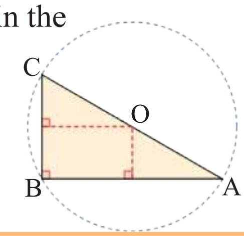
[5-5-1] Triangle and Circle

(the median of the triangle)
(The axes of the three sides of the triangle intersect in one point)
And by them, we can draw the circumscribed circle of the triangle.
The circumscribed circle:of each triangle has one circumscribed circle, its center is a point of intersection the three axes.



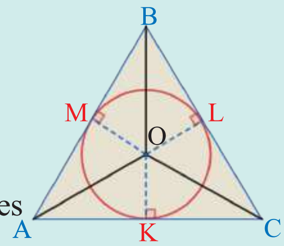
The axes: are the perpendiculars on sides of a triangle from their midpoint, and they meet in one point (O) which has the same distance from its vertices, this point is the center of the circle which passes through the vertices of the triangle.

Example (1): Find the point of intersection of the axes of the triangle ABC as in the nearby figure, and draw the circumscribed circle.

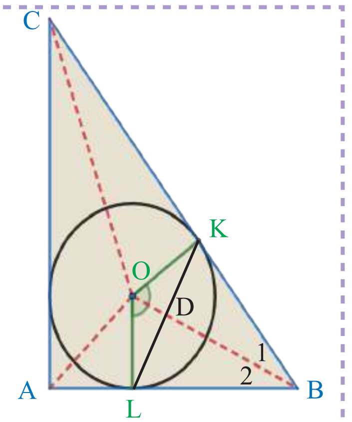


The axis of $\overline{AB}$ passes through the midpoint of $\overline{AB}$ and parallels $\overline{BC}$
The axis of $\overline{BC}$ passes through the midpoint of $\overline{BC}$ and parallels $\overline{AB}$
Then the three axes meet in the midpoint of $\overline{AC}$ which represent the center of the circumscribed circle of the triangle.

We can benefit from the theorem of the bisectors of the triangles angles to draw the inscribed circle of the triangle
The bisectors of the triangles angles intersect in one point.
The point of intersection of the bisectors of the angles locates on the same distance from the three sides.
In every triangle, there is a circle inside the triangle which is tangent to its three sides and it is called the inscribed circle. OL = OK = OM



Example (2): The circle which its center is O surrounded by the triangle ABC, prove that $\overline{BO}$ bisects $\angle LOK$ and bisects $\overline{KL}$



The theorem of the two tangents BK=BL
The two radius of the circle. OK=OL
$\because \triangle BOK, \triangle BOL$ congruent (s.s.s)
$m\angle 1 = m\angle 2$
$\overline{BO}$ **bisects** $\angle LOK$
The triangles LDB, KDB are congruent (S.A.S)
$\overline{KL} \perp \overline{BO}$
$\therefore \overline{BO}$ bisects $\overline{KL}$

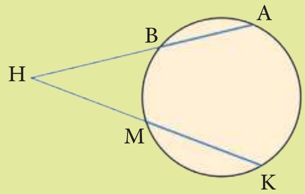
[5-5-2] Line Segments and Circle

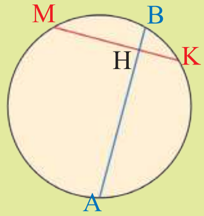
In lesson (5- 4), you have learned how to find the lengths of a chord parts which intersects with a perpendicular diameter on it. But how can we find the lengths of other intersected chords.

Theorem of the two secants of the circle

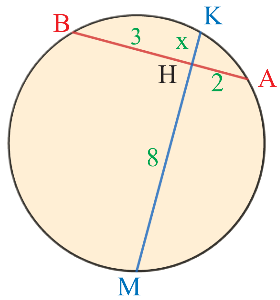
The theorem

If two intersected lines intersect a circle then the product of the length of the two segments from the point to the circle is constant along any line through the point and circle. $HM \times HK = HB \times HA$





Example (3): Find the value of x and the length of each chord.



$HM \times HK = HB \times HA$ A theorem of the two secants in the circle

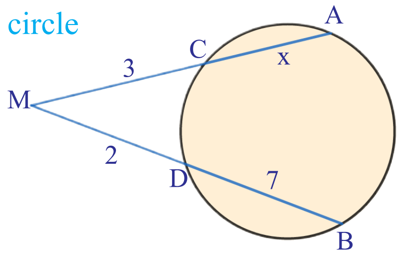
$8 \times x = 3 \times 2$ By substitution and simplifying

$x = \frac{6}{8} = \frac{3}{4}$

$AB = AH + HB = 2 + 3 = 5$ The chord length AB

$MK = MH + HK = 8 + \frac{3}{4} = 8\frac{3}{4}$ The chord length MK

Example (4): Find the value of x and the length of $.\overline{AM}, \overline{BM}$



$MD \times MB = MC \times MA$ A theorem of the two secants in the circle

$2 \times 9 = 3 \times (3 + x)$ simplifying

$18 = 9 + 3x$ By substitution

$3x = 18 - 9 = 9$ Length of $\overline{AM} = 6$

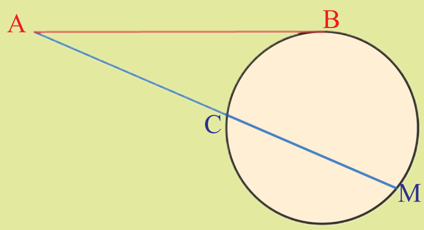
$x = \frac{9}{3} = 3$ Length of $\overline{BM} = 9$

We can use the result of multiplying the two parts of the secant by the theorem of the secant and the tangent, and in this case, the tangent will be the external and total part of the segment itself.

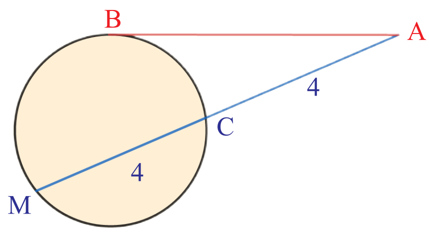
The theorem of the tangent and the secant in circle

The theorem

If a line has a tangency with a circle and another line intersects it, two line segments will be formed on the secant, and the result of multiplying their two lengths will be equaled to the square of the tangent length. $AC \times AM = (AB)^2$



Example (5): Find the length of the tangent AB .

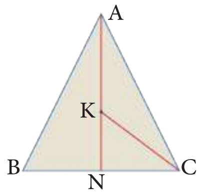


$AC \times AM = (AB)^2$ The theorem of the tangent and the secant in the circle

$4 \times 8 = (AB)^2$ By substitution

$\therefore AB = 4\sqrt{2}$ The length of the tangent AB

Make sure of your understanding



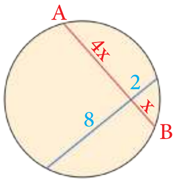
1 The triangle ABC is an isosceles triangle $AB = AC$, N is the midpoint of $\overline{BC}$, $\overline{KA} \cong \overline{KC}$, prove that K is the point of intersection of axes of the triangle ABC, then draw the circumscribed circle.

Questions 1- 2 are similar to example 1

2 ABC is a regular triangle, the length of its side is 12cm, determine the point of intersection of its axes, then draw the circumscribed circle, and find the length of its diameter.

Questions 3 - 5 are similar to examples 3,4

Find the value of x and the length of each unknown segments of the following:

3


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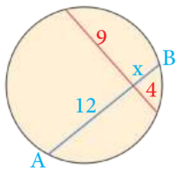

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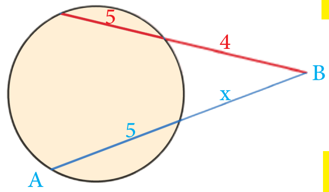
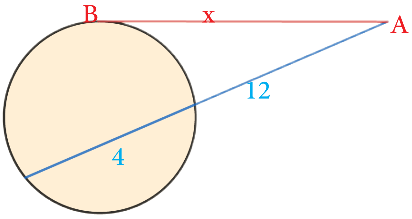


Questions 6- 7are similar to example 5

Find the value of x and lengh of $\overline{AB}$:

6


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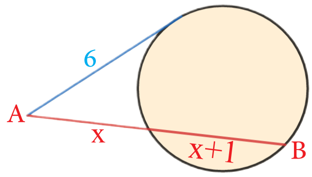
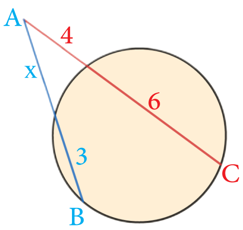


Solve the Exercises

8 ABC is a right-angled and isosceles triangle, the length of each of two sides is 6cm, draw the circle that surrounded by the triangle ABC and find the area of the circle.

9 ABC is a right-angled and isosceles triangle, its chord is $\overline{BC}$. Determine the intersection point of the axes of this triangle and draw circumscribed of it.

Find the value of x and the length of the unknown segments of the following:

10


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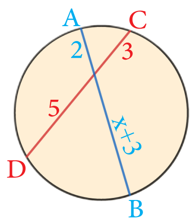

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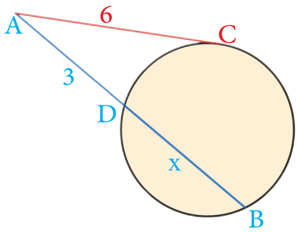

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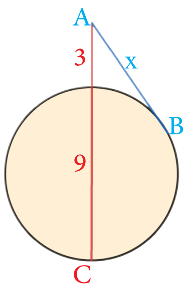
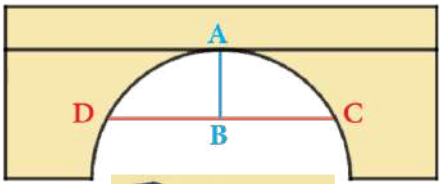
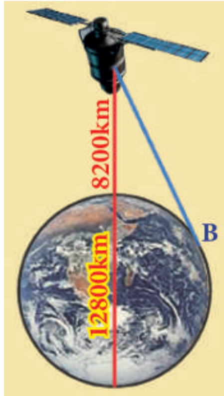


Solve the problems

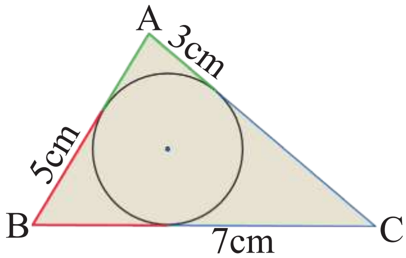


14 Construction: A bridge bases on an arc a circle, as it is shown in nearby figure, $\overline{AB}$ axis of $\overline{DC}$, AB=60m, DC=150m, what is the diameter of the circle?

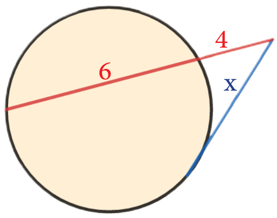


15 Space: A satellite rotates around the earth in a height of 8200km, if the diameter of the earth is about 12800 km, what is the distance which separates the satellite from the point B in the nearby figure.

16 Geometry: find the perimeter of the triangle ABC by using the nearby figure.



Think



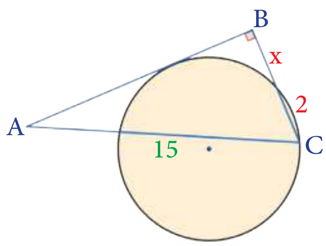
17 Discover the mistake: Below are two solutions to find the value of X in the nearby figure, which solution is wrong? Explain your answer.

The theorem of the tangent and the secant.

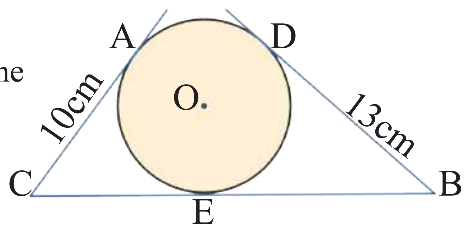
i) $4 \times 6 = x^2$

$24 = x^2 \Rightarrow x = 2\sqrt{6}$

ii) $x^2 = 40 \Rightarrow x = 2\sqrt{10}$



18 Challenge: In the nearby figure, AB=10 and it is the tangent of the circle,find the value of x.



19 Open problem: In the nearby figure, a circle its center O and $\overline{AC}, \overline{BC}, \overline{BD}$ are tangents of the circle, find the length of the segment BC.

Write

A problem which uses the axes and the bisectors of the angles for a triangle to draw a the circumscribed ciricle.

Lesson [5-6] Angles and Circle

Idea of the lesson:
-Finding the measure of inscribed angles and the tangent ones.
-Finding the measure of angles which their sides intersect with a circle.

Vocabulary:
-Inscribed angles
-Tangent angles

Learn

The screwdriver is used as a tool to fix or open the screws, and the gap in this tool takes a hexahedron shape inside a metal cylinder, and every angle in the hexahedron shape represents an inscribed angle inside the circle.

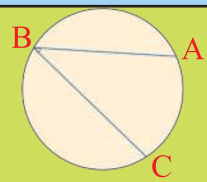
[5-6-1] The Inscribed Angle

You have previously learned the arc by the indication of the centeral angle and how to measure the arc. In this lesson, we will learn the **inscribed angle** it is an angle which its vertices is a point of the circle points and its two sides are chords in the circle.
We will also learn how to measure it by using the arc which opposite it by the following theorems which are without reasoning.

The theorem of the inscribed angles

The measurement of the inscribed angle equals the half of its intercepted arc.

$$m\angle B = \frac{1}{2} m\,\overset{\frown}{AC}$$



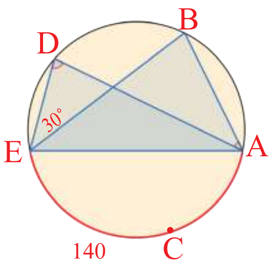
Example (1): **Find the measure of the following inscribed angles in the nearby figure.**

i) $\angle D$

$m\angle D = \frac{1}{2} m\,\overset{\frown}{ECA}$

$= \frac{140}{2} = 70^{\circ}$

$m\angle D = 70^{\circ}$



ii) $\angle BAD$

The theorem of the inscribed angles $m\angle BAD = \frac{1}{2} m\,\overset{\frown}{BD}$

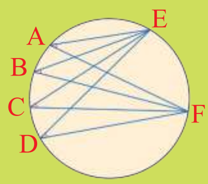
By substitution $m\angle BED = \frac{1}{2} m\,\overset{\frown}{BD}$

$\therefore m\angle BED = m\angle BAD = 30^{\circ}$

The theorem of the inscribed angles which intercepted the same arc

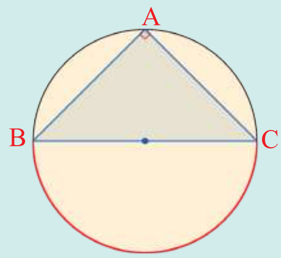
All the inscribed angles which intercept the same arc are congruent.

$$m\angle A \cong m\angle B \cong m\angle C \cong m\angle D = m\,\overset{\frown}{EF}$$



There is a special case for the inscribed angle when it is a right angle
each inscribed angle faces a semicircle is a right angle.
each right- inscribed angle faces opposite diameter.
each inseribed angle facing adiagonal is aright angle

$m\angle A = 90^{\circ}$



Example (2): A circle ,its diameter is $\overline{KH}$, intersects $\overline{HL}$ in N and intersects $\overline{KL}$ in M, as in the nearby figure, prove that $\overline{HM}$ and $\overline{KN}$ are altitudes in the triangle HKL.

