

Republic of Iraq
Ministry of Education
General Directorate of Curricula



Series of Maths Books for Intermediate Stage

Mathematics

Third Intermediate

Revised by

A specialized Committee in the Ministry of Education

Fourth Revised Edition

2023

This series (Maths for Intermediate stage) has been edited by a special team of specialists in Ministry of Education/ General Directorate of curricula with participation of specialists from universities professors in Ministry of higher Education according to international standards to achieve the goals of designing modern syllabus which helps the students to be:

Successful learners long life

Self-esteem individuals

Iraqi citizens feeling proud

Scientific supervisor

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استناداً إلى القانون يوزع مجاناً ويمنع بيعه وتداوله في الأسواق

INTRODUCTION

The Maths subject is considered one of the basic courses that helps students to acquire educational abilities to develop his thinking and solving problems and it helping to deal with difficult situations in his life.

As a starting point of attention by the Ministry of Education represented by the General Directorate of curricula to develop the curricula in general and specially of Maths in order to go a long with the technological and scientific development in different fields of life. A plan has set up to edit the series of Maths books for the three stages. Primary stage has been achieved and the work started to continue the series by editing the books of intermediate stage.

The series of new Iraqi Maths Books as a part of General frame work of curricula that reinforces the basic values as Iraqi identity, forgiveness, respecting different opinions, social justice and offering equal chance for creativity and it also reinforces abilities of thinking and learning, self-efficiency, action and citizenship efficiency.

The series of Iraqi Maths books has been built on student- centered learning according to international standards.

The series of Iraqi maths books for intermediate stage has been built on six items: learn, make sure of your understanding, solve the exercises, solve the problems, think and write. The Maths book for third intermediate stage contains four basic fields: The numbers and the operations, Algebra, Geometry and Measurement, Statistics and Probabilities for each field. The books.

The maths books have distinguished by presenting material in modern styles that attract and help the student to be active through presenting drills, exercises and environmental problems in addition there are extra exercises at the end of the book that are different from the exercises and drills in the lessons because they are objective so the student can answer through multiple choices and that prepare the student to participate the international competitions.

This book is an expansion for the series of developed Maths books for primary stage and it is also considered as support for the developed syllabus in maths and it also has a teachers book so we hope in applying them, the student will gain scientific and practical skills and develop their interest to study Maths.

We hope God help us to serve our country and our sons

Authors

Relations and Inequalities in Real Numbers

lesson 1-1 Ordering Operations in Real Numbers.

lesson 1-2 Mapping.

lesson 1-3 The Sequences.

lesson 1-4 Compound Inequalities.

lesson 1-5 Absolute Value Inequalities.

Tsunami wave is moved in a great rapidity in the Seas, but its rapidity becomes greater when it reaches to beach due to the effect of its huge energy. It terribly strikes beach to cause mass destruction. We can calculate speed of Tsunami by using ($v = \sqrt{9.6 d}$ m/s, where (d) represents the deep of water in metre).

Pretest

Classify the number if it is rational or irrational number:

1 $\sqrt{25}$

2 $\sqrt{7}$

3 $\frac{0}{\sqrt{3}}$

4 $\sqrt{\frac{16}{25}}$

5 $\sqrt{\frac{49}{5}}$

6 $\frac{30}{4}$

7 $-6\frac{3}{2}$

8 $-\sqrt{8}$

Estimate the following square roots by near them to the nearest tenth, then represent them on the straight line of numbers:

9 $\sqrt{2} \approx \dots\dots$

10 $-\sqrt{3} \approx \dots\dots$

11 $\sqrt{\frac{6}{25}} \approx \dots\dots$

12 $\sqrt{\frac{81}{49}} \approx \dots\dots$

Compare between the real numbers by using the symbols ($<$, $>$, $=$):

13 $\sqrt{5}$ $2\frac{1}{3}$

14 1.25 $\sqrt{2.25}$

15 $\sqrt{\frac{0}{3}}$ $\frac{0}{6}$

16 $\frac{\sqrt{12}}{\sqrt{3}}$ $\frac{\sqrt{5}}{\sqrt{20}}$

17 Ordering the following real numbers from the least to the greatest.

$\sqrt{7}$, 2.25 , $\sqrt{5}$

18 Ordering the following real numbers from the greatest to the least.

$-3\frac{1}{5}$, $-\frac{7}{3}$, -3.33

Solve the following inequalities in \mathbb{R} by using properties of inequalities in the real numbers:

19 $3x + \frac{2}{5} \geq 4x - \frac{3}{5}$

20 $\frac{3}{7} > z - \frac{9}{14}$

21 $\frac{3y}{8} \geq \frac{2}{7}$

22 $\frac{-4m}{11} < \frac{9}{22}$

23 $6(z - 3) > 5(z + 1)$

24 $4\left(\frac{1}{2}v + \frac{3}{8}\right) > 0$

Simplify the following numerical sentences by using ordering operations on the real numbers:

25 $\sqrt{2}(1 - \sqrt{18}) = \dots\dots\dots$

26 $3\sqrt{12} + 2\sqrt{3} - 4\sqrt{3} = \dots\dots\dots$

27 $\frac{\sqrt{7} - 8\sqrt{7}}{2\sqrt{7}} = \dots\dots\dots$

28 $\frac{6\sqrt{44}}{\sqrt{5}} \div \frac{18\sqrt{11}}{\sqrt{5}} = \dots\dots\dots$

Lesson [1-1]

Ordering Operations in Real Numbers

Learn

Idea of the lesson:

* Simplify the numerical sentences which contain real numbers by using ordering operations.

Vocabulary:

- * Real number.
- * Rationalizing the Denominator
- * Conjugate

Tsunami quake, which occurred in Japan in 2011 is considered one of the greatest quake which happened over the ages. Its speed can be calculated by using the law $v = \sqrt{9.6 d}$ m/s, where d represents the deep of water. What's the approximate speed of Tsunami if the deep of water is 1000m?



[1-1-1] Using ordering operations to simplify numerical sentences .

You have previously learned the natural numbers, whole numbers, integers, rational numbers and real numbers. We can ordering them in the following:

$$N \subset W \subset Z \subset Q \subset R$$

You have also learned how to simplify the numerical sentences by using the ordering of operations in these numbers. We will develop your skills in simplifying the numerical sentences which contain different real numbers include real roots and perfect squares roots, and also simplifying fractions contain roots by applying the properties on them and by using the ordering of operations in the real numbers. We use also (Rationalizing) denominator to simplify sentences by multiplying the conjugate numbers (the result of multiplying two conjugate numbers is a rational number, (the number $2 - \sqrt{3}$ is conjugate to $2 + \sqrt{3}$ because the product them is rational number).