An inscribed angle HNK opposite the diameter $\overline{KH}$ is a right angle.

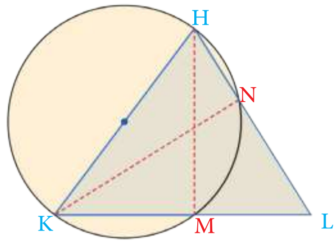
$$\therefore m\angle HNK = 90^\circ$$

$\overline{KN}$ is an altitude in the triangle HKL.

An inscribed angle HNK opposite the diameter $\overline{KH}$ is a right angle

$$\therefore m\angle HMK = 90^\circ$$

$\overline{HM}$ is an altitude in the triangle HKL



[5-6-2] Tangential Angle

Tangential angle: Is the angle which is formed by the tangent of the circle with another line passes through the point of tangency.

The theorem of the tangent angle	
If the circle tangent intersects with a line passes through the point of tangency, the measure of the angle between them will be the half of the intersected arc. $m\angle A = \frac{1}{2} m\,\overset{\frown}{ADC}$	

Example (3): **By using the theorem of the tangent angles and the nearby figure, find the measurement of the following:**

i) $\angle BAC$

The theorem of the tangent angles $\quad m\angle BAC = \frac{1}{2} m\,\overset{\frown}{CA}$

By substitution $\quad = \frac{144}{2} = 72$

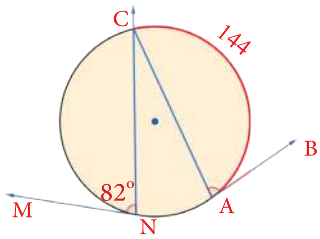
$\therefore m\angle BAC = 72^\circ$

ii) $\overset{\frown}{NC}$

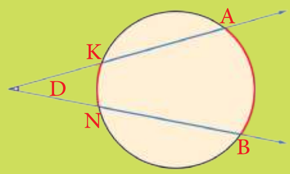
$m\angle CNM = \frac{1}{2} m\,\overset{\frown}{CN}$

$82^\circ = \frac{1}{2} m\,\overset{\frown}{CN}$

$\therefore m\,\overset{\frown}{CN} = 164$



[5 -6- 3] Internal and Exterior Angels in the Circle

The theorem of the exterior angle in the circle	
If two lines intersected outside a circle, the measure of the angle between them equals the half of the difference between the two intersected arcs. $m\angle D = \frac{1}{2}(m\,\overset{\frown}{AB} - m\,\overset{\frown}{KN})$	

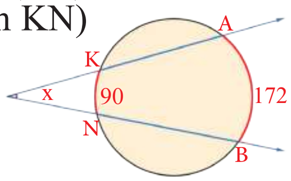
Example (4): **Find the measure of the exterior angle x in each of the following:**

i)By using the theorem of the exterior angle in the circle and by the substitution of the arcs value, in the figure, we find the measure of the angle x.

$$m\angle x = \frac{1}{2}(m\,\overset{\frown}{AB} - m\,\overset{\frown}{KN})$$

$$= \frac{1}{2}(172 - 90)$$

$$\therefore m\angle x = \frac{82^\circ}{2} = 41^\circ$$



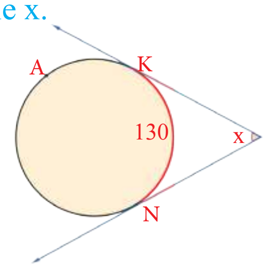
ii) By using the theorem of the exterior angle in the circle and by the substitution of the value $\overset{\frown}{KANK}$ by 360, we find the measure of the angle x.

$$m\overset{\frown}{KAN} = 360^\circ - 130^\circ = 230^\circ$$

$$m\angle x = \frac{1}{2}(m\,\overset{\frown}{KAN} - m\,\overset{\frown}{KN})$$

$$= \frac{1}{2}(230 - 130)$$

$$\therefore m\angle x = \frac{100^\circ}{2} = 50^\circ$$

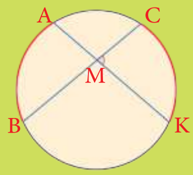


The theorem of the interior angle in a circle

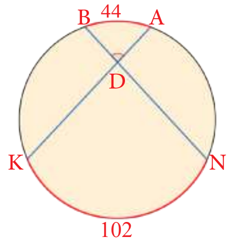
If two lines intersected inside a circle, the measure of the angle between them equals a half of the sum of the measure for the cutting arches.

$m\angle CMK = \frac{1}{2}(m\,\overset{\frown}{CK} + m\,\overset{\frown}{AB})$

$m\angle CMA = \frac{1}{2}(m\overset{\frown}{AC} + m\overset{\frown}{BK})$



Example (5): Find the measure $\angle ADB$,using theorem of the interior angle in the circle.



The theorem of the interior angle in a circle $\quad m\angle ADB = \frac{1}{2}(m\,\overset{\frown}{KN} + m\,\overset{\frown}{AB})$

By substituting and simplifying $\quad = \frac{1}{2}(102 + 44)$

$\therefore m\angle ADB = \frac{146^\circ}{2} = 73^\circ$

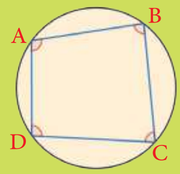
We can draw a circle passes through the four vertices a quadrinomial this quadrinomial is called the circular quadrinomial.

The theorem of the circular quadrinomial

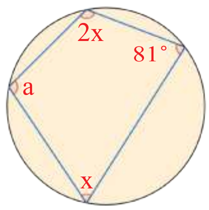
In each circular quadrinomial, the sum of measuring each two opposite angles equals 180^0 .

$m\angle A + m\angle C = 180^\circ$

$m\angle B + m\angle D = 180^\circ$



Example (6): Find the value of x, and (a) in the nearby figure.



The theorem of the circular quadrinomial

$\because a + 81^\circ = 180^\circ$

$\therefore a = 180^\circ - 81^\circ = 99^\circ$

$\because x + 2x = 180^\circ \Rightarrow 3x = 180^\circ$

$\therefore x = 60^\circ$

Make sure of your understanding

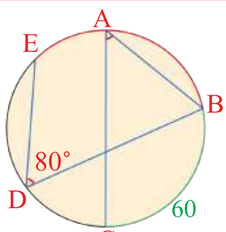
Find the measure of each of the following:

Questions 1,5 are similar to example 1

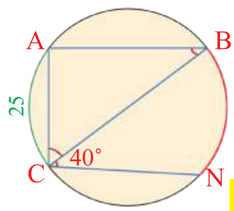
Questions 6,7,10 are similar to example 2

Questions 8, 9 are similar to example 3

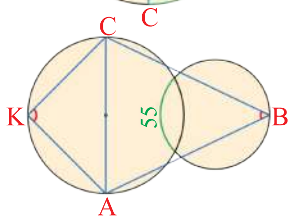
1. $m\,\overset{\frown}{BE}$
3. $m\angle CAB$



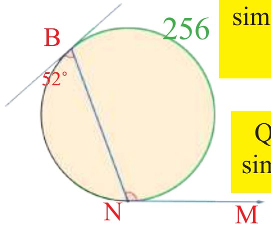
2. $m\angle ABC$
4. $m\angle ACB$
5. $m\,\overset{\frown}{BN}$



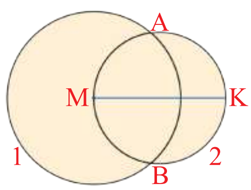
6. $m\angle CKA$
8. $m\angle CBA$



7. $m\angle MNB$
9. $m\,\overset{\frown}{BN}$

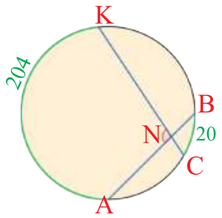


10. If you knew that M is a center of the circle 1 and $\overline{MK}$ is the diameter of the circle 2, prove that $\overline{KA}$ and $\overline{KB}$ are two tangents to the circle1.

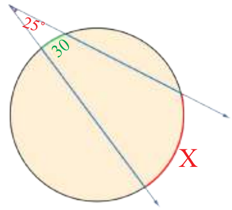


Find the measure for each of the following:

11 $m\angle KNA$



12 $m\widehat{x}$

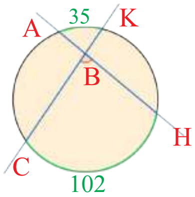


Questions 11 - 12 are similar to examples 4,5,6 respectively

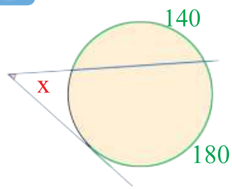
Solve the Exercises

Find the measure for each of the following:

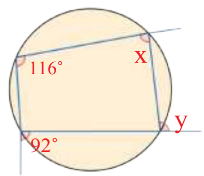
13 $m\angle HBC$



14 $m\angle x$

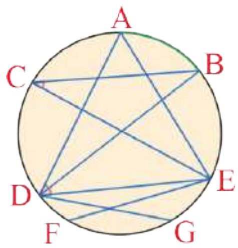


15 $m\angle x, m\angle y$

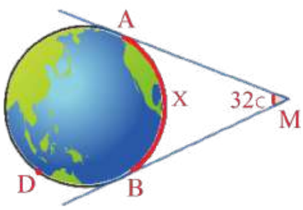


Solve the problems

16 Glass: one of the artists had drawn the nearby diagram on glass, find the measure of $\angle$ ADE if you knew that m $\angle$ BCE=30^0 and the measure $\widehat{AB} = 42$



17 Space: A satellite rotates around the earth, and when it reaches to the point M, its height will be 14000 km above the earth. What is the measure of the arch that we can see by the camera of the satellite on the earth?

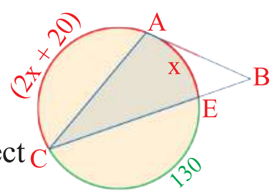


Think

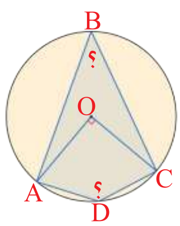
18 Discover the mistake: Saeed wrote

$$m\angle CAB = \frac{160^\circ}{2} = 80^\circ$$

show the mistake and find the correct answer.

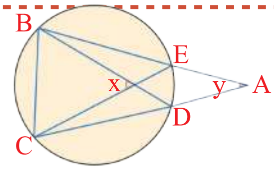


19 Numerical sense: Find the value of the unknown angles



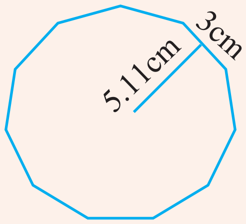
Write

The theorem of the interior and exterior angles to compare the two angles x,y ?



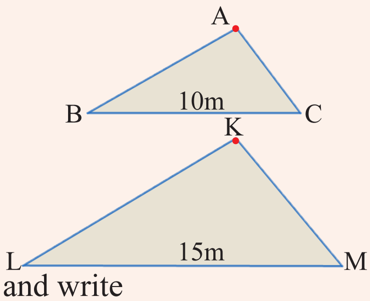
Chapter Test

1 Find the area and the perimeter of a regular polygon, if you were given the data in the nearby figure.

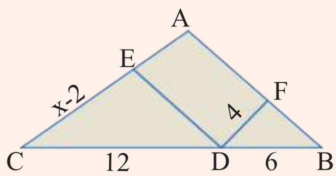


2 Find the lateral area and the volume of the cone if you knew that the area of its base is 9π cm^2 and its lateral height is 5 cm.

3 The two triangles ABC, KLM are similar, the area of the triangle ABC=24 m^2 ,
What is the area of the triangle KLM?

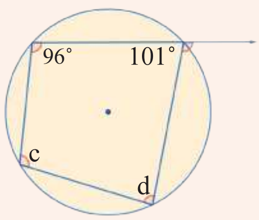


4 Show if the two triangles ABC, FBD in the nearby figure are similar, and write the ratio of the similarity if $\overline{AC} // \overline{FD}$, and find the value of x.

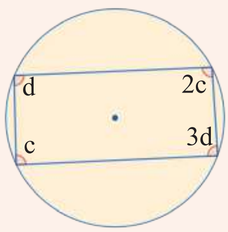


5 **Find the measure of the unknow angles in the following figures:**

i)

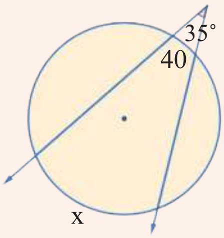


ii)

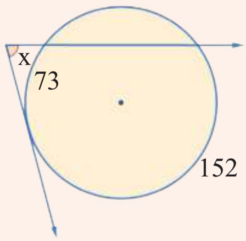


6 **Find the value of x in each of the following:**

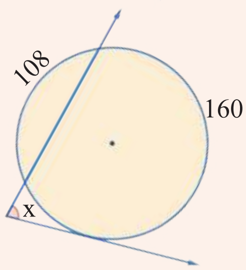
i)



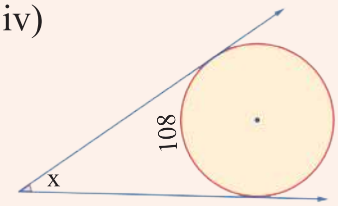
ii)



iii)

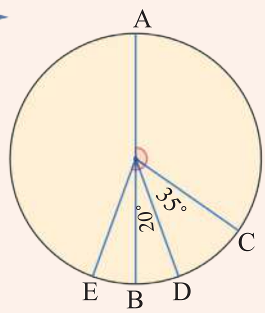


iv)



7 **Find the measure of the unknown angles and arcs in the nearby figure.**

i) m∠AOC ii) m $\widehat{DC}$
iii) m $\widehat{DB}$ iv) m∠DOA



Chapter 6

Statistics And Probilities

Before marketing its production, the cars factories usually investigate some issues to make sure of the quality, such as the durability of cars engines, the quality of cars electricties, colors and affairs related to design, like cars lights and others.

Pretest

Find the arithmetic mean, median, the mode and the range for each of the following:

1 9,6,8,5,5,8,7,6,9,7

2 20,17,42,26,27,12,13

3 8,7,5,8,2,8,9,1,4,3,3,5

4 Represent the following graphs by points, then find the arithmetic mean,median, the mode and the range.

0,2,5,3,1,4,5,3,4,3

Write each fraction as percentage :

5 $\frac{1}{4}$

6 $\frac{13}{20}$

7 $\frac{27}{100}$

8 $\frac{3}{25}$

9 **A box contains five red balls, and three white balls. Find a probability of withdrawal.**

i) One red ball.
ii) One white ball after returning the red ball to the box.
iii) One white ball in case the red ball without returning to the box.

10 **a,b two perfect events, find:**

i) p (a) if $P(b) = \frac{2}{7}$

ii) p (b) , p (a) if p (a) three times p(b).