Example (1)

Find the approximate speed of Tsunami if the water deep is 1000m.

$$\begin{aligned} v &= \sqrt{9.6 d} && \text{Law of calculating Tsunami speed, where } d \text{ represents the deep of water} \\ &= \sqrt{9.6 \times 1000} = \sqrt{9600} = 98 \text{ m/sec} && \text{The approximate speed of Tsunami} \end{aligned}$$

Example (2)

Simplifying the following numerical sentences by using the ordering of operations on the real numbers:

$$\begin{aligned} \text{i)} & (\sqrt{12} - \sqrt{18}) (\sqrt{12} + \sqrt{18}) = (2\sqrt{3} - 3\sqrt{2}) (2\sqrt{3} + 3\sqrt{2}) && \text{by using the distribution} \\ &= 2\sqrt{3} (2\sqrt{3} + 3\sqrt{2}) - 3\sqrt{2} (2\sqrt{3} + 3\sqrt{2}) = 12 + 6\sqrt{6} - 6\sqrt{6} - 18 = -6 \\ \text{ii)} & \left(\sqrt[3]{\frac{8}{27}} - \sqrt{\frac{2}{3}} \right) \div \left(\frac{3\sqrt{2} - 2\sqrt{3}}{\sqrt{27}} \right) = \left(\frac{2}{3} - \sqrt{\frac{2}{3}} \right) \div \left(\frac{3\sqrt{2} - 2\sqrt{3}}{3\sqrt{3}} \right) \\ &= \frac{2\sqrt{3} - 3\sqrt{2}}{3\sqrt{3}} \times \frac{-3\sqrt{3}}{2\sqrt{3} - 3\sqrt{2}} = -1 \end{aligned}$$

Example (3)

Simplifying the following numerical sentences by using the ordering of operations on the real numbers, then write the result to nearest tenth:

$$\begin{aligned} \text{iii)} & \sqrt{12} (\sqrt{3} - \sqrt{8}) - 6 = 2\sqrt{3} (\sqrt{3} - 2\sqrt{2}) - 6 = 2\sqrt{3} \times \sqrt{3} - 2\sqrt{3} \times 2\sqrt{2} - 6 \\ &= 6 - 4\sqrt{3 \times 2} - 6 = -4\sqrt{6} \approx -4 \times 2.4 = -9.6 \\ \text{iv)} & (-27)^{\frac{1}{3}} \left(\frac{1}{9} \sqrt{7} - \frac{1}{9} \sqrt{28} \right) = \sqrt[3]{-27} \left(\frac{1}{9} \sqrt{7} - \frac{2}{9} \sqrt{7} \right) = -3 \left(\frac{1}{9} \sqrt{7} - \frac{2}{9} \sqrt{7} \right) \\ &= -\frac{1}{3} \sqrt{7} + \frac{2}{3} \sqrt{7} = \frac{1}{3} \sqrt{7} \approx 0.9 \end{aligned}$$

Note : $a^{\frac{n}{m}} = \sqrt[m]{a^n}$

Example (4)

Simplify the following numerical sentences by using rooting the denominator and ordering operations on the real numbers.

$$\text{i) } \frac{7 - \sqrt{5}}{\sqrt{5}} = \frac{7 - \sqrt{5}}{\sqrt{5}} \times 1 = \frac{7 - \sqrt{5}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}(7 - \sqrt{5})}{\sqrt{5}\sqrt{5}} = \frac{7\sqrt{5} - \sqrt{5}\sqrt{5}}{5} = \frac{7\sqrt{5} - 5}{5}$$

$$\begin{aligned} \text{ii) } \frac{\sqrt{21}}{2\sqrt{3} - \sqrt{7}} &= \frac{\sqrt{21}}{2\sqrt{3} - \sqrt{7}} \times \frac{2\sqrt{3} + \sqrt{7}}{2\sqrt{3} + \sqrt{7}} = \frac{\sqrt{3}\sqrt{7}(2\sqrt{3} + \sqrt{7})}{(2\sqrt{3} - \sqrt{7})(2\sqrt{3} + \sqrt{7})} && \text{Multiplying by conjugates} \\ &= \frac{6\sqrt{7} + 7\sqrt{3}}{12 - 7} = \frac{6\sqrt{7} + 7\sqrt{3}}{5} && \text{The denominator is a difference between two squares} \end{aligned}$$

1-1-2] Using calculator and approximation to Simplify Numerical Sentence

You have previously learned how to simplify the numerical sentences which contain integer negative powers and scientific form for number by using calculator. Now, you will develop your skills by simplify the numerical sentences which contain numbers raising to rational powers, in addition to the integers by using calculator to write the result in approximated way.

Example (5)

Calculate the powers for each of the following, then write the result which should be approximated to two decimal places, if it is not an integer:

$$\begin{aligned} \text{i) } 9^{-\frac{3}{2}} &= (3^2)^{-\frac{3}{2}} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27} \approx 0.04 && \text{ii) } (\sqrt{7})^2 = (7^{\frac{1}{2}})^2 = 7 \\ \text{iii) } 2^{\frac{5}{3}} \times 2^{\frac{1}{3}} \times 2^{-\frac{3}{2}} &= 2^{\frac{10+2-9}{6}} = 2^{\frac{1}{2}} = \sqrt{2} \approx 1.41 && \text{iv) } 5^2 \div 5^{\frac{3}{2}} = 5^{\frac{4}{2} - \frac{3}{2}} = 5^{\frac{1}{2}} = \sqrt{5} \approx 2.24 \end{aligned}$$

Use the ordering of operations and write the result which should be approximated to two decimal places by using a calculator for each of the following:

$$\begin{aligned} \text{v) } \left(\frac{1}{2}\right)^2 + 3^{-2} - 2^{\frac{3}{2}} &= \frac{1}{2^2} + \frac{1}{3^2} - \sqrt{2^3} = \frac{1}{4} + \frac{1}{9} - \sqrt{8} \approx 0.25 + 0.11 - 2.83 = -2.47 \\ \text{vi) } 8^{\frac{1}{3}} - (-8)^0 + 3^2 \times 3^{\frac{1}{2}} &= \sqrt[3]{8} - 1 + 3^{\frac{5}{2}} = \sqrt[3]{8} - 1 + \sqrt{3^5} \approx 2 - 1 + 9 \times 1.73 = 16.57 \end{aligned}$$

Example (6)

Use calculator to write the result in the scientific form for the number which should be approximated to the nearest two decimal places:

$$\begin{aligned} \text{i) } 7.6 \times 10^{-4} - 0.41 \times 10^{-3} &= 7.6 \times 10^{-4} - 4.135 \times 10^{-4} = 3.465 \times 10^{-4} \approx -3.47 \times 10^{-4} \\ \text{ii) } 0.052 \times 10^4 + 7.13 \times 10^2 &= 5.2 \times 10^2 + 7.13 \times 10^2 = 12.33 \times 10^2 \approx 1.23 \times 10^3 \\ \text{iii) } (7.83 \times 10^{-5})^2 &= (7.83 \times 10^{-5})(7.83 \times 10^{-5}) = 61.3089 \times 10^{-10} \approx 6.13 \times 10^{-9} \\ \text{iv) } 4.86 \times 10^2 \div 0.55 \times 10^5 &= (4.86 \div 0.55) \times 10^2 \times 10^{-5} \approx 8.84 \times 10^{-3} \end{aligned}$$

Make sure of your understanding

Simplify the following numerical sentences :

1 $(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3}) = \dots$

2 $(\sqrt{7} - \sqrt{2})^2 = \dots$

3 $(\sqrt{125} - \sqrt{20})(\sqrt[3]{\frac{8}{27}}) = \dots$

4 $\frac{4\sqrt{12}}{5\sqrt[3]{-27}} \div \frac{2\sqrt{24}}{\sqrt{8}} = \dots$

Questions 1-4
are similar
to example 2

Simplify the following numerical sentences , and write the result to the nearest tenth:

5 $\sqrt{7}(\sqrt{28} - \sqrt{2}) - 5 \approx \dots$

6 $(-125)^{\frac{1}{3}}(\frac{1}{10}\sqrt{3} - \frac{1}{4}\sqrt{12}) \approx \dots$

Questions 5-6
are similar
to example 3

Simplify the following numerical sentences by rooting the denominator and ordering operations on the real numbers:

7 $\frac{1 - \sqrt{3}}{4\sqrt{3}} = \dots$

8 $\frac{1 - \sqrt{20}}{\sqrt{5}} = \dots$

9 $\frac{\sqrt{50} - \sqrt{3}}{2\sqrt{3}} - \frac{10 - \sqrt{6}}{2\sqrt{6}} = \dots$

Questions 7-9
are similar
to example 4

Use the ordering of operations and write the result which should be approximated to two decimal places by using the calculator for each of the following:

10 $(\frac{1}{3})^2 + 3^{-3} - 3^{\frac{3}{2}} \approx \dots$

11 $27^{\frac{1}{3}} - (-9)^0 + 3^2 \times 5^{\frac{1}{2}} \approx \dots$

Questions 10-11
are similar
to example 5

Use the calculator to write the result in the scientific form of the number which should be approximated to the nearest two decimal places:

12 $6.43 \times 10^{-5} - 0.25 \times 10^{-3} = \dots$

13 $(9.23 \times 10^{-3})^2 = \dots$

Questions 12-13
are similar
to example 6

Solve the Exercises

Simplify the following numerical sentences:

14 $(\sqrt{18} - \sqrt{50})(\frac{-27}{64})^{\frac{1}{3}} = \dots$

15 $\frac{\sqrt{12}}{3\sqrt[3]{125}} \div \frac{5\sqrt[3]{8}}{\sqrt{25}} = \dots$

Simplify the following numerical sentences , and write the result to the nearest tenth.

16 $7\sqrt{\frac{2}{49}} - 3\sqrt{\frac{8}{81}} + \sqrt{\frac{18}{36}} \approx \dots$

Simplify the following the numerical sentences by using rooting the denominator and the ordering of operations in the real numbers :

17 $\frac{\sqrt{7} - 3\sqrt{5}}{\sqrt{7} + 3\sqrt{5}} = \dots$

18 $\frac{\sqrt{33} - \sqrt{11}}{\sqrt{99}} - \frac{\sqrt{60} - \sqrt{5}}{5\sqrt{15}} = \dots$

Solve the problems

19 Satellites: We essentially use satellite in communications, such as TV signals, telephone calls in all over the world, weather forecasts and tracking of hurricanes. The satellites rotate around earth in limited speed and special orbits. The orbital speed of moon is calculated by the following relation $v = \sqrt{\frac{4 \times 10^{14}}{r}}$ m/sec, where r represents the radius of orbit (the distance of moon from the earth centre), what is the speed of moon if the orbit radius is 300 km?



20 Fighting fires: We can calculate the speed of flowing water which releases by fire trucks by using the following law $v = \sqrt{2hg}$ foot/sec, where h represents the maximum height of water, and (g) represents the acceleration speed of earth (32 foot/sec^2). To fire fighting in the forests, the firefighters in the Civil Defence need to pump water in height of 80 foot. Is it enough to use a pumper releases water in a speed of 72 foot/sec,?



1 foot = 30cm
Measuring unit in French system

21 Geometry: Find the area of triangle which topped a front of house if its height $(\sqrt{18} - \sqrt{3})$ meter and its base length is $(3\sqrt{2} + \sqrt{3})$ meter.



Think

22 Challenge: Prove the correct of the following:

$$(7^{\frac{1}{3}} - 5^{\frac{1}{3}}) (7^{\frac{2}{3}} + 7^{\frac{1}{3}} 5^{\frac{1}{3}} + 5^{\frac{2}{3}}) = 2$$

23 Correct the mistake: Shaker wrote the result of adding two numbers as follow:

$$8.4 \times 10^{-3} + 0.52 \times 10^{-2} = 1.36 \times 10^{-3}$$

Determin Shaker's mistake and correct it.

24 Numerical sense: Does the number $\sqrt{125}$ locate between the two numbers 10.28 and 11.28?

Write

The result of adding by approximation to the nearest tenth:
 $6^{\frac{3}{2}} + 5^{\frac{3}{2}} = \dots$

Lesson [1-2] Mappings

Idea of the lesson:

*Identify the mapping and its types and how it can be represented graphically in the coordinate plane and Identify the composition of mappings

Vocabulary:

- *The relation.
- *Ordered pair .
- *Cartesian product .
- *The mapping.
- *Domain and Co-domain and the Range.
- *The composition of mapping.