Determine if the two events are independent or dependent:

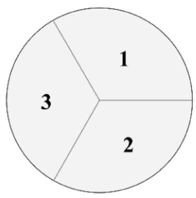
11 After flapping a coin, writting (tail) was appeared, while a picture (head) was appeared after the second flapping

12 Drop a yellow ball, then red ball from a sack which contains 3 yellow balls and 5 red balls.

13 The number 5 was a appeared when the dice was thrown for the first time, but when it was thrown for the second time the number 6 appeared.

14 Drop a card, which a name of Jamana was written on, from a sack without returning it, then another card, which contains the name of Sally, was droped from the same sack.

15 A disc spinner was stopped on the number 3, and appearance of number 3 when a dice was thrown for once.



16 Three cards contain the letters A ,B, C, in how many ways we can arrange the cards on a straight line.

Lesson [6-1] Design a Survey Study and Analysis its Results

Idea of the lesson:
*Designing a survey study
*Analyzing the results

Vocabulary:
*A survey study
*Society
*Sample

Learn

Al- Najaf factory for men's suits, one of most important factories. people who are concerned in the factory affairs interested in quality of production by checking the type of clothes, colors, modern designs and others. The process of checking all cloths will be illogical , so samples of suits would be checked instead to see If the product needed to be developed or not.

[6- 1- 1]] Design a Survey Study

Sample:- It is a subest from the society. By analyzing the results of the sample, we can get conclusions about the society as a whole. The conclusions would be more representative to the society in any of the following two cases:

• Size of sample is biggest.

• Using more samples.

To the type of the sample has an effect on its conclusions which be got, and they are two types:

The unbiased sample: If each one has the same probability of choice.

The unbiased sample: If each one has different probabilities of choice.

Example (1):

A school headmaster distributed 100 questionnaires to the students to know their opinions about the quality of food in school shop stall.

i)Determine the sample and the society which the sample was taken from.

ii)Describe the method of gathering data which the headmaster used.

iii)Determine if the sample was biased or unbiased.

i)The sample: The students who received the questionaires, their number was 100 students.

Society: All students of school

ii)Method of gathering data is a survey study, where they would be taken from the answers of students who represent the sample.

iii)The sample is unbiased: because this sample consists of the students who were chosen randomly.

Example (2):

A shop seller wants to give a gift for every customer shopping from his shop, so he stood in front of the shop and asked 20 customers about the type of gift that everyone wish to be presented to him.

i)Determine the sample and the society that the shop seller chose.

ii)Determine the method of gathering data which the shop seller used.

iii) Determine if the sample was biased or unbiased.

i)The sample: The customers who were asked, their number was 20.

Society: The customers who visited the shop.

ii)The method of gathering data is a survey study, where the answers were taken from the selected sample (20 people)

iii)The sample is unbiased because people who visited the shop were chosen randomly.

Example (3):

Ten persons, who were in a restaurant for kabab, were asked about their favorite meals

i)Determine the sample and the society who were chosen by the owner of the restaurant.

ii)Describe the method of gathering data which the owner of restaurant used

iii)Determine if the sample was biased or unbiased.

i)The sample: The ten persons who entered the restaurant.

Society: All people who entered the restaurant.

ii)The method of gathering data is a survey study, where the answers were taken from the selected sample.

iii)The sample is braised: Because the favorite meal for the persons who were sitting in the restaurant is kabab.

[6-1-2] Analysing the results

After gathering the data by the survey study, the data would be summarized to be meaningful. All that will be done by using the measurements of the central tendency (arithmetic mean, median, the mode) which was studied previously in different ways and choosing the suitable measurement to represent the data.

The type	When it better to be used
Arithmetic mean	When there are no extreme values in the data set
Median	When there are extreme values in the data set, but there are no big gaps in the middle of data
The Mode	When there are repeated numbers in data set

Example (4):

Which of the measurements of central tendency is (if there is) more suitable to represent the data in each of the following:

i) The near by data show the weights of 10 boxes in kg:3,2,3,6,5,5,21,4,3,5

Arithmetic mean: is not suitable to represent the data because there is a big extreme value which is: 21

The mode: is unsuitable to represent the data because there are more then one

Mode: 3, 5

Median : is the suitable measurement to represent the data because there is no big gap in the middle of set 2,3,3,3,4,5,5,5,6,21

ii) Mohammed had got the following marks in five tests of Maths: 90,93,85,86,91

Arithmetic mean $\frac{90+93+85+86+91}{5} = \frac{445}{5} = 89$

Arithmetic mean: 89 is a suitable measurement to represent the data because there is no extreme value

Median: 90 is a suitable measurement for representing the data because it comes in the middle of data and no big gap is there.

Both of them suitable measurement for representing data.

The Mode: there is no mode because there is no repetition in the data.

Make sure of your understanding

Determine the sample and the society, then describe the style of gathering data and differentiate the biased sample from the unbiased one in each of the following, illustrate your answer :

1. 30 persons entered a library, the sixth person from each six persons was asked about his or her favorite hobby.
2. 100 questionnaires were distributed to a group of workers in a factory. They were asked about the conditions of work inside the factory.
3. In a zoo, animals were distributed; the animal from each group was chosen randomly to make tests on it.

Which measurement of central tendency (if there is) is suitable represent the following data? Illustrate your answer :

4. 8, 10, 14, 8,13,6
5. 8, 10, 8, 9,11,4,6,54
6. 8, 9, 8, 6,10,9,11,13,14,8,6,7,19

Solve the Exercises

Determine the sample and the society, then describe the style of gathering the data and differentiate the biased sample and the unbiased one in each of the following. Illustrate your answer :

7. A factory owner wants to check if the workers work well or not. He watches a worker for two hours.
8. A number of students stands at the main gate of school; they were asked the tenth placed student who entered school about her favourite hobby.

Which measurement of the central tendency (if there is) is more suitable to represent the following data? Illustrate your answer.

9. 34,47,41,49,39,26,40
10. 6,2,4,4,3,2,6,2,4,4,20
11. 5,3,5,8,5,3,6,7,4,5

Solve the problems

Hospital: Medical city hospital is a complete medical center that offers its services to all citizens in Baghdad and other governorates. In an introductive seminar, a doctor was selected randomly to present an overview about the services that his department offers within the hospital.

12 Describe the sample and the socity.

13 Is the sample biased or not? Illustrate that.

14 **Shopping:** The table below shows the number of customers who visit a shop for selling electric appliances in each hour of a day. Which measurement of the central tendency is more suitable to represent the data.

Number of customers			
79	71	86	86
88	32	79	86
71	69	82	70
85	81	86	86

15 **Feeding:** the table below shows the calories for some vegetables in the plate for each sample. Which measurement of the central tendency is more suitable to represent the data.

vegetables	calories	vegetables	Calories
onion	16	cucumber	13
pepper	20	corn	66
cabbage	17	spinach	9
carrot	28	pumpkin	17

Think

16 **Challenge:** Find a set of numbers in which its median is smaller than its arithmetic mean.

17 **Correct the mistake:** Snaria says that the arithmetic mean is the more suitable measurement of the central tendency to represent data 3,5,4,8,20. Determine Snaria,s mistake, then correct it.

18 **Numerical sense:** In a survey study about the daily going to a secondary school, a questionnaire was distributed to 50 students, the result was that 74% prefer to go to school in the morning . It this study reliable? Explain that.

Write

A question about a meaning whcih you want to get an answer about it by a survey study.

Lesson [6-2] Data and Misleading statistic

* Discrimination the misleading data
* Discrimination the misleading statistics

* Misleading data
* Misleading statistics

Learn

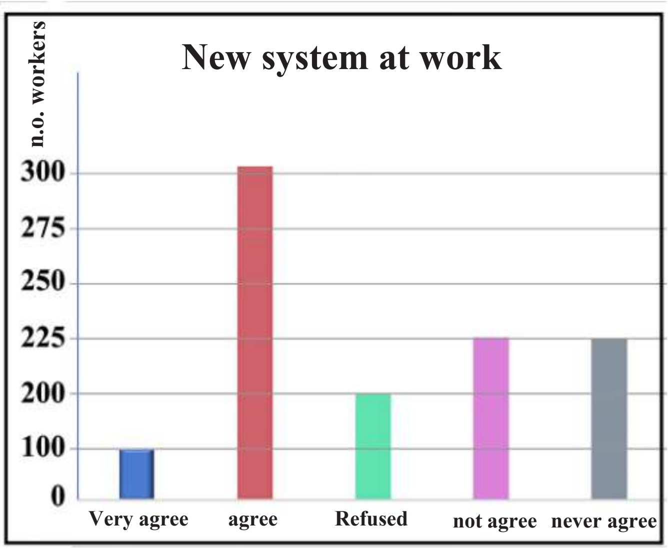
We often see advertisements for sale on the shopwindow of commercial shops at the end of each season.
These advertisements attract the customers to buy goods

[6- 2- 1]

Represent the data which focus on a certain item of the , goods in an extreme ways by showing facts in a way that would misleading the customers and encourage them to buy goods.

Example (1):

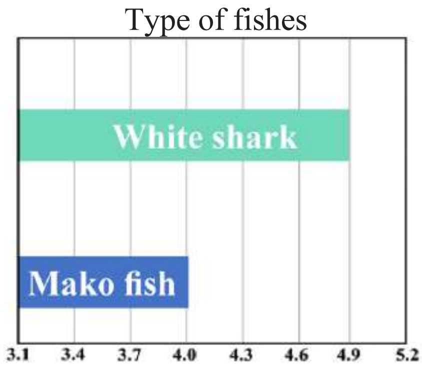
An owner of a factory thinks to apply a new system at work. He distributed a questionnaire in which he asked the workers their opinions in the new system.
Does the representation, in the nearby figure, give the correct form about the outcomes of the questionnaire?
At the first time, it seems that most workers are agree with the new system, although it is knowing that the longest period of grading is unconstant.



Note: there are 450 workers either not agree with this new system or they are never agree with this system, while the number of workers who said yes is more than 300 workers. So, the nearby graphic diagram considers misleading one, and the conclusion is untruthful.
Note: (the graphic diagram may be misleading, by extending or shortening the periods among the values of data to give a certain impression)

Example (2):

The nearby graphic diagram shows the relation between the two lengths of the big white shark and the Mako fish. Show if the graphic diagram is misleading, explain that.



- From the nearby figure, we see that the length of the upper column is twice the length of the lower column double.
- But the corresponding value to the length of the upper column is 4.9 and value to the length of the lower column is 4, and certainly the value 4.9 is not double the value 4. So the nearby graphic diagram is misleading.

Note: (when the graphic diagram started from zero, it will not be misleading)

[6-2-2] Discrimination Misleading Statistics

The misleading statistics: in addition to the misleading diagrams, the misleading statistics use to promote of a company or certain goods. By looking carefully at the given informations of the advertisement we can discriminate the misleading statistics.

Example (3):

An owner of a men's suits shop put the following advertise ment (New mens suits, the average of price is 45000 dinars) There are five types of the suits inside the shop which their prices reach up to (in thousands dinars) 53, 48,20,50,54

$$\frac{54+50+20+48+53}{5}=45$$

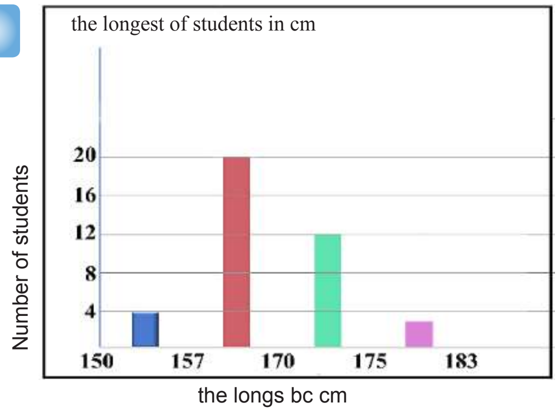
-The average price of suits is 45000 dinars, but there is only one suit which its price 20000 dinars less than this average, and that will make the customer pay more than this price for the suit.

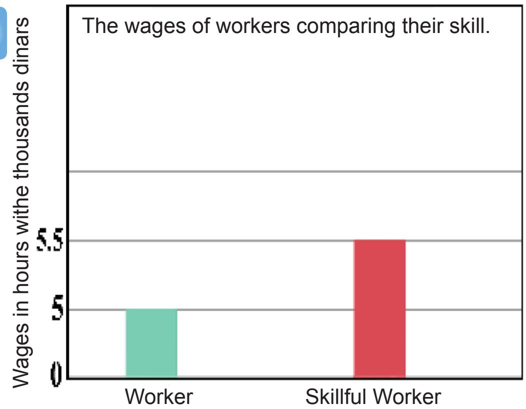
Example (4):

A survey had been done on 800 students from the preparatory stage, 70 students wished to join the engineering college, while 50 student wished to enter the college of medicine. The results reveal that students prefer engineering college to the college of medicine. The students who participated in the servy is (50 + 70) = 120 from 800 students, which means that the random sample was small. The percentage of students who participated in the servey equale $\frac{120}{800}\times 100$ thats equale 15% .

Make sure of your understanding

Clarify how can the following two graphic diagrams give a misleading impression?

1


2


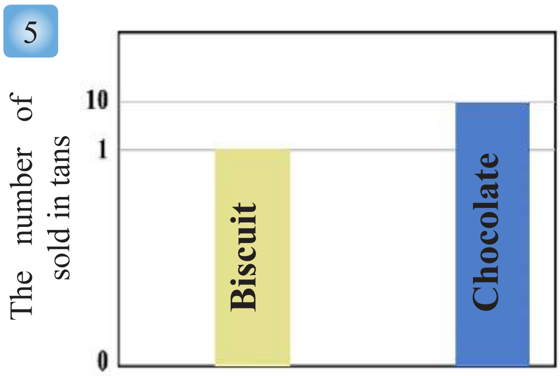
Illustrate why the following statistics are misleading:

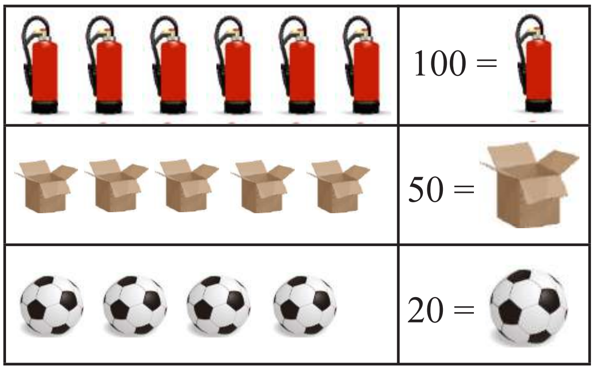
3 An article was shown to 20 persons to evaluate it, 13 persons were admired in the article. According to that, the person who wrote the article announced that the article is durable for publishing because the ratio of people who support the article is 13 to 7.