Learn

the group X represents the archaeological locations in Iraq $x = \{\text{Ishtar gate, Awr, Al- Hadar}\}$. Assume the group B represents some Iraqi cities $y = \{\text{Baghdad, Al- Hila, Al- Nasiriya, Al-Mosul, Arbil}\}$. The relation $R: X \rightarrow y$ which represents the connection of each archaeological location with the city which it sits in: $R = \{(\text{Al-Nasiriya, Awr}), (\text{Al-Mosul, Al-Hadar}), (\text{Babylon, Istar gate})\}$, X represents its domain and Y represents its Co-domain.



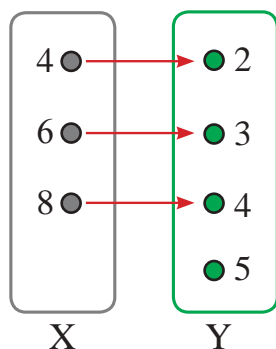
[1-2-1] Mapping and its representation in the coordinate plane

You have previously learned the relation from set X to the set Y and it is subset (set of ordered pairs (x,y) , where the first projection (the first coordinate) belongs to the set X and the second projection (the second coordinate) belongs to the set Y according to the Cartesian product $X \times Y$. you will learn the mapping $R: X \rightarrow Y$ and how to represent it in an arrowy diagram and represent it (graphically) and identify its types.

The mapping: Let R relation from the set X to the set Y and each element in set X has one from in Y then the relation R can be called the mapping from X to Y , $R: X \rightarrow Y$. We called the set X the (domain) , and the set Y (Co-domain) , each element in Y connected with element from X and represents a from for it the set of all form in the Co-domain is called the (Range) , and rule which transfers the element into its form is called the connection rule (mapping rule) and we refer to it by $R (X) , (X, Y)$.

Example (1)

If $R: X \rightarrow Y$ represents a mapping of connection rule $(y = \frac{1}{2}x)$ from the set $X = \{4, 6, 8\}$ to the set, $Y = \{2, 3, 4, 5\}$, and write the mapping in ordered pairs form ,then represent the mapping in an arrowy diagram and determine the domain and the range of the mapping.



The arrowy diagram explains the relation of connecting the elements of the two sets within the connection rule

$$Y = R(X) = y = R(x) = \frac{1}{2} x$$

$$4 \rightarrow 2, 6 \rightarrow 3, 8 \rightarrow 4$$

So the set of mapping $R = \{(4,2), (6,3), (8,4)\}$ Domain: is the set of first coordinates of the ordered pairs in R, and it is the set $\{4, 6, 8\}$

Range: is the set of the second coordinates of the ordered pairs in R, and it is the set $\{2, 3, 4\}$

Note: the range is a subset from the Co-domain of the mapping we see that the range \neq Co-domain ($R_p \neq (Y)$)

Example (2) The following table represents the relation between the weight (kg) and the price of fish ($y = f(x)$).

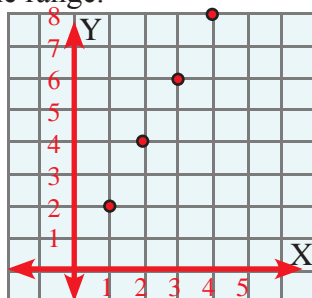
Does the relation represent a mapping?

If it is a mapping, then write a connection rule

and Determine the domain and the range.

and represent in the plane.

x = weight (kg)	y = price in thousands dinars
1	2
2	4
3	6
4	8



The connection rule $y = 2x$

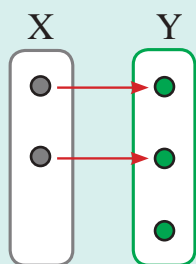
The domain = $\{1, 2, 3, 4\}$ the range = $\{2, 4, 6, 8\}$

[1-2-2] The types of mappings

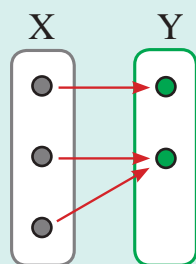
The mapping will be $f: X \rightarrow Y$

i) Surjective mapping

If the range = the co-domain



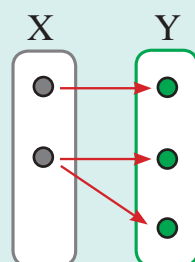
not Surjective mapping and injective



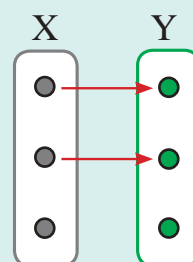
not Injective mapping and surjective

ii) Injective mapping : If each element in Y connects with only one element only in X

iii) Bijective mapping If the mapping is surjective and Injective at the same time



Relation not mapping



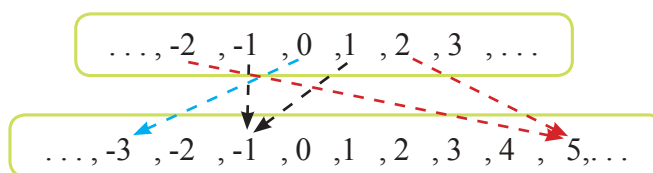
Bijective mapping surjective and injective

Example (3) If $f: \mathbb{Z} \rightarrow \mathbb{Z}$, where $f(x) = 2x^2 - 3$, show the type of the mapping, where \mathbb{Z} represents the set of the integers.

$$f(x) = 2x^2 - 3 \quad f(-2) = 5, \quad f(-1) = -1, \quad f(0) = -3, \quad f(1) = -1, \quad f(2) = 5$$

First: The mapping is not surjective because the range does not equal the co-domain.

Second: The mapping is not injective because $f(1) = f(-1) = -1$ while $1 \neq -1$

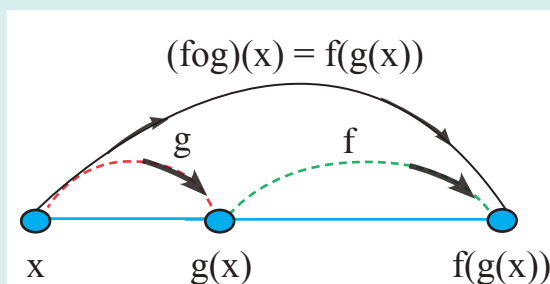


[1-2-3] Composition of mappings

We study a method to find a new mapping from two known mappings which are $f(x)$, $g(x)$ and they are:-

i) The mapping $(f \circ g)(x) = f(g(x))$ and it can be read as f composite g (f after g), and it is a result of finding $g(x)$ at first and then finding its image in the mapping f .

ii) The mapping $(g \circ f)(x) = g(f(x))$ and it can be read as g composite f , and it is the result of finding $f(x)$ at first, and then finding its image in the mapping g .



Example (4) If $f: \mathbb{N} \rightarrow \mathbb{N}$, $f(x) = 2x + 1$, and $g: \mathbb{N} \rightarrow \mathbb{N}$, $g(x) = x^2$, Find the two mappings composition, i) $(f \circ g)(3)$ ii) $(g \circ f)(3)$, what you see? iii) if $(f \circ g)(x) = 33$, find the value of x

i) Find $(f \circ g)(3)$

$$\begin{aligned}(f \circ g)(3) &= f(g(3)) = f(3^2) \\ &= f(9) = 2 \times 9 + 1 \\ &= 19\end{aligned}$$

$$\begin{aligned}\text{ii) } (g \circ f)(3) &= g(f(3)) \\ &= g(2 \times 3 + 1) \\ &= g(7) = 7^2 = 49\end{aligned}$$

Note: $(f \circ g)(3) \neq (g \circ f)(3)$

iii) $(f \circ g)(x) = f(g(x)) = f(x^2) = 2x^2 + 1$

$$2x^2 + 1 = 33 \Rightarrow 2x^2 = 32 \Rightarrow x^2 = 16 \Rightarrow x = 4 \text{ or } x = -4 \text{ neglect}$$

Make sure of your understanding

Write a connection rule for the mapping and represent it in an arrowy diagram and write the domain and the range of it:

1 $f = \{(1,2), (2,3), (3,4), (4,5)\}$

2 $g = \{(1,3), (2,5), (3,7), (4,9)\}$

Questions 1-2
are similar
to example 1

Write a connection rule for the following mappings and represent them in the coordinate plane and write their domain and range:

3 $f = \{(1,0), (2,0), (3,0), (4,0)\}$

4 $g = \{(0,0), (1,-1), (2,-2), (3,-3)\}$

Questions 3-4
are similar
to example 2

5 If the mapping $f: \mathbb{N} \rightarrow \mathbb{N}$, where $f(x) = 3x + 2$. Show if the mapping is surjective or not.

Questions 5
are similar
to example 3

6 Assume the two mappings $f: \mathbb{Z} \rightarrow \mathbb{Z}$, where $f(x) = 3x + 1$ and $g: \mathbb{Z} \rightarrow \mathbb{Z}$, where $g(x) = 2x + 5$. Find the value of x if $(f \circ g)(x) = 28$.

Questions 6-7
are similar
to example 4

7 If $f: \mathbb{N} \rightarrow \mathbb{N}$, where $f(x) = 5x + 2$ and $g: \mathbb{N} \rightarrow \mathbb{N}$, where $g(x) = x + 3$.

Write the mapping $(f \circ g)$ by writing its ordered pairs.

Solve the Exercises

8 If $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$, and $f: A \rightarrow B$ is defined as follow:

$f = \{(1,4), (2,5), (3,6)\}$, Draw the arrowy diagram of the mapping and represent it in the coordinate plane.

9 If $f: A \rightarrow \mathbb{Z}$, where $f(x) = x^2$ and the set $A = \{-2, -1, 0, 1, 2\}$. Represent the mapping in the coordinate plane, and show if the mapping is injective or not?

10 Assume $f: \mathbb{N} \rightarrow \mathbb{N}$, and $g: \mathbb{N} \rightarrow \mathbb{N}$, $f(x) = x^2$, and $g(x) = x + 1$. It is required to find:

i) $(g \circ f)(x)$, $(f \circ g)(x)$, ii) $(f \circ g)(2)$, $(g \circ f)(2)$

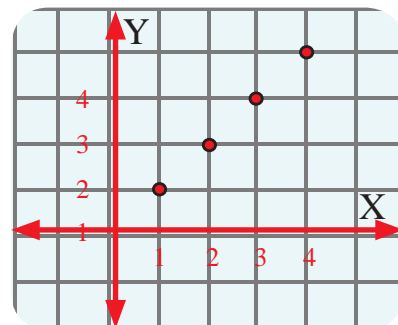
Solve the problems

11 Temperatures: In a day of winter, the temperatures recorded as shown in the following relation $R = \{(6,-2), (9,-3), (12,-4), (15,-5)\}$, where the first coordinate represents the time in hours, and the second coordinate represents the temperature in celsius degrees. Represent the relation in a table and represent it in the coordinate plane. Does the relation represent a mapping or not? Explain your answer.



12 Coordinate plane: The nearby graphic figure represents the mapping $f:N \rightarrow N$.

Write the coordinates of the ordered pairs which can be represented by the mapping points in the graphic. Write a connection rule of the mapping, is the mapping an injective or not?



13 Health: The relation $W_r = 2\left(\frac{W_b}{3}\right)$ represents the mass of water in human body, where W_r represents the mass of water and W_b represents the mass of human body. Hassan's mass is 150 kg, he follows a diet to reduce the mass for three months, he lost 6kg in the first month and 12kg in the second month and 12kg in the third month. Write all the ordered pairs for the relation between Hassan's mass and the mass of water in his body. Does it represent a mapping or not?



Think

14 Challenge: If $A = \{1,2,3\}$ and $g:A \rightarrow A$ is a defined mapping, as follow:

$g = \{(3,1), (1,2), (2,3)\}$, $f = \{(1,3), (3,3), (2,3)\}$
show does $f \circ g = g \circ f$?

15 Correct the mistake: Yaseen saide that the relation $f:Z \rightarrow Z$, where $f(x) = x^3$ does not represent an injective mapping. Determine Yaseen's mistake and correct it.

16 Numerical sense: Determine if each of the following relations $f:X \rightarrow Y$ represents a mapping or not? Explain that.

X	1	2	3	4	5
Y	3	6	11	18	27

Write

The value of x, if the mapping $f: N \rightarrow N$, and where $f(x) = 4x - 3$, $(f \circ f)(x) = 33$.

Lesson [1-3] The Sequences

Learn

Idea of the lesson:

*Identify the sequence and the arithmetic sequence and their properties.