4 A sport store sold for a certain period 320 sport suits, while a store for entertainment sold for the same period 90 sport suits

Solve the Exercises

Clarify how can the following two graphic diagrams give a misleading impression:

5


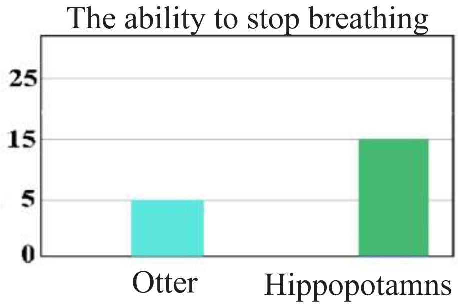
6


7 In a survey for 6 persons about reading a daily newspaper, 4 of them said that they prefer reading the newspaper (x), at the end of the survey the following sentence was mentioned:
2 from 3 persons prefer reading the newspaper (x) , why does this advertisement consider misleading?

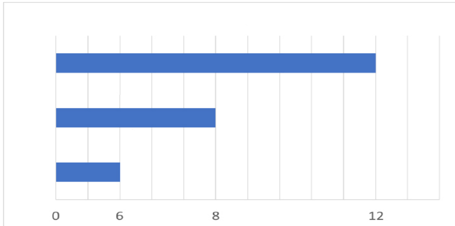
8 100 students were asked about the way they prefered to come to school. 60 students of them thier answer were as the followings. 32 of them prefer coming by taxis, 18 students prefer walking and 10 students prefer their own cars. conclude that half of the students prefer taxis. Illustrate why the statistics are misleading.

Solve the problems

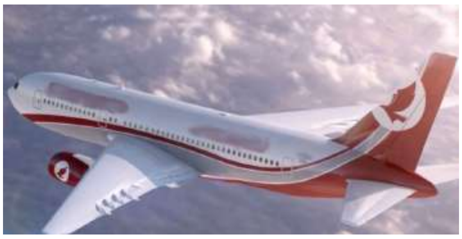
9 **Biology:** the nearby graphic diagram represents the ability to stop breathing for Hippo and other.
Why the data in the diagram are misleading? Clarify that.



10 **Reading:** The nearby graphic diagram represents people who prefer reading litarate, sceintific and art books. Illustrate why the data are misleading ?



11 **Transportation:** the profits of the aviation company A reached to 5500 million dinars in the two months July and August. While the profits of the aviation company B were 7500 million dinars in the two months April and May.
Illustrate why the statistics are misleading ?



12 **Feeding:** the pack of broccoli contains 477mg of potassium, a big carrot contains 230mg, while the cauliflower contains 803mg of potassium.
Illustrate why the statistics are misleading ?

Think

13 **Discover the mistake:** Mohammed says that the diagram will not be misleading if it started from zero for columns regardless of proving the long of periods.
Discover Mohammed›s mistake.

14 **Numerical sense:** A seller had got the following amounts of thousands dinars:
February 965, March 170, April 120, July 125, May 100
He told his friends that the average of his monthly commission is 265000 dinars.
Illustrate why this statistic is misleading.

15 What is the thing that you should be sure of to decide that the diagram is misleading or not?

Write

A question from everyday life which we need to make misleading diagrams.

Lesson [6-3] Experimental Probability and Theoretical Probability

Idea of the lesson:
* Counting the experimental probability
* Counting the theoretical probability

Vocabulary:
* The experimental Probability
* The theoretical Probability
-Sample space

Learn

Mohanned tossed two coins 13 times and he wrote down the results which shown in the nearby table.

results	frequency
H,H	7
H,T	3
T,H	1
T,T	2

1- find the ratio $\frac{\text{number of appearance (H.T)}}{\text{number of element of the sample space}}$

2-Find the ratio $\frac{\text{number of appearance (H,T)}}{\text{number of experiments}}$

Does the ratio in question one equal the ratio in the second question? Explain that.

[6- 4- 1] Experimental probability and Theoretical probability.

You have learned counting the experimental and theoretical probabilities, where the determing of the probability in the paragraph (learning) by implementing the experimental and the results in this way is called the experimental probability. While the probabilities which based on facts and well-known properties are called the theoretical probabilities.

Example (1):

The sample space for the experiment of tossed two coins is: $\Omega = \{(H,H),(H,T),(T,H),(T,T)\}$

Then, the elements number of the sample space equals 4

From the table, the number of times, in which event HT appeared, equals 3

Then, the theoretical probability $P(H,T) = \frac{\text{number of appearance (H.T)}}{\text{number of element of the sample space}} = \frac{3}{4}$

The ratio in the second question. $\frac{3}{13}$ In the table, the number of times, in which the event HT appeared, equals 3

The number of times of the experiment equals 13

Then the experimental probability $P(H,T) = \frac{\text{number of appearance (H,T)}}{\text{number of experiments}} = \frac{3}{13}$

The theoretical probabilities: provide us with the results of experiment without implementing it (depending on the sample space by experiment)

experimental probability: provide us with the results of experiment by repeating it many times (depending on the repeating of experiment)

Example (2):

A researcher found in a factory of cars batteries that probability in which the battery is not good is $\frac{3}{20}$, is the probability theoretical or experimental?

and if the factory want to get 240 batteries which are not good. How many batteries the factory has to produce? this probability is experimental, because it depends on what had really happened, we use proportionality for the all second part of the example.

And each 3 batteries out of 20 are not good.

Then, 240 batteries are not good out of x batteries produced by if factory.

$\frac{3}{20} = \frac{240}{x}$ write the proportion

$3x = 4800$ cross product

$x = \frac{4800}{3}$ dividing the equation on 3

$x = 1600$ the factory must produce 1600 batteries

Example (3):

When rolled the two dices at once, Find the probability of :

i) The event: we get the sum 5 on the two faces of the dices.
ii) The event: the digit on the face of the first dice is the double of the digit on the face of the second dice.
This is a theoretical probability: because the two dices were at once.
The number of digits of the first dice=6, the number of digits of the second dice=6
Then, according to the fundamental counting principle: the elements number of the sample space equals 6x6=36.

$$\Omega = \left\{ \begin{array}{l} (1,1).......(1,6) \\ (2,1).......(2,6) \\ . \\ (6,1).......(6,6) \end{array} \right\}, \quad n = 36$$

i) $E_1 = \{(1,4),(4,1),(2,3),(3,2)\}, m = 4, \ n = 36$ Event :The sum on the two faces of the dices is 5

$P(E_1) = \frac{m}{n}$ Probability formula

$P(E_1) = \frac{4}{36} = \frac{1}{9}$ Substititution and simplify

ii) $E_2 = \{(2,1),(4,2),(6,3)\}, m = 3, n = 36$ Event :digit on first dice is double digit of second dice

$\therefore P(E_2) = \frac{3}{36} = \frac{1}{12}$

[6-4-2] Disjoint Events

The two disjoint events: are two events which can not be happened together in one experiment.
Example:- when we rolled the dice for once, the opportunity of getting an odd number and an even number at the same time impossible.
Then they are two disjoint events
Calculate the probability of the two disjoint events
If E_1, E_2 are two disjoint events, then the probability of happening E_1 or E_2 equals the sum of the two events probability
That means $P(E_1 \text{ or } E_2) = P(E_1) + P(E_2)$

Example (4):

When rolled the dice for one time, find the probability of getting the number 3 or an even number.
Because it is impossible that the number 3 appears on the face of dice, and at the same time an even number appears too! then these two events are disjoint.

$\Omega = \{1,2,3,4,5,6\}$ the sample space

$P(E_1) = \frac{m}{n} \Rightarrow m=1, n=6 \Rightarrow P(E_1) = \frac{1}{6}$ Probability of getting the number 3

$P(E_2) = \frac{m}{n} \Rightarrow m=3, n=6 \Rightarrow P(E_2) = \frac{3}{6}$ Probability of getting the even number

$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2)$ Probability of the disjoint events

$P(E_1 \text{ or } E_2) = \frac{1}{6} + \frac{3}{6} = \frac{4}{6} = \frac{2}{3}$ By substitution and simplifying

Then, the Probability of appearing the number 3 or an even number when the dice was rolled equals $\frac{2}{3}$

Example (5):

If we rolled the two dice for once, find the Probability of getting two equaled numbers or the sum of two equaled numbers is 3

The number of the sample space elements when the two dice are rolled is 36

$E_1 = \{(1,1), (2,2),(3,3),(4,4),(5,5),(6,6)\}$

$$P(E_1) = \frac{\text{number of elements of } E_1}{\text{sample space}} = \frac{6}{36}$$

$E_2 = \{(1,2), (2,1)\}$

$$P(E_2) = \frac{\text{number of elements of } E_2}{\text{sample space}} = \frac{2}{36}$$

E_1, E_2 are two disjoint events

$P(E_1 \text{ or } E_2) = P(E_1)+P(E_2)$ Probability of the disjoint events

$= \frac{6}{36} + \frac{2}{36} = \frac{8}{36} = \frac{2}{9}$ By substitution and simplifying

Make sure of your understanding

In the experiment of rolled the two dices for once, find the probability of happening the following events:

1 The two numbers on the face of the two dice are equaled.

2 The number on the face of the first dice represents the half of the second dice number

3 The sum of the two numbers on the faces of the two dices is 10.

4 The sum of the two numbers on the faces of the two dices is less than 5.

5 Are the previous probabilities experimental or theoretical?

6 A sack contains 4 red balls and green ball, how many blue balls should be added to the sack to make the probability of pulling a red ball is $\frac{2}{3}$? is the probability experimental or theoretical?

7 A man was standing at one of Baghdads squares, he counted 25 cars: 13 cars are yellow, 7 cars are white and 5 cars are grey. Guess that the next car which crosses the squar is yellow one, and what is the probability experimental or theoretical? Write the ratio in form of decimal fraction and percentage.

8 When two dices are rolled, find the probability of getting two numbers which their sum is 5 or 11, are the two events disjoint? Explain that.

Solve the Exercises

In the experiment of rolled the two dices at the once, find the probability of happening the following events:

9 The sum of the two numbers on the two faces of the two dices is more than 8.

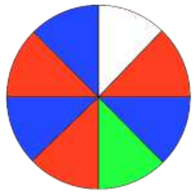
10 The sum of the two numbers on the two faces of the two dices equal 12.

11 A study includes 100 persons had been done, 15 persons side that they use their left hands. If this study involves 400 persons, then according to your expectation how many persons use their left hands?

Solve the exercises

12 Find the probability of pulling a card with an odd number or with a number which represents the multiples of 2 from a group of cards which were numbered from 1 to 9.

Solve the problems



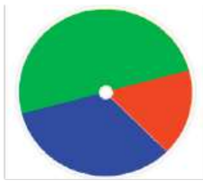
13 **Entertainment:** In which colour we can paint the space to say that the probability of coming the spinner to this colour is $\frac{1}{4}$?

14 **Stamps:** Mohannad likes to collect stamps; among 60 stamps he collected 25 stamps from the Arab countries, 15 stamps from African countries and 20 stamps from European countries. Guess the probability that the next stamp will be European one

15 **Sport:** In atraining of basketball, the player scored 15 balls from 25 throws, what is the experimental probability in which the basketball player will score the next throw? Write the answer as a fraction, a decimal number and percentage.

16 **Study:** A man said that there are three persons in his family have blue eyes from each 22 persons. If he expected a new born, what is the probability that the new baby's eyes will be blue?

Think



17 **Challenge:** A disc with a spinner. It was divided into three parts as in the nearby figure: Its half is green, third of it is red and sixth of it is blue . what is the probability in which the spinner will refer to the green or red colour?

18 **Discover the mistake:** Sarah and Mohannad want to determine the probability of choosing red or blue ball randomly from a sack contains 5 blue balls. 4red balls and 6 yellow balls. Which answer is correct? Illustrate your answer.

Mohannad	Sarah
$P(R \text{ or } B) = P(R) \times P(B) = \frac{4}{15} \times \frac{5}{15} = \frac{4}{45}$	$P(R \text{ or } B) = P(R) + P(B) = \frac{4}{15} + \frac{5}{15} = \frac{9}{15} = \frac{3}{5}$

Write

Explain what each number in $\frac{2}{9}$ which represents in both, the experimental and theoretical probability.

Lesson [6-4] Compound Events

Idea of the lesson:
-Calculate the probability of independent events.
-Calculate the probability of depended events.

Vocabulary:
-Independent events.
Dependent events.

Learn

The reports of the Iraqi airlines company refer to the accurateness of the arrival of its airplanes which $\frac{19}{20}$ as a ratio, and the not 2% refer to lose baggaes. What is the probability of arrival an airplane in its accurate time and without losing baggaes?

[6 - 5- 1] Independent Events

You have previously learned the concept of the independent events (the result of one event doesn't affect on the result of the other). In this lesson, we will learn how to calculate the probability of independent events, If E_1,E_2 were two independent events, then the probability of that they can be happened together equals the result of multiplying the probability E_1 by the event probability E_2.
That means $P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2)$

Example (1):

In the paragraph of learn:-
The probability of arrival the airplane in its accurate time is $P(E_1) = \frac{19}{20}$
The probability of the baggages will be lost is $P(E_2) = \frac{1}{50}$
The arrival of the airplane in its accurate time
will not affect on the losing of baggages, which means that the two events are independent.

$P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2)$ The probability of the independent events

$= \frac{19}{20} \times \frac{1}{50}$ By substitution and simplifying

$= \frac{19}{1000} = 0.019 = 1.9\%$

Example (2):

A sack contains 3 red balls, 4 green balls and 5 blue balls. A ball was taken randomly, then it was returned and another ball was taken. Find the probability of taking a red ball at first, and then a green ball.

$P(R) = \frac{\text{number of red ball}}{\text{total number of balls}} = \frac{3}{12} = \frac{1}{4}$ Taking red ball

$P(G) = \frac{\text{number of green ball}}{\text{total number of balls}} = \frac{4}{12} = \frac{1}{3}$ Taking green ball

The two events are independent

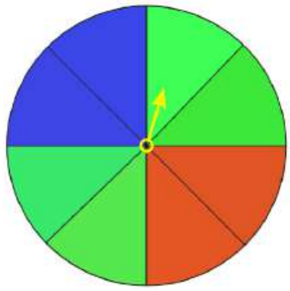
$P(R \text{ and } G) = P(R) \times P(G)$ The probability of the independent events

$= \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$ By substitution and simplifying

then the probability of taking a red ball and then a green ball with returing the red ball equal $\frac{1}{12}$

Example (3):

If one of the numbered cards was chosen, and then the spinner of disc was rotated, as shown in the nearby figure.
What is the probability that the result will be an even number and the colour will be blue?



-We suppose that P(E_1) is a probability of the even number

$$P(E_1) = \frac{2}{4} = \frac{1}{2}$$

-We suppose that P(E_2) is a probability of stopping the spinner at the blue colour. $P(E_2) = \frac{1}{4}$

The probability of the independent events

$$P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2)$$

$$= \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

By substitution and simplifying

So, the probability (an even number and blue colour) is $\frac{1}{8} = 12.5\ \%$

[6-5-2] Dependent Events

The dependent events (they affected by the results of each other)

If E_1 and E_2 are two dependent events, the probability of happening them together is the result of multiplying the probability of the first event E_1 by (the probability of the event E_2 after happening the event E_1)

That means:- $P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2 \text{ after } E_1)$

Example (4):

In example (2) if we didn't return the red ball into the sack.
What is the probability of taking a red ball and then a green ball?

$P(R) = \frac{\text{number of red ball}}{\text{total number of balls}} = \frac{3}{12} = \frac{1}{4}$ Taking the red ball

If we don't return the red ball to the sack. That means the number of red balls becomes 2 and the total number of the balls, in this case, is 11 instead of 12.

$P(G \text{ after } R) = \frac{\text{number of green ball}}{\text{total number of balls}} = \frac{4}{11}$ Taking a green ball

The two event are dependent

$P(R \text{ and } G) = P(R) \times P(G \text{ after } R)$ The probability of the dependent event

$= \frac{1}{4} \times \frac{4}{11} = \frac{1}{11}$ By substitution and simplifying

Then, the probability of taking a red ball and then a green ball without returning the red ball aqual . $\frac{1}{11}$

Example (5):

A box contains 5 red balls, 3 blue balls and 8 yellow balls. A ball was taken from the box, and then another one was taken without returning the first one. Find P (yellow, and then red).

Suppose P (Y) is taking yellow ball $\Rightarrow P(Y) = \frac{8}{16} = \frac{1}{2}$

Without returning the yellow ball, the total number of balls in the box becomes 15 balls, as follow:- 5 red balls,3 blue balls and 7 yellow balls.

A red ball was taken from the box

$P(R \text{ after} Y) = \frac{5}{15} = \frac{1}{3}$ The two events are dependent

$P(Y \text{ and } R) = P(Y) \times P(R \text{ after } Y)$ The probability of the dependent events

$= \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ By substitution and simplifying

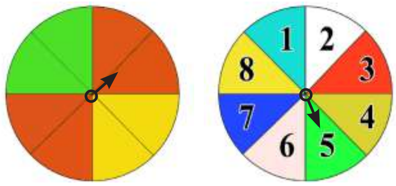
Then, the probability of taking a yellow ball and then a red ball without returning the yellow ball is $\frac{1}{6}$

Conclusion

1. We find $P(E_2), P(E_1)$
2. If E_1, E_2 are independent, then $P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2)$
3. If E_1, E_2 are dependent, then $P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2 \text{ after } E_1)$

Make sure of your understanding

1 A box contains 3 red balls and 3 green balls, what is the probability of taking two green balls without returning the first ball?



2 Spinner in the two opposite discs were released at the same time, what is the probability in which the first spinner comes to the red colour and the second spinner comes to number 5?

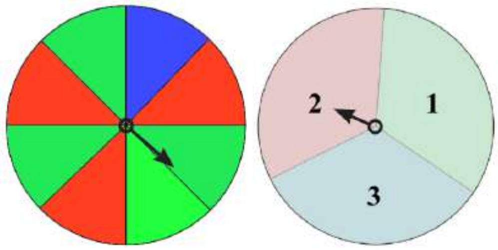
3 If we flap two coins at once, what is the probability of appearing a picture (head) in the first coin and writing (tail) in the second coin?

Solve the Exercises

4 A box contains 5 red cards, 4 black cards and 6 green cards.
A card was taken and then a second card was taken without returning the first one. What is the probability that the first card was red and the second was black?

5 A spinner was released in the two nearby discs at once, what is the probability in which the spinner of the first disc comes to the green colour and the second spinner comes to the number of 3?

6 The two dices were rolled at once, what is the probability of appraring a number which can be divided on 3 in the first dice and a number which can be divided on 5 in the second dice?



Solve the problems

7 **Sweets:** A box contains 10 pieces of sweets with strawberry flavor, 15 pieces with chocolate flavor and 5 pieces with lemon flavor. What is the probability of choosing two pieces randomly, the first with chocolate flavor without returning and the second with lemon flavor?

8 **Books:** Suha chose a book from a shelf in her room and returned it to take another one, what is the probability in which the chosen book would be from Maths? It is worth to be mentioned that the shelf contains 5 books of Maths, 2 books of English language and 3 books of science.

Think

9 **Discover the mistake:** Jumana and her sister Sally want to determine the probability of choosing a red ball and a yellow ball randomly by taking them from a sack contains 4 red balls and 5 yellow balls without returning the ball after taking it.

Jumana	Sally
P(red and yellow)	p(red and yellow)
P(yellow) $\times$ P(red)	p(yellow) $\times$ p(red)
$\frac{4}{9} \times \frac{5}{9}$	$\frac{4}{9} \times \frac{5}{8}$

Which one of them is correct?

10 **Challenge:** A dice and a coin were thrown, what is the probability of appearing a digit which is greater than 2 and less than 6 on the dice and the writing (tail) on the coin?

11 **Open problem:** 10 cards with three different shapes. Write a problem related to pull two cards randomly without returning them, the probability have to be $\frac{1}{15}$.

Write

An example on two independent events and another example on two dependent events.

Chapter Test

1 **Distributed questionnaires to 30 students from 100 students answer the following:**

i) Detrmine the sample and society which the sample was taken from.
ii) Describe the method of the questionnaires ditributed.
iii) Detrmine if the sample was baised or unbaised.

2 How to distinguish between the misleading data graphics and unmisleading data graphics.

3 **When throwing the disce for 25 times, the results shows in the following table:**

Result	1	2	3	4	5	6
Count	2	6	3	5	2	7

i) What is the type of the probability?
ii) Find the probability to gettinhg the number 8.

4 **In the experiment of rolled the dice for once, find:**
i) Type of the probability experiment or theretical.
ii) Probability to getting a number divided by 4.

5 Mohannad was standing at one of baghdads squares, he counted 20 cars, 10 cars are salon, 7 cars are small transportation, 3 cars are truck, guess that the next car which crosses the squares is salon car.

Exercises of chapters

Multiple Choice

[1-1] Ordering Operations in Real Numbers

Choose the correct answer for each of the following:

Simplify the following numerical sentences by using the ordering of operations in the real numbers:

1 $(\sqrt{2}+\sqrt{7})(\sqrt{2}+\sqrt{7}) = ...$ a) $2+9\sqrt{7}$ b) $2+9\sqrt{2}$ c) $9+2\sqrt{14}$ d) $2+9\sqrt{14}$

2 $\frac{6\sqrt{50}}{3\sqrt[3]{-8}} \div \frac{2\sqrt{14}}{\sqrt{7}} = ...$ a) $\frac{-5}{2}$ b) $\frac{-2}{2}$ c) $\frac{\sqrt{2}}{5}$ d) $\frac{-\sqrt{2}}{5}$

3 $(-27)^{\frac{1}{3}}(\frac{1}{6}\sqrt{2} - \frac{1}{4}\sqrt{32}) = ...$ a) $\frac{-5}{\sqrt{2}}$ b) $\frac{5}{\sqrt{2}}$ c) $\frac{\sqrt{2}}{5}$ d) $\frac{-\sqrt{2}}{5}$

Simplify the following numerical sentences by using ratioralizing the denominator and ordering theoperations in the real numbers:

4 $\frac{(\sqrt{2}-\sqrt{3})}{(\sqrt{2}+\sqrt{3})} = ...$ a) $5+6\sqrt{2}$ b) $5-6\sqrt{2}$ c) $2\sqrt{6}-5$ d) $2\sqrt{6}+5$

Use the ordering of operations and write the result neared to two decimal places using the calculator for each of the following:

5 $(\frac{1}{3})^2 - 3^{-2} - (5)^{\frac{3}{2}} \approx ...$ a) -18.11 b) 18.11 c) 11.18 d) -11.18

Multiple Choice

[1-2] Mappings

Choose The Correct Answer for each of the following:

1 If , $f: Z \to R$ where $f(x) = 3x - 2$ then the number 10 is the image of the number:

a) 5 b) 4 c) 3 d) 2

2 If , $f: A \to B$, where $A = \{2, 3, 4, 5\}$, $B = \{4, 6, 8\}$ and $f = \{(2, 4), (3, 6), (4, 8), (5, 8)\}$

then f is surjection mapping because:

a) Range $\neq$ co-domain b) f does not injection

c) The range is the domain of A d) Range = co-domain

3 If , $f: Z \to Z$ where $f(x) = 2x - 3$, $g: Z \to Z$ where $g(x) = x + 1$ then the mapping (gof) (x) is:

a) $2x - 2$ b) $2x - 4$ c) $2x + 2$ d) $2x + 4$

4 Let $f: \{2, 3, 5\} \to N$ where $f(x) = 3x - 1$

and $g: N \to N$ where $g(x) = x + 1$ then the range of (gof) is:

a) $= \{5, 8, 14\}$ b) $= \{5, 6, 9\}$

c) $= \{6, 12, 15\}$ d) $= \{6, 9, 12\}$

5 If the mapping $f: Q \to Q$ where $f(x) = 4x + 1$ and the mapping $g: Q \to Q$ where

$g(x) = \frac{1}{3} x^2 - 1$ if $(fog)(x) = 45$ then the value of x is :

a) ± 5 b) ± 6 c) ± 7 d) ± 8

Multiple Choice

[1-3] Compound Inequalities

Choose The Correct Answer for each of the following:

Solve the compound inequalities which include (and) algebraically:

1 $-10 < x$ and $x \leq -2$

a) $\{x: -10 \leq x\} \cap \{x: x \leq -2\}$ b) $\{x: -10 < x\} \cap \{x: x \leq -2\}$

c) $\{x: -10 \leq x\} \cup \{x: x \leq -2\}$ d) $\{x: -10 < x\} \cup \{x: x \leq -2\}$

2 $16 < 3z + 9$ and $3z + 9 < 30$

a) $\{z: \frac{3}{7} \leq z < 7\}$ b) $\{z: \frac{7}{3} < z \leq 7\}$

c) $\{z: \frac{3}{7} < z < 7\}$ d) $\{z: \frac{7}{3} < z < 7\}$

Solve the compound inequalities which include (or) algebraically:

3 $\frac{y+5}{3} < \frac{1}{3}$ or $\frac{y+5}{3} > \frac{7}{3}$

a) $\{y: y < 4\} \cap \{y: y > 2\}$ b) $\{y: y > -4\} \cup \{y: y < 2\}$

c) $\{y: y < -4\} \cap \{y: y > -2\}$ d) $\{y: y < -4\} \cup \{y: y > 2\}$

Write the compound inequality which shows the range of the third side length in a triangle which the lengths of its other two sides are known:

4 8cm , 2cm

a) $6 \leq x < 10$ b) $6 \leq x \leq 10$ c) $6 < x < 10$ d) $6 < x \leq 10$

Write the inequalities represent the solution set in the line of numbers:

5

a) $y \leq -3$ or $y > 5$ b) $y \leq -3$ or $y \geq 5$ c) $y < -4$ or $y \geq 5$ d) $y < -3$ or $y > 5$

Multiple Choice

[1-4] Absolute Value Inequalities

Choose The Correct Answer for each of the following:

Solve the following absolute value inequalities:

1. $|y - 8| < 13$

a) $5 < y < -21$ b) $-5 \leq y \leq 21$

c) $-5 < y < 21$ d) $-5 < y \leq 21$

2. $|3x| - 7 < 1$

a) $-\frac{8}{3} \leq x < \frac{8}{3}$ b) $-\frac{8}{3} < x \leq \frac{8}{3}$

c) $-\frac{8}{3} \leq x \leq \frac{8}{3}$ d) $-\frac{8}{3} < x < \frac{8}{3}$

3. $|6 - 3y| \geq 9$

a) $y \leq 1$ or $y \geq -5$ b) $y < -1$ or $y > 5$

c) $y > -1$ or $y < 5$ d) $y < -1$ or $y \geq 5$

4. $|\frac{7 - 2y}{3}| \geq 3$

a) $y \leq -1$ or $y \geq 8$ b) $y < -1$ or $y \geq 8$

c) $y < -1$ or $y > 8$ d) $y < -1$ or $y > 8$

Multiple Choice

[2-1] Multiplying Algebraic Expressions

Choose The Correct Answer for each of the following:

Find the result of multiplying and algebraic expression by another algebraic expression:

1. $(x + 5)^2$ a) $x^2 - 10x + 25$ b) $x^2 + 10x + 25$ c) $x^2 + 5x + 25$ d) $x^2 - 5x + 25$

2. $(z - \sqrt{7})^2$ a) $z^2 - 7z + 49$ b) $z^2 + 7y + 49$ c) $z^2 - \sqrt{7}\, z + 7$ d) $z^2 - 2\sqrt{7}\, z + 7$

3. $(x + 8)(x - 8)$ a) $x^2 - 64$ b) $x^2 + 64$ c) $x^2 + 16$ d) $x^2 - 16$

4. $(y + \sqrt{6})(y - \sqrt{6})$ a) $y^2 - \sqrt{12}$ b) $y^2 - 6$ c) $y^2 + \sqrt{12}$ d) $y^2 + 6$

5. $(y - 2)(y^2 + 2y + 4)$ a) $y^3 + 8$ b) $y^3 - 8$ c) $y^3 - 4$ d) $y^3 - 16$

6. $(y + \frac{1}{5})^3$ a) $y^3 - \frac{3}{3}y^2 + \frac{3}{25}y - \frac{1}{125}$ b) $y^3 + \frac{3}{5}y^2 - \frac{3}{25}y + \frac{1}{125}$

   c) $y^3 + \frac{3}{5}y^2 + \frac{3}{25}y + \frac{1}{125}$ d) $y^3 - \frac{3}{5}y^2 - \frac{3}{25}y - \frac{1}{125}$

Multiple Choice

[2-2]Factoring the Algebraic Expression by using agreater Common Factor .

Choose The Correct Answer for each of the following:

Factoring each expression by using the greatest common factor:

1 $6y^2(3y - 4) + 36y$

a) $6y(3y^2 + 4y + 6)$ b) $6y(3y^2 + 4y - 6)$

c) $6y(3y^2 - 4y - 6)$ d) $6y(3y^2 - 4y + 6)$

Factoring each expression by using the binomial term as a greatest common factor:

2 $\frac{1}{4}(x + 9) - \frac{1}{2}x^2(x + 9)$

a) $(x + 9)(\frac{1}{4} - \frac{1}{2}x^2)$ b) $(x - 9)(\frac{1}{4} - \frac{1}{2}x^2)$

c) $(x + 9)(\frac{1}{4} + \frac{1}{2}x^2)$ d) $(x + 9)(\frac{1}{2} - \frac{1}{4}x^2)$

3 $\sqrt{2}\, v(x - 1) - \sqrt{3}\, t(x - 1)$

a) $(x + 1)(\sqrt{2}\, v - \sqrt{3}\, t)$ b) $(x - 1)(\sqrt{2}\, v - \sqrt{3}\, t)$

c) $(x - 1)(\sqrt{2}\, v + \sqrt{3}\, t)$ d) $(x + 1)(\sqrt{2}\, v + \sqrt{3}\, t)$

Factoring each expression by using the property of grouping, then check the correct of solution:

4 $3y^3 - 9y^2 + 5y - 15$

a) $(y + 3)(3y^2 + 5)$ b) $(y - 3)(3y^2 - 5)$

c) $(y - 3)(3y^2 + 5)$ d) $(y - 3)(3y^2 - 5)$

Factoring the expression by using the property of grouping with the inverse:

5 $20y^3 - 4y^2 + 3 - 15y$

a) $(5y + 1)(4y^2 - 3)$ b) $(5y - 1)(4y^2 + 3)$

c) $(5y - 1)(4y^2 - 3)$ d) $(5y + 1)(4y^2 + 3)$

Multiple Choice

[2-3] Factoring the Algebraic Expression by using Special Identities.