Vocabulary:

*Sequence
*Arithmetic sequence
*General term
*Constant sequence
*Common difference of sequence

Bashar works in a gallery for five days a week. He produces one painting each three days. Arrange a table to connect between the number of days and the number of paintings which were painted by Bashar, if you know that Bashar worked for 4 weeks. Write set of the ordered pairs from the table. Does the table represent a pattern? Does it represent a sequence?



[1-3-1] Sequence and Function

You have previously learned the function and how to determine its domain and range. Now, you will learn the sequence as a function and how to express it and how to write its terms, as follow: The sequence

$f: \mathbb{N} \rightarrow \mathbb{R}$ is a function represented by a set of the ordered pairs $\{(1, f(1)), (2, f(2)), (3, f(3)), \dots, (n, f(n)), \dots\}$, where the first projection represent the natural numbers set (infinite sequence) and we can refer to it by $\{f(n)\}_{n=1}^{\infty}$ or as a subset (finite sequence) and we can refer to it by $\{f(n)\}_{n=1}^m$, so it is enough to write the second projection (images) $\{f(1), f(2), f(3), \dots, f(n), \dots\}$ and it is called u_n the general term of the sequence $f(n) = u_n$.

The sequence can be written $\{u_1, u_2, u_3, u_4, \dots, u_i, \dots\}$.

Example (1) Arrange a table to connect between the number of days and the number of paintings. Write the ordered pairs set from the table. Does the table represent a pattern? Does it represent a sequence?

Ordered pairs :

$\{(1,3), (2,6), (3,9), (4,12), (5,15), (6,18)\}$

Number of paintings	1	2	3	4	5	6
Number of days	3	6	9	12	15	18

Yes it represents a pattern and the relation represents “three times”, and the relation represents a sequence, its general term is $u_n = 3n$, $n \in \{1, 2, 3, 4, 5, 6\}$

and it can be written as follow $\{u_n\} = \{3n\} = \{3, 6, 9, 12, 15, 18\}$

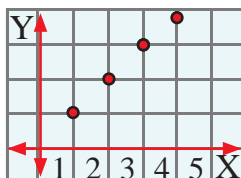
Example (2) Write the first five ordered pairs of the sequence $\{u_n\}$ and represent it in the coordinate plane:

i) $\{n\} = \{1, 2, 3, 4, 5, \dots\}$

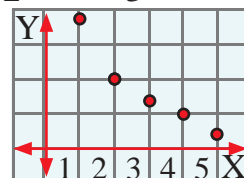
ii) $\{\frac{1}{n}\} = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots\}$

$\{(1,1), (2,2), (3,3), (4,4), (5,5), \dots\}$

$\{(1,1), (2, \frac{1}{2}), (3, \frac{1}{3}), (4, \frac{1}{4}), (5, \frac{1}{5}), \dots\}$



$$u_n = n$$



$$u_n = \frac{1}{n}$$

[1-3-2] Arithmetic Sequence

Arithmetic sequence is the sequence in which the difference between each two consecutive terms represents a constant number, and it is called the base of the sequence (the common difference of sequence), its symbol is $d = u_{n+1} - u_n$. We can write a sequence by knowing its first term $a = u_1$ and the base d . The general term law of the arithmetic sequence is $u_n = a + (n-1)d$, where $n \in \mathbb{N}$. and in general we can define the kind of the sequence as following :

- i) Increasing sequence in which $d > 0$, example $\{1, 3, 5, 7, 9, \dots\}$,
- ii) Decreasing sequence in which $d < 0$, example $\{4, 2, 0, -2, -4, \dots\}$
- iii) Constant sequence in which $d = 0$, example $\{5, 5, 5, 5, 5, \dots\}$.

Example (3) Write the first five terms for each of the following arithmetic sequences:

- i) An arithmetic sequence in which the first term is 3 and its common difference is 6.
 $\{3, 9, 15, 21, 27, \dots\}$
- ii) An arithmetic sequence in which the first term is 1 and its common difference is -3.
 $\{1, -2, -5, -8, -11, \dots\}$
- iii) An arithmetic sequence in which the seventh term is 36 and its common difference is 4
 $u_n = a + (n-1)d \Rightarrow u_7 = a + 6d \Rightarrow 36 = a + 6 \times 4 \Rightarrow a = 12$
 $\{12, 16, 20, 24, 28, \dots\}$

$$u_1 \xrightarrow{+d} u_2 \xrightarrow{+d} u_3 \xrightarrow{+d} \dots \xrightarrow{+d} u_n$$

Example (4) Write terms to the following sequences:

- i) An arithmetic sequence in which the third term is 8 and $d = -3$. Find the terms between u_7 and u_{11} .
 $u_n = a + (n-1)d \Rightarrow u_3 = a + 2d \Rightarrow 8 = a - 6 \Rightarrow a = 8 + 6 = 14$
 $u_n = a + (n-1)d \Rightarrow u_7 = a + 6d \Rightarrow u_7 = 14 + 6(-3) \Rightarrow u_7 = -4$
 $u_8 = u_7 + d = -4 - 3 = -7$
 $u_9 = u_8 + d = -7 - 3 = -10$
 $u_{10} = u_9 + d = -10 - 3 = -13$
 $\{-7, -10, -13\}$
- ii) Write the twentieth term of the arithmetic sequence $\{6, 1, -4, -9, \dots\}$, and determine if it is decreasing or increasing.
 $d = u_{n+1} - u_n \Rightarrow d = 1 - 6 = -5, a = 6$
 $u_n = a + (n-1)d \Rightarrow u_{20} = a + 19d \Rightarrow u_{20} = 6 + 19(-5) \Rightarrow u_{20} = -89$
 $\because d < 0$, so the sequence is decreasing

By finding the values of (a), we can get the value of the term 7 and the terms which follow

Example (5) Write the first five terms for each of the following sequences:

- i) $\{2n - 1\} = \{1, 3, 5, 7, 9\}$, ii) $\{(-1)^n\} = \{-1, 1, -1, 1, -1\}$
- iii) $\{7\} = \{7, 7, 7, 7, 7\}$, iv) $\{\frac{n}{3}\} = \{\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}\}$
- v) $\{n^2\} = \{1, 4, 9, 16, 25\}$, vi) $\{n^3\} = \{1, 8, 27, 64, 125\}$

Make sure of your understanding

Write the first four ordered pairs for the sequence which its general term is given:

1 $u_n = 3n$

2 $u_n = n - 4$

3 $u_n = n^2$

4 $u_n = \frac{1}{2n}$

5 $u_n = 3n - 1$

Questions 1-5
are similar
to example 2

Write the first five terms of each of the following sequences:

6 An arithmetic sequence in which the first term is 1 and its common difference is 5.

7 An arithmetic sequence in which the first term is -5 and its common difference is 2.

8 An arithmetic sequence in which the first term is -3 and its common difference is -4.

Questions
6-8
are similar
to example 3

Write the terms for the following sequences:

9 Find the terms between u_8 and u_{12} for an arithmetic sequence in which the third term is (9) and $d = -2$.

10 Find the terms between u_{10} and u_6 for an arithmetic sequence in which the second term is -11 and $d = -3$.

11 Write 23rd term for the arithmetic sequence $\{3, -1, -5, -9, \dots\}$

Questions 9-11
are similar
to example 4

Write the first five terms for each of the following sequences:

12 $\{4n\} = \dots\dots\dots$

13 $\{2n - 5\} = \dots\dots\dots$

14 $\{\frac{1}{n+1}\} = \dots\dots\dots$

15 $\{9\} = \dots\dots\dots$

Questions 12-15
are similar
to example 5

Solve the Exercises

Write the first four ordered pairs for the sequence which its general term is given:

16 $u_n = 10 - 4n$

17 $u_n = \frac{1}{3n+1}$

Write the first five terms for each of the following sequences:

18 An arithmetic sequence in which the seventh term is $\frac{1}{24}$ and the common difference is $\frac{1}{3}$

Write terms for the following sequences:

19 Find the terms between u_{10} and u_{13} for an arithmetic sequence in which the seventh term is $\frac{13}{2}$ and $d = 1$.

20 Find the terms between u_{20} and u_{23} for an arithmetic sequence in which the second term is (0) and $d = -1$.

Determine the type of sequences (increasing, decreasing, constant) for each of the following:

21 $\{u_n\} = \{3 - 2n\}$

22 $\{u_n\} = \{n^3 - 1\}$

Solve the problems

- 23 Running sport:** In one of running competitions of the times of the first winner are recorded in the following table

Distance in km	1	2	3	4	5
Time in minutes and seconds	3.12	6.32	9.52	12.72	15.92

Write the ordered pairs set from the table. Does the table represent a pattern? Does it represent a sequence? Explain your answer.



- 24 Sport of pole vault:** The following table shows the attempts of one of the world champions in the sport of pole vault.

Attempts	1	2	3	4	5
Height in metre	5.90	5.95	6.00	6.05	6.10

Write the ordered pairs set from the table. Does the table represent a pattern? Does it represent a sequence? Explain your answer.



- 25 Agriculture:** Hassan had bought a farm for breeding cows, and after one year which had 20 cows. This number was increased year by year as a result to the new births which were in a constant rate. After six years, the number of cows had become the double. Represent the problem in a table and Write the ordered pairs set from the table. Does the table represent a pattern? Does it represent a sequence? Explain your answer..



Think

- 26 Challenge:** Find the value of x which makes the first three terms of the arithmetic sequences as follow:
 $\{2x, x + 1, 3x + 11, \dots\}$
- 27 Correct the mistake:** Rabiha said that the sequence which general term is $u_n = 8 - 2n$, considered an increasing sequence because $d > 0$, discover the mistake of Rabiha and then correct it.
- 28 Numerical sense:** What is the eleventh term for a sequence in which the third term is (4) and the common difference is $-\frac{1}{2}$.

Write

The term which occupies the place 101 in the arithmetic sequence which its fifth term is -4 and the common difference is 12.

Lesson [1-4]

Compound Inequalities

Idea of the lesson:

*Solving the inequalities which contain connecting tools (and) , (or), then representing the solution on the numbers line

Vocabulary:

*Compound inequalities

*Intersection

*Union

*Solution set

Learn

We use the minimum and maximum celsius degrees to measure the weather temperature during a day because it is variable from time to time. If the minimum celsius temperature in Baghdad is 8°C and the maximum one is 15°C during December month, write an inequality represents the temperature in Baghdad , then find its solution



[1-4-1] Compound inequalities which contain “and”

You have previously learned the algebraic inequalities and their properties. You have also learned how to find the solution set and how to represent it in the line of numbers. Now, you will learn the compound inequalities which contain the connecting tool “and” and how to find their solution set and how to represent them in the line of the real numbers. As the compound inequality contains the connecting tool “and” and it consists of two inequalities, so it will be true only when the two inequalities are true.

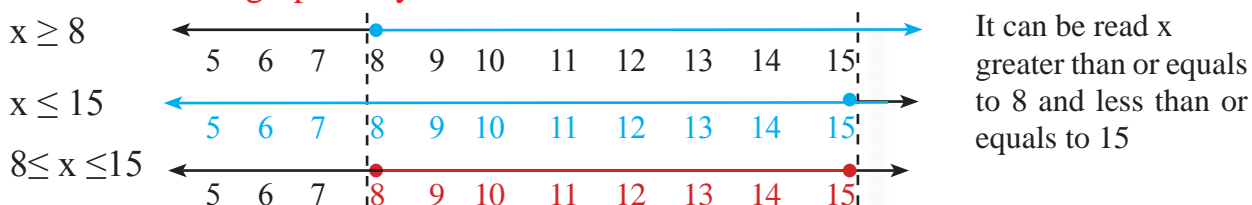
According to that, its solution set will be a set of intersection for solution of the two inequalities. We can do that by two methods, the first is graphically by representing the solution of the two inequalities in the line of numbers and then determine the intersection area. The second method is algebraically by finding the solution set for each inequality, then take their intersection set ($S = S_1 \cap S_2$).

Example (1)

Write the compound inequality which represents the small and big celsius temperatures in Baghdad, then find its solution.