Choose The Correct Answer for each of the following:

Factor each of the following algebraic expressions:

1. $12y^3z - 3yz^3$
   a) $3y(2y - z)(y + 2z)$ b) $3z(2y - z)(2y + z)$
   c) $3yz(2y - z)(2y + z)$ d) $3yz(y - 2z)(y + 2z)$

2. $\frac{1}{6}x^3 - x\frac{1}{24}$
   a) $\frac{x}{6}(x + \frac{1}{2})(x - \frac{1}{2})$ b) $\frac{x}{6}(x + \frac{1}{4})(x - \frac{1}{4})$
   c) $\frac{x}{3}(\frac{1}{2}x + \frac{1}{2})(\frac{1}{2}x - \frac{1}{2})$ d) $\frac{x}{6}(\frac{1}{4}x + \frac{1}{4})(\frac{1}{4}x - \frac{1}{4})$

3. $4x^2 + 24x + 36$
   a) $(x + 6)^2$ b) $(x - 6)^2$ c) $4(x - 3)^2$ d) $4(x + 3)^2$

Determine which of the following algebraic expressions represents a perfect square:

4. $64 - 48y + 9y^2$
   it is not a perfect square because a) $2(4)(3y) \neq - 48y$
   a perfect square because b) $2(8)(4y) = 48y$
   it is a perfect because c) $-2(8)(3y) \neq - 48y$
   It is not a perfect square because d) $-4(4)(3y) = 48y$

Write the missing term in the algebraic expression $ax^2 + bx + c$ to become a perfect square:

5. $z^2 + …. + 49$
   a) $14z$ b) $- 10z$ c) $7z$ d) $-7z$

6. $36 - 24x + ….$
   a) $2x^2$ b) $-2x^2$ c) $4x^2$ d) $- 4x^2$

Multiple Choice

[2-4] Factoring the Algebraic Expression of three terms by Trial and Error (Experiment).

Choose The Correct Answer for each of the following:

Factor each of the following algebraic expressions:

1 $x^2 + 7x + 12$

a) $(x - 3)(x + 4)$ b) $(x + 3)(x + 4)$

c) $(x - 1)(x + 7)$ d) $(x - 3)(x - 4)$

2 $x^2 - 5x - 36$

a) $(x - 6)(x + 6)$ b) $(x + 12)(x - 3)$

c) $(x - 9)(x + 4)$ d) $(x + 9)(x - 4)$

3 $y^2 + 4y - 21$

a) $(y - 7)(y + 3)$ b) $(y + 7)(y - 3)$

c) $(y - 7)(y - 3)$ d) $(y + 7)(y + 3)$

Put signs between the terms inside brackets to make the factoring of the algebraic expression correct:

4 $4y^2 -2y -12 = (2y…3)(2y…4)$

a) $(2y - 3)(2y + 4)$ b) $(2y + 3)(2y + 4)$

c) $(2y - 3)(2y - 4)$ d) $(2y + 3)(2y - 4)$

5 $48 - 30z + 3z^2 = (6…3z)(8… z)$

a) $(6 - 3z)(8 - z)$ b) $(6 + 3z)(8 + z)$

a) $(6 - 3z)(8 + z)$ b) $(6 + 3z)(8 - z)$

Multiple Choice

[2-5] Factoring the Algebraic Expressions Sum of Two Cubes or difference Between Two Cubes.

Choose The Correct Answer for each of the following:

Factor each of the following algebraic expressions in simplest form:

1 $8 + x^3$

a) $(2 - x)(4 + 2x + x^2)$
b) $(2 + x)(4 - 2x + x^2)$
c) $(2 - x)(4 - 2x + x^2)$
d) $(2 + x)(4 + 2x + x^2)$

2 $\frac{1}{z^3} + \frac{1}{64}$

a) $(\frac{1}{z} + \frac{1}{4})(\frac{1}{z^2} + \frac{1}{4z} + \frac{1}{16})$
b) $(\frac{1}{z} - \frac{1}{4})(\frac{1}{z^2} - \frac{1}{4z} + \frac{1}{16})$
c) $(\frac{1}{z} - \frac{1}{4})(\frac{1}{z^2} + \frac{1}{4z} + \frac{1}{16})$
d) $(\frac{1}{z} + \frac{1}{4})(\frac{1}{z^2} - \frac{1}{4z} + \frac{1}{16})$

3 $\frac{27}{125} + \frac{8}{x^3}$

a) $(\frac{3}{5} - \frac{2}{x})(\frac{9}{25} + \frac{6}{5x} + \frac{4}{x^2})$
b) $(\frac{3}{5} - \frac{2}{x})(\frac{9}{25} - \frac{6}{5x} + \frac{4}{x^2})$
c) $(\frac{3}{5} + \frac{2}{x})(\frac{9}{25} - \frac{6}{5x} + \frac{4}{x^2})$
d) $(\frac{3}{5} + \frac{2}{x})(\frac{9}{25} - \frac{6}{5x} - \frac{4}{x^2})$

4 $9 - \frac{1}{3}z^3$

a) $\frac{1}{3}(3 - z)(9 + 3z - z^2)$
b) $\frac{1}{3}(3 - z)(9 + 3z + z^2)$
c) $\frac{1}{3}(3 + z)(9 + 3z + z^2)$
d) $\frac{1}{3}(3 - z)(9 - 3z + z^2)$

5 $0.008x^3 - 1$

a) $(0.02x - 1)(0.04x^2 + 0.002x + 1)$
b) $(0.02x - 1)(0.04x^2 + 0.02x + 1)$
c) $(0.2x + 1)(0.4x^2 - 0.2x + 1)$
d) $(0.2x - 1)(0.04x^2 + 0.2x + 1)$

Multiple Choice

[2-6] Simplifying Rational Algebraic Expressions

Choose The Correct Answer for each of the following:

Write each of the following expressions in simplest form:

1. $\frac{x+3}{4x} \times \frac{4x-12}{x^2-9}$ a) $\frac{3}{x}$ b) $\frac{x}{4}$ c) $\frac{1}{4}$ d) $\frac{1}{x}$

2. $\frac{z^2-2z-15}{9+3z} \times \frac{5}{z^2-25}$ a) $\frac{5}{z+5}$ b) $\frac{3}{5(z+5)}$ c) $\frac{5}{3(z+5)}$ d) $\frac{3}{z+5}$

3. $\frac{1-z^3}{1+z+z^2} \div \frac{(1-z)^2}{1-z^2}$ a) $1-z$ b) $1+z$ c) $1+z+z^2$ d) $1-z+z^2$

Write each of the following expressions in simplest form:

4. $\frac{2y^2+1}{y^3-1} - \frac{y}{y^2+y+1}$ a) $\frac{y}{y+1}$ b) $\frac{1}{y+1}$ c) $\frac{1}{y-1}$ d) $\frac{y}{y-1}$

5. $\frac{3y+1}{y+4} - \frac{y-4}{3y-1} - \frac{10+8y^2}{3y^2+11y-4}$

a) $\frac{5}{(y+4)(3y-1)}$ b) $\frac{3}{(y+4)(3y-1)}$

c) $\frac{-3}{(y+4)(3y-1)}$ d) $\frac{-5}{(y+4)(3y-1)}$

Multiple Choice

[3-1] Solving system of two linear equations with two variables

Choose The Correct Answer for each of the following:

Find the set of solution of the system graphically in R:

1. $\left.\begin{array}{l} y = 4x - 6 \\ y = x \end{array}\right\}$

a) {(-2 , -2)} b) {(-2 , 2)} c) {(2 , -2)} d) {(2 , 2)}

Find the set of solution of the system in R by using the substitution for each of the following:

2. $\left.\begin{array}{l} 3x + 4y = 26 \\ 5x - 2y = 0 \end{array}\right\}$

a) {(2 , 5)} b) {(-2 , -5)} c) {(2 , -5)} d) {(-2 , 5)}

3. $\left.\begin{array}{l} \frac{3x}{4} - \frac{y}{2} = 4 \\ \frac{y}{2} - \frac{x}{4} = 2 \end{array}\right\}$

a) {(12 , -10)} b) {(-12, -10)} c) {(12 , 10)} d) {(-12 , 10)}

Find the set of solution of the system in R by using the elimination for each of the following:

4. $\left.\begin{array}{l} 7x - 4y = 12 \\ 3x - y = 5 \end{array}\right\}$

a) $\{(-\frac{8}{5}, \frac{1}{5})\}$ b) $\{(-\frac{8}{5}, -\frac{1}{5})\}$ c) $\{(\frac{8}{5}, \frac{1}{5})\}$ d) $\{(\frac{8}{5}, -\frac{1}{5})\}$

5. $\left.\begin{array}{l} 6y - 2x - 8 = 0 \\ y + x - 12 = 0 \end{array}\right\}$

a) {(8 , -4)} b) {(8, 4)} c) {(-8 , 4)} d) {(-8 ,-4)}

Multiple Choice

[3-2] Solving Quadratic Equations with one variable

Choose The Correct Answer for each of the following:

Solve the following equations in R by using the greatest common factor and the difference between two squares:

1. $7z^2 - 21 = 0$ a) s = {7 , -7} b) s = {3 , -3} c) s = $\{\frac{1}{3}, -\frac{1}{3}\}$ d) s = $\{\sqrt{3}, -\sqrt{3}\}$

2. $4(x^2 - 1) - 5 = 0$ a) s = $\{\frac{3}{2}, -\frac{3}{2}\}$ b) s = $\{\frac{1}{2}, -\frac{1}{2}\}$ c) s = $\{\frac{3}{2}, \frac{3}{2}\}$ d) s = $\{\frac{1}{2}, \frac{1}{2}\}$

3. $(y + 7)^2 - 81 = 0$ a) s = {2 , -2} b) s = {16 , -16} c) s = {2 , -16} d) s = {-2 , 16}

Solve the following equations in R by using the rule of square root:

4. $4(y^2 - 1) = 45$ a) s = $\{\frac{7}{2}, -\frac{7}{2}\}$ b) s = $\{\frac{7}{2}, \frac{7}{2}\}$ c) s = $\{\frac{2}{7}, -\frac{2}{7}\}$ d) s = $\{\frac{7}{4}, -\frac{7}{4}\}$

5. $x^2 - \frac{13}{16} = \frac{3}{16}$ a) s = $\{\frac{3}{4}, -\frac{3}{4}\}$ b) s = $\{\frac{\sqrt{3}}{4}, -\frac{\sqrt{3}}{4}\}$ c) s = {2 , - 2} d) s = {1 , - 1}

Multiple Choice

[3-3] Solving the quadratic equations by the experiment

Choose The Correct Answer for each of the following:

Solve the following equations in R by factoring in the experiment:

1 $y^2 + 10y + 21 = 0$ a) s ={3,-7}   b) s ={-3,7}   c) s ={-3,-7}   d) s ={3,7}

2 $x^2 - 5x - 36 = 0$ a) s ={7, -8}   b) s ={-4,9}   c) s ={4,-9}   d) s ={-4,-9}

3 $32 + 12x - 9x^2 = 0$ a) s $=\{\frac{4}{3}, \frac{8}{3}\}$ b) s $=\{\frac{-4}{3}, \frac{-8}{4}\}$ c) s $=\{\frac{4}{3}, \frac{-8}{3}\}$ d) s $=\{\frac{-4}{3}, \frac{8}{3}\}$

4 What is the number which its square increases in 42?

a) s ={7, 6}   b) s ={7, - 6}   c) s ={-7, 6}   d) s ={-7, - 6}

5 Two numbers its product is 54 , one of them increases in 3 to the other number , what are the two numbers?

a) s ={6 , 9}   b) s ={6 , -9}   c) s ={-6 , 9}   d) s ={-6 , - 9}

Multiple Choice

[3-4] Solving the quadratic equations by the perfect sequre

Choose The Correct Answer for each of the following:

Solve the following equations in R by the perfect Square:

1. $x^2 + 6x + 9 = 0$ a) $x = 6$ b) $x = -3$ c) $x = 4$ d) $x = 3$

2. $4z^2 - 20z + 25 = 0$ a) $z = \frac{-5}{2}$ b) $z = \frac{-2}{5}$ c) $z = \frac{5}{2}$ d) $z = \frac{2}{5}$

3. $\frac{1}{16} - \frac{1}{2}x + x^2 = 0$ a) $x = \frac{1}{4}$ b) $x = \frac{-1}{4}$ c) $x = \frac{1}{2}$ d) $x = \frac{-1}{2}$

Solve the following equations in R by completing the square:

4. $x^2 - 12x = 13$ a) $s = \{13, 1\}$ b) $s = \{13, -1\}$ c) $s = \{-13, 1\}$ d) $s = \{-13, -1\}$

5. $y^2 - \frac{1}{3}y = \frac{2}{9}$
   a) $\{\frac{3}{2}, \frac{1}{3}\}$ b) $\{\frac{-3}{2}, \frac{1}{3}\}$
   c) $\{\frac{2}{3}, \frac{-1}{3}\}$ d) $\{\frac{-2}{3}, \frac{1}{3}\}$

6. $z^2 + 2\sqrt{5}\,z = 4$
   a) $s = \{3 + \sqrt{5}, 3 - \sqrt{5}\}$ b) $s = \{\sqrt{5} - 3, 3 - \sqrt{5}\}$
   c) $s = \{3 - \sqrt{5}, -3 - \sqrt{5}\}$ d) $s = \{\sqrt{5} + 3, \sqrt{5} - 3\}$

Multiple Choice

[3-5] Using General Law to Solve the Equations.