The temperature (minimum) is not less than 8° ($8 \leq x$), while the temperature (maximum) is not greater than 15° ($x \leq 15$). The temperature is not less than 8 and not greater than 15° ($8 \leq x$ and $x \leq 15$). It can be solved in any of the two methods:

The first method : **graphically**



The second method: **Algebraically** : $8 \leq x \leq 15 \Rightarrow 8 < x$ and $x \leq 15$

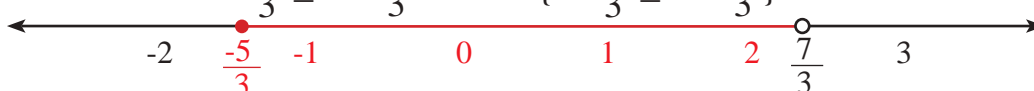
$$\Rightarrow S = S_1 \cap S_2 = \{x: x \geq 8\} \cap \{x: x \leq 15\} = \{x: 8 \leq x \leq 15\}$$

Example (2)

Solve the compound inequality which contains (and) $-3 \leq 3x+2 < 9$ algebraically, then represent the solution on the straight line of numbers.

$$-3 \leq 3x+2 < 9 \Rightarrow -3 - 2 \leq 3x+2 - 2 < 9 - 2 \Rightarrow -5 \leq 3x < 7 \Rightarrow \frac{-5}{3} \leq \frac{3x}{3} < \frac{7}{3}$$

$$\Rightarrow \frac{-5}{3} \leq x < \frac{7}{3} \Rightarrow S = \{x: \frac{-5}{3} \leq x < \frac{7}{3}\}$$

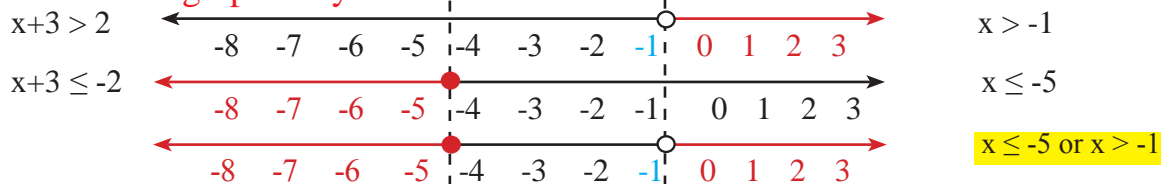


[1-4-2] Compound inequalities which contain “or”

After you have already learned the compound inequality which contains the connecting tool (and), you will learn the compound inequality which contains the connecting tool (or) and it will be true only when, at least one of its two inequalities is true. Accordingly its solution set is a set of the two inequalities solution union. It can be found in two methods, first graphically by representing the two inequalities solution in the line of numbers, then determining union area. Second method, algebraically by finding the solution set for each inequality, then taking their union set ($S = S_1 \cup S_2$).

Example (3) Solve the compound inequality $2 < x+3$ or $x+3 \leq -2$ graphically and algebraically.

First method: **graphically**



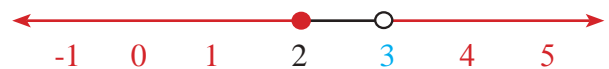
Second method: **algebraically**

$$x+3 \leq -2 \text{ or } x+3 > 2 \Rightarrow \begin{cases} x+3 > 2 \text{ or } x+3 \leq -2 \\ x > -1 \text{ or } x \leq -5 \end{cases} \Rightarrow S = S_1 \cup S_2 = \{x: x > -1\} \cup \{x: x \leq -5\}$$

Example (4) Solve the compound inequality which contains (or) algraphically and represent the solution on the line of numbers.

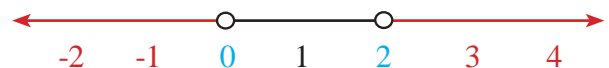
i) $y-3 \leq -1$ or $y+3 > 6 \Rightarrow y \leq 2$ or $y > 3$

$$\Rightarrow S = S_1 \cup S_2 = \{y: y \leq 2\} \cup \{y: y > 3\}$$



ii) $\frac{2v+1}{3} > \frac{5}{3}$ or $\frac{2v+1}{3} < \frac{1}{3} \Rightarrow v > 2$ or $v < 0$

$$\Rightarrow S = S_1 \cup S_2 = \{v: v > 2\} \cup \{v: v < 0\}$$



[1-4-3] Triangular Inequality

One of the subjects which connects algebra to geometry is the triangular inequality “in each triangle, the sum of two sides length is greater than the length of the third side”, it is used in the geometrical constructions and designs. If the lengths side of a triangle is (A,B,C), then the following three inequalities should be true : $A+B > C$, $A+C > B$, $B+C > A$

Example (5) i) Can the three sides of a triangle with length 13cm ,10cm and 2cm compose a triangle ?

No, they can't because: $2 + 10 \nless 13$ is false , $10 + 13 > 2$ is true , $2 + 13 > 10$ is true .

ii) Write a compound inequality which shows the length of the third side in a triangle which has two sides with length 8cm and 10cm.

Suppose that the length of the third side is x, then:

$$\left. \begin{aligned} 8 + 10 > x &\Rightarrow 18 > x \Rightarrow \text{The third side is less than 18} \\ 8 + x > 10 &\Rightarrow x > 2 \Rightarrow \text{The third side is greater than 2} \\ 10 + x > 8 &\Rightarrow x > -2 \Rightarrow \text{Doesn't give any useful data} \end{aligned} \right\} \Rightarrow$$

So the length of this side must be less than 18 and greater than 2 and by the compound inequality, we see that the range of the third side length is $2 < x < 18$

Make sure of your understanding

Solve the compound inequalities which include (and) graphically:

1 $-4 \leq y - 1 < 3$

2 $8 \geq z + 2 \geq -4$

Questions 1-2
are similar
to example 1

Solve the compound inequalities which include (and) algebraically, then represent the solution set on the line of numbers:

3 $12 \leq x + 6$ and $x + 6 < 15$

4 $-9 < 2x - 1 \leq 3$

Questions 3-4
are similar
to example 2

Solve the compound inequalities which includes (or) graphically:

5 $8y \leq 32$ or $8y \geq 64$

6 $\frac{2z}{3} < \frac{2}{3}$ or $\frac{2z}{3} \geq \frac{8}{9}$

Questions 5-6
are similar
to example 3

Solve the compound inequalities which include (or) algebraically, then represent the solution on the line of numbers:

7 $3n - 7 > -5$ or $3n - 7 \leq -9$

8 $x + 15 < 22$ or $x + 15 \geq 30$

Questions 7-8
are similar
to example 4

Can the three sides, which shown below, compose a triangle?

9 1cm, 2cm, $\sqrt{3}$ cm

10 5cm, 4cm, 9cm

11 1cm, $\sqrt{2}$ cm, $\sqrt{2}$ cm

12 3cm, 