Choose The Correct Answer for each of the following:

The solution set for the following equations by using the general law in R:

1 $y^2 - 5y - 5 = 0$

a) $s = \{\frac{3+5\sqrt{5}}{2}, \frac{3-5\sqrt{5}}{2}\}$ b) $s = \{\frac{5+3\sqrt{5}}{4}, \frac{3-5\sqrt{5}}{4}\}$

c) $s = \{\frac{5+3\sqrt{5}}{2}, \frac{5-3\sqrt{5}}{2}\}$ d) $s = \{\frac{5+3\sqrt{3}}{2}, \frac{3-3\sqrt{3}}{2}\}$

2 $2x^2 - 8x = -3$

a) $s = \{\frac{4+\sqrt{10}}{2}, \frac{4-\sqrt{10}}{2}\}$ b) $s = \{\frac{2+\sqrt{10}}{2}, \frac{4+\sqrt{10}}{2}\}$

c) $s = \{\frac{4+\sqrt{5}}{4}, \frac{4-\sqrt{5}}{4}\}$ d) $s = \{\frac{2+\sqrt{5}}{2}, \frac{2-\sqrt{5}}{2}\}$

3 $3x^2 - 6(2x+1) = 0$

a) $s = \{2+\sqrt{3}, 2-\sqrt{3}\}$ b) $s = \{2+\sqrt{2}, 2-\sqrt{2}\}$

c) $s = \{2+\sqrt{6}, 2-\sqrt{6}\}$ d) $s = \{6+\sqrt{6}, 6-\sqrt{6}\}$

Determine the roots of equation by using the distinctive:

4 $x^2 - 6x - 7 = 0$

a) Two rational real roots b) Two irrational real roots

c) Two equale real root $(\frac{-b}{2a})$ d) Two unreal roots (the solution set in R $= \varnothing$)

5 What is the value of the constant K which makes the two roots of the equation

$y^2 - (k + 10)y + 16 = 0$ are equaled?

a) k = 2 , -18 b) k = -2 , -18 c) k = 6 , 18 d) k = -6 , -18

Multiple Choice

[3-6] Solving the Fractional Equations

Choose The Correct Answer for each of the following:

Find the solution for each of following equations in R:

1. $\frac{2}{12x^2} - \frac{1}{6} = \frac{1}{4x}$ a) $s = \{2, \frac{1}{2}\}$ b) $s = \{-2, \frac{1}{2}\}$ c) $s = \{2, \frac{-1}{2}\}$ d) $s = \{-2, \frac{-1}{2}\}$

2. $\frac{8x}{5} = \frac{5}{8x}$ a) $s = \{\frac{5}{8}, \frac{-8}{5}\}$ b) $s = \{\frac{5}{8}, \frac{8}{5}\}$ c) $s = \{\frac{5}{8}, \frac{-5}{8}\}$ d) $s = \{\frac{8}{5}, \frac{-8}{5}\}$

3. $\frac{16x - 64}{x^2} = 1$ a) $x = -8$ b) $x = 8$ c) $x = -6$ d) $x = 6$

Find the solution for each of following equations in R:

4. $\frac{2}{x-2} - \frac{3}{x-1} = 1$ a) $s = \{2 + \sqrt{7}, 2 - \sqrt{7}\}$ b) $s = \{1 + \sqrt{3}, 1 - \sqrt{3}\}$ c) $s = \{1 + \sqrt{7}, 1 - \sqrt{7}\}$ d) $s = \{2 + \sqrt{3}, 2 - \sqrt{3}\}$

5. $\frac{3y}{y-4} + \frac{y}{y-2} = \frac{5y^2 - 4y + 8}{y^2 - 6y + 8}$ a) $s = \{4, -2\}$ b) $s = \{-4, -2\}$ c) $s = \{-4, 2\}$ d) $s = \{4, 2\}$

Multiple Choice

[4 - 1]

Graphical Representation of the Equation in the Coordinate Plane

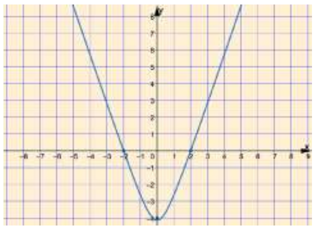
Choose the correct answer for each of the following:

1. The line which its equation is $y = \frac{3}{2}$

a) Intersects two axes **b)** Parallels Y-axis **c)** Parallels X-axis

d) Does not intersect any of the two axes

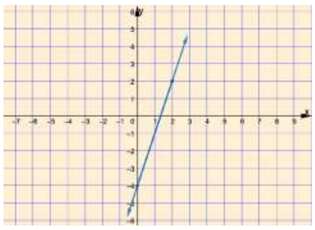
2. Which of the following equations express the equation which is graphically represented in the nearby figure?



a) $y = -3x^2$ b) $y = 2x^2 + 4$

c) $y = x^2 - 4$ d) $y = 3x^2 - 4$

3. Which of the following equations express the equation which is graphically represented in the nearby figure?



a) $y = 3x + 4$ b) $y = 4x + 3$

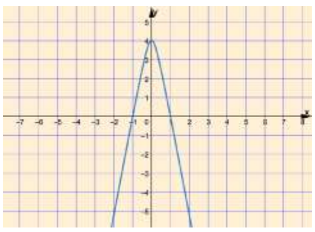
c) $y = - 3x + 4$ d) $y = 3x - 4$

4. Which of the following equations express a linear equation?

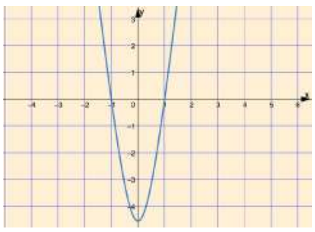
a) $y = x^2 + 1$ b) $y^2 = x + 1$ c) $y^2 = x^2 + 1$ d) $y = x + 1$

5. Which of the following graphic equations express the equation $y = -x^2 + 4$

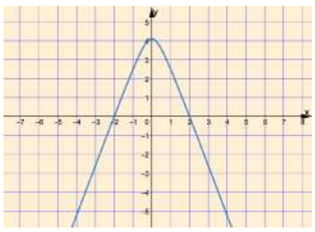
a)



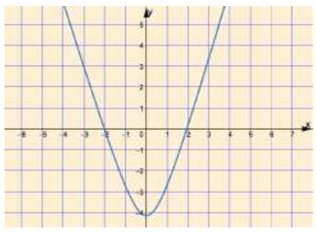
b)



c)



d)



6. We need one of the following to represent the nonlinear equation:

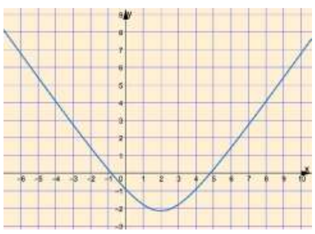
a) One point at least

b) Maximum ,two points

c) Only two points

d) At least, three points

7. What are the coordinates of the curved vertex which is represented in the nearby figur



a) (2,-1) b) (1,2) c) (2,-2) d) (0,2)

[4 - 2]

Slop of the a Line

Choose the correct answer for each of the following:

1 Which slop expresses the slope of the line which passes through the two points (-1,3),(5,-2)

a) $\frac{5}{6}$ b) $-\frac{6}{5}$ c) $\frac{-5}{6}$ d) $\frac{6}{5}$

2 The slop of the line which parallels the y_ axis will be:

a) Zero   b) Undefind   c) Negative   d) Positive

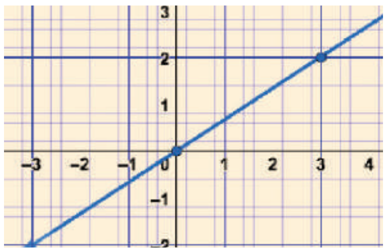
3 The Y- Intercept of the line which its equation 3x-5y=15 is:

a) -5   b) 3   c) 5   d) -3

4 The intersection point of the line which its equation x + y = 6 with the X- axis is.

a) (0,6)   b) (-6,0)   c) (6,0)   d) (0,0)

5 Which of the following lines expresses the line which is represented aside?



a) 2x - 3y = 0   b) 3y + 2x = 0   c) 3y - 2x = 0   d) 2x + 3y = 0

6 The slop of the line which parallels the X- axis is :

a) Zero   b) Undefind   c) Negative   d) Positive

7 What is the slope of the line 3x-2y= -6 ?

a) $-\frac{3}{2}$ b) $-\frac{2}{3}$ c) 3   d) $\frac{3}{2}$

8 The slope of the line which passes through the two points (8,-3),(5,-3) is:

a) Positive   b) Negative   c) Zero   d) Undefind

[4 - 3]

The Equation of the Line

Choose the correct answer for each of the following:

1 The line equation which passes through the two points (-2,-3), (-1,-7) is:

a) $y - 4x = -11$ b) $y - 4x = 11$ c) $4y + x = -11$ d) $y + 4x = -11$

2 The line which its equation is $y + x = 0$, its slope and one of its points are:

a) $m = -1$, (4,4) b) $m = 1$, (4,4) c) $m = -1$,(4,-4) d) $m = 1$, (-4,-4)

3 Use the line equation $y = mx + k$ and find the value of K,m of the line $7y-3x = 21$

a) $m = \frac{3}{7}, k = -3$ b) $m = \frac{7}{3}, k = 3$ c) $m = \frac{3}{7}, k = -3$ d) $m = \frac{3}{7}, k = 3$

4 Which of the following points locates on the line which its equation is: $y + 4x = 0$

a) (1,4) b) (4,-1) c) (4,1) d) (1,-4)

5 The line equation which it is slope is (-1) and its Y-Intercept equals (-2) is :

a) $y + x - 2 = 0$ b) $y + x + 2 = 0$ c) $y + x - 2 = 0$ d) $y - x - 2 = 0$

6 Which equation in form slope - intercept to line through two points (-1,-2), (1,6)

a) $y = -3x + 6$ b) $y = 4x - 2$ c) $y = 4x + 2$ d) $y = 2x + 4$

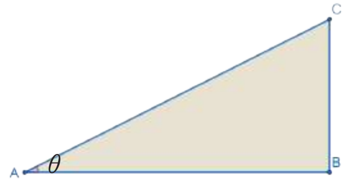
7 The cost of a meal a restaurant is 25,000 dinars, in addition to 3,000 dinars for any extra type of appetizers, which of the equation represents the cost of a meal with (x) of the appetizers?

a) $y = 25x + 3$ b) $y = 25x - 3$ c) $y = 3x + 25$ d) $y = 3x - 25$

Multiple Choice

[4 - 4]
Trigonometric Ratios

Choose the correct answer for each of the following:



1 In the nearby figure, the triangular ratio $\sin\theta$ writes as:

a) $\frac{AB}{AC}$ b) $\frac{BC}{AB}$ c) $\frac{BC}{AC}$ d) $\frac{AB}{AC}$

2 ABC is a right _angled triangle in B , if $\cos A = \frac{3}{5}$ Then tan C equals:

a) $\frac{4}{5}$ b) $\frac{5}{4}$ c) $\frac{4}{3}$ d) $\frac{3}{4}$

3 If $\tan\theta = \frac{1}{\sqrt{3}}$ Then the value of the angle θ equals:

a) $45°$ b) $60°$ c) $90°$ d) $30°$

4 The numerical value of the expression $\sin 30° \cos 30°$ equals:

a) $\frac{1}{\sqrt{3}}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{\sqrt{3}}{4}$ d) $\frac{2}{\sqrt{3}}$

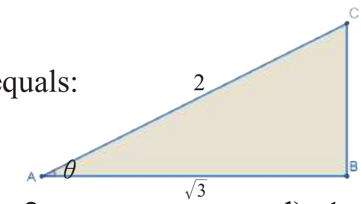
5 The inverse of the ratio $\cos\theta$ is:

a) $\sin\theta$ b) $\sec\theta$ c) $\csc\theta$ d) $\cot\theta$

6 The numerical value of the expression $(\sec 60°)^2 - (\tan 60°)^2$ equals:

a) -1 b) 0 c) 2 d) 1

7 ABC is a right_ angled triangle in B, as in the nearby figure:
The numerical value of the expression $(\sin\theta)^2 + (\cos\theta)^2$ equals:



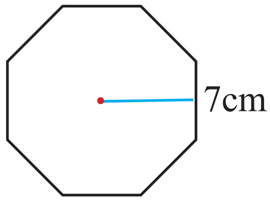
a) -1 b) 0 c) 2 d) 1

8 If $\csc\theta = 2$, then the value of the angle θ is :

a) $45°$ b) $60°$ c) $90°$ d) $30°$

[5 - 1]
Polygons and Polyhedrons (Pyramid and Cone)

Choose the correct answer for each of the following:



1 What is the perimeter of the nearby regular octagonal?

a) 45.5 cm b) 48 cm c) 38.3 cm d) 56 cm

2 The perimeter of a square which its area is $225m^2$ is :

a) 25m b) 20 m c) 15 m d) 60 m

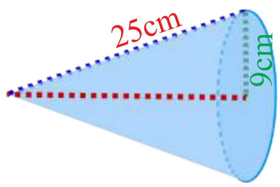
3 The perimeter of a regular pentagonal which its apothem is 3m and the radius of its circle is 5m,is:

a) 16.2 m b) 40 m c) 16 m d) 10.49 m

4 The area of a regular 7-gon which its apothem is 6cm and the length of its side is 7.5 cm, is:

a) $157.5\ cm^2$ b) $28.5\ cm^2$ c) $28\ m^2$ d) $9975\ m^2$

5 The lateral area of the cone in the nearby figure is:



a) $360\pi\ cm^2$ b) $450\pi\ cm^2$ c) $369\pi\ cm^2$ d) $1640\pi\ cm^2$

6 The volume of a pyramid which the length of each side of its square base is 18cm and its height is 20cm, is:

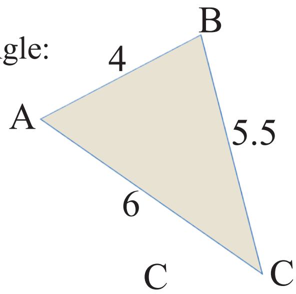
a) $2160cm^3$ b) $120\ cm^3$ c) $260\ cm^3$

d) $134\ cm^3$

[5 - 2]
Triangles

Choose the correct answer for each of the following:

1 Arrange the angles from the smallest to the greatest in the nearby triangle:



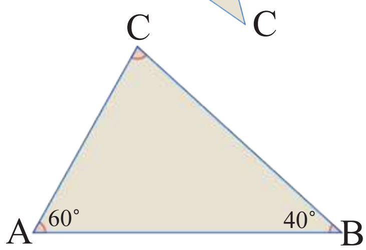
a $m\angle C, m\angle A, m\angle B$

b. $m\angle A, m\angle B, m\angle C$

c. $m\angle B, m\angle C, m\angle A$

d. $m\angle C, m\angle B, m\angle A$

2 Arrange the sides from the longest to the shortest in the nearby triangle



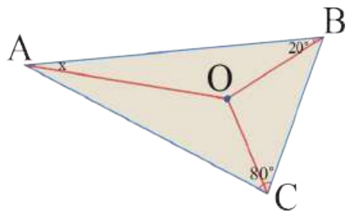
a. $\overline{BC}, \overline{AC}, \overline{AB}$

b. $\overline{AB}, \overline{BC}, \overline{AC}$

c. $\overline{AC}, \overline{BC}, \overline{AB}$

d. $\overline{AB}, \overline{AC}, \overline{BC}$

3 If O is the meeting point of the angles bisectors of the triangle ABC in the nearby figure, then the value of x is:



a) 20° b) 40° c) 30° d) 50°

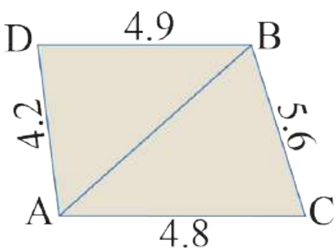
4 The triangle ABC has two median segments $\overline{AD}, \overline{CE}$ Which meet in point O (AD=36cm, CE=24cm) , B is vertex of ABC, then the value of $\overline{OE}$ is:

a) 8 cm b) 24 cm c) 16 cm d) 12 cm

5 In the question (4), the value of $\overline{AO}$ is:

a) 6 cm b) 12 cm c) 24 cm d) 14 cm

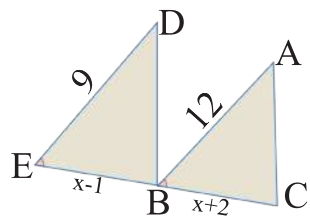
6 The similarity ratio between the two triangles ADB,ACB is:



a. $\frac{8}{7}$ b. $\frac{7}{8}$

c. 7 d. 8

7 If the two triangles DBE, ABC are similar and there were the two angles, $m\angle ABC = m\angle DEB$ then the value of x is:

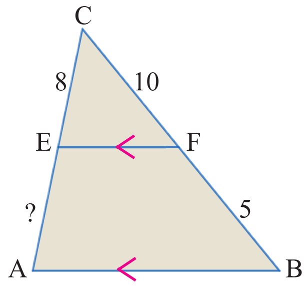


a) 8 b) 12 c) 10 d) 6

[5 - 3]
Proportion and measurement in Triangles

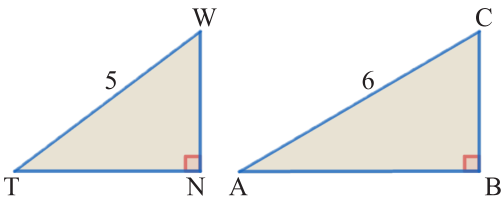
Choose the correct answer for each of the following:

1 If $\overline{AB} \parallel \overline{EF}$, then the length of the line segment AE is:



a) 4 b) 5 c) 2 d) 10

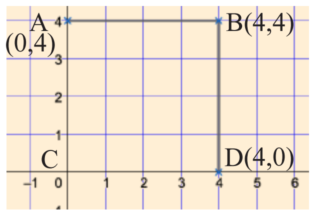
2 If $\Delta ACB \sim \Delta TWN$,If you knew that the height of the triangle TWN is (3), then area of the triangle ABC



a) 42.3 cm^2 b) 43.2 cm^2 c) 40.2 cm^2 d) 40.3 cm^2

After dilation it in a geometric proportion its ratio is $\frac{4}{3}$

A picture was drawn to be as it is shown in the nearby figure:



Choose the correct answer for the questions (3_6)

3 The coordinates of the point A before the dilation are:

a) (0,3) b) (3,0) c) (3,3) d) (0,0)

4 The coordinates of the point B before the dilation are:

a) (0,3) b) (3,0) c) (3,3) d) (0,0)

5 The coordinates of the point C before the dilation are:

a) (0,3) b) (3,0) c) (3,3) d) (0,0)

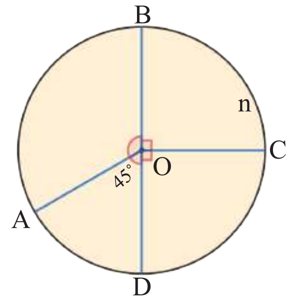
6 The coordinates of the point D before the dilation are:

a) (0,3) b) (3,0) c) (3,3) d) (0,0)

[5 - 4]

The Circle

Look at the nearby figure and choose the correct answer for the questions (1_4) :



1 The measure of $\angle AOB$ is:

a) 180° b) 135° c) 90° d) 45°

2 The measure of $\overset{\frown}{AB}$ is:

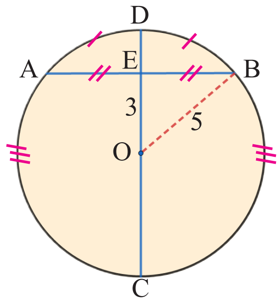
a) 180 b) 90 c) 135 d) 45

3 The measure of $\overset{\frown}{ABC}$ is:

a) 180 b) 90 c) 225 d) 135

4 The measure of $\overset{\frown}{BC}$ is:

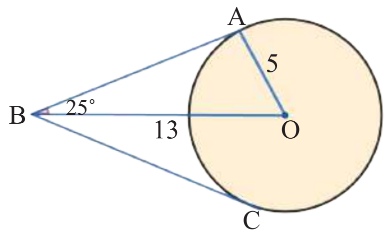
a) 90 b) 42 c) 45 d) 135



5 The length of the chord AB in the nearby figure is:

a) 12 b) 10 c) 6 d) 8

Look at the nearby figure and choose the correct answer for the questions (6 - 7):



6 The measure of $\angle AOB$ is:

a) 115° b) 120° c) 65° d) 90°

7 The length of the segment BC is:

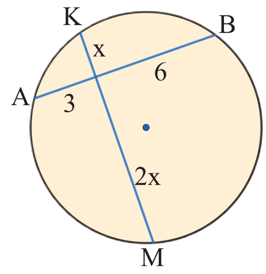
a) 10 b) 14 c) 12 d) 5

Multiple Choice

[5 - 5]

Triangle and Circle, Line Segments and Circle

Look at the nearby figure and choose the correct answer for questions (1- 2) :



1 The value of x is:

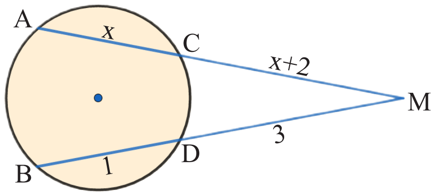
a) 2 b) 6

c) 9 d) 3

2 The length of $\overline{MK}$ Is:

a) 12 b) 9 c) 5 d) 4

Look at the nearby figure and choose the correct answer for questions (3_5)



3 The value of x is:

a) 2 b) 3 c) 1 d) 4

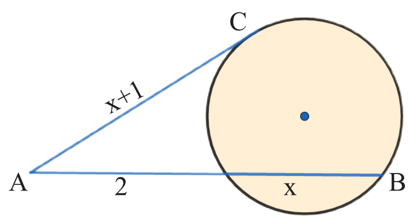
4 The length of $\overline{BM}$ is:

a) 4 b) 6 c) 5 d) 2

5 The length of $\overline{AM}$ is:

a) 4 b) 2 c) 6 d) 3

Look at the nearby figure and choose the choose the correct answer for the questions (6_8):



6 The value of X is:

a) 1 b) $\sqrt{2}$ c) $\sqrt{3}$ d) 0

7 The length of the tangent is:

a) $\sqrt{2}+1$ b) $\sqrt{3}+1$ c) 4 d) $\sqrt{5}+1$

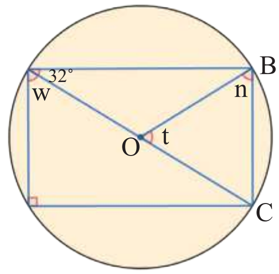
8 The length of $\overline{AB}$ is:

a) $6+\sqrt{3}$ b) $2+\sqrt{3}$ c) $5+\sqrt{3}$ d) $4+\sqrt{3}$

[5 - 6]

Angles and Circle

Look at the nearby figure and choose the correct answer for questions (1- 3) :



1 The measure of the angle (w) is:

a) 45° b) 58°

c) 90° d) 32°

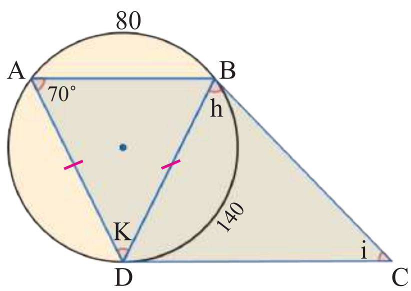
2 The measure of the angle (t) is:

a) 45° b) 64° c) 32° d) 48°

3 The measure of the angle (n) is:

a) 45° b) 64° c) 32° d) 84°

Look at the nearby figure and choose the correct answer for questions (4_6)



4 The measure of the angle (h) is:

a) 70° b) 72° c) 90° d) 80°

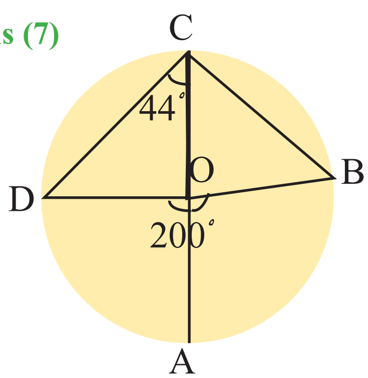
5 The measure of the angle (i) is:

a) 39° b) 70° c) 40° d) 45°

6 The measure of the angle (k) is:

a) 70° b) 30° c) 40° d) 78°

Look at the nearby figure and choose the correct answer for questions (7)



7 The measure of $\overset{\frown}{ABC}$ is:

a) 56 b) 112 c) 65 d) 82

[6 - 1]

Design a Survey Study and analysis its results

Choose the correct answer for each of the following:

1 Which one of the measurements of the centeral tendency (if it is found) is suitable to the following data: 8, 8, 12, 11, 15, 15, 16, 21, 23, 27, 31, 70?

a) Range b) Mode c) Median d) Arithmetic mean

2 Which of the centeral tendency measurements (if it is found) for the following data 2, 3, 4, 5,6, 7?

a) Range b) Mode c) Median d) Arithmetic mean

3 Which of the centeral tendency measurements (if it is found) 18, 1, 3, 16, 23, 3, 2?

a) Range b) Mode c) Median d) Arithmetic mean

4 The range of the following data 24, 18, 32,24,22,18 is:

a) 18 b) 32 c) 14 d) 50

5 Which measurements is not from the measurements of the centeral tendency?

a) Range b) Mode c) Median d) Arithmetic mean

6 The extreme value of these data 4, 30, 3, 5, 5, 6, 5, 3 is:

a) 3 b) 5 c) 5 d) 30

7 The median will be the best for the measurements of the centeral tendency for the data which:

a) Have extreme value with great gaps in the middle

b) Don't have extreme value there are not great gaps in the middle

c) Have extreme value and there are not great gaps in the middle

d) Don't have extreme value there are not great gaps in the middle

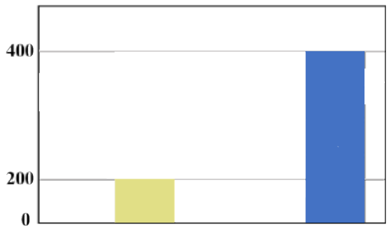
Multiple Choice

[6 - 2]
Data and misleading statics

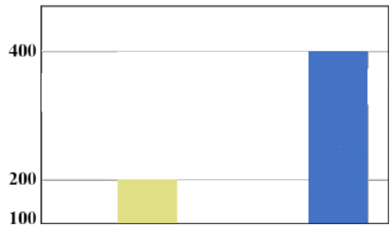
Choose the correct answer for each of the following:

1 Which graphic diagram is best in representing certain data:

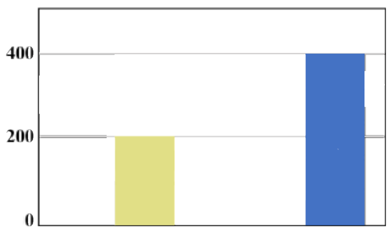
a)



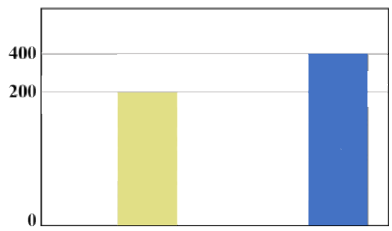
b)



c)



d)



2 The graphic diagram will be misleading if:

a) Starts from zero and the intervals are not equal
b) Does not start from zero and the intervals are not equal
c) Does not start from zero and the intervals are equal
d) Starts from zero and the intervals are equal

3 In a questionnaire includes 6 teachers about the work, 4 of them prefer to work in the morning, the person who does the questionnaire had written the following (2 teachers out of 3 prefer to work in the morning) why does this announcement consider misleading?

a) The sample is so large
b) The sample should include building workers
c) The sample is so small
d) The sentence should be (a teacher should be separated out of 2 teachers)

4 In atrade of shop offer kind the cheese to 12 persons to evaluate before viewing it , 6 persons were a dmired it , According to that the product said the product is good because the ratio whose prefer it was 6 to 3

a) Data is not misleading because the ratio 6 to 3 is high ratio .

b) Data is not misleading because the people who prefer it of the rest

c) Misleading data , al though people whose prefer it double of rest .

d) Misleading data because the sample was medium size .

Multiple Choice

[6 - 3]

Experimental Probability and theoretical Probability

Choose the correct answer for each of the following:

1 If E_1, E_2 are two disjoint events, then P (E_1 or E_2) equals:

a) $P(E_1) - P(E_2)$ b) $P(E_1) \times P(E_2)$ c) $P(E_1) + P(E_2)$ d) $\frac{P(E_1)}{P(E_2)}$

2 Ahmed had scored 20 goals out of 25 attempts, which percentage for the experimental probability that Ahmed will record in the next attempt?

a) 50% b) 60% c) 70% d) 80%

3 Tamara had released the disc spinner once, which percentage for the theoretical probability that the pointer will indicate to white number ?

a) 35% b) 30% c) 12.5% d) 20%

4 By throwing the two dices once, the probability of getting two number their sum is 3 or the result of multiplying them is 3,is:

a) $\frac{1}{3}$ b) $\frac{1}{9}$ c) $\frac{2}{3}$ d) 1

5 E_1 , E_2 are two disjoint event, if P $P(E_1 or E_2) = \frac{5}{6}$ and $P(E_2) = \frac{2}{3}$, then P (E_1) equals:

a) $\frac{1}{3}$ b) $\frac{1}{6}$ c) $\frac{1}{4}$ d) $\frac{1}{5}$

6 By throwing the two dices, the probability of getting two number their sum is 13,is:

a) 3 b) 2 c) 1 d) 0

[6 - 4]
Compound Events

Choose the correct answer for each of the following:

1. E_1, E_2 are two independent events, where $P(E_1)$= 0.3 and $P(E_2)$ = 0.9, the probability of happening E_1,E_2 together is:

a) 1.2 b) 0.6 c) 0.27 d) 0.3

2. Mustafa had thrown a dice and a coin , the probability of appearing a number greater than 5 on the dice and the writing (tial) on the coin is :

a) $\frac{2}{3}$ b) $\frac{1}{3}$ c) $\frac{1}{12}$ d) 3

3. A box has 5 red balls and 4 green balls.
E_1 represent pulling a red ball, E_2 represents pulling a green ball without returning the red one, the probability of happening them together is :

a) $\frac{10}{9}$ b) $\frac{5}{18}$ c) $\frac{19}{18}$ d) $\frac{1}{18}$

4. E_1, E_2 are two dependent events, the probability of happening them together is:

a) $P(E_1 \text{ and } E_2) = P(E_1) + P(E_2 \text{ after } E_1)$

b) $P(E_1 \text{ and } E_2) = P(E_1) + P(E_2 \text{ before } E_1)$

c) $P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2 \text{ after } E_1)$

d) $P(E_1 \text{ and } E_2) = P(E_1) \times P(E_1 \text{ after } E_2)$

5. The relation $P(E_1 \text{ and } E_2) = P(E_1) \text{ X } P(E_2)$ between the two events E_1, E_2, where they are:

a) No relation between them b) Independent c) dependent d) Another thing

6. E_1, E_2 are two disjoint events, $P(E_2) = 0.45$ $P(E_1) = 0.15$ the probability of happening E_1 or E_2 is:

a) 0.0675 b) 3 c) 0.6 d) 0.3

CONTENTS

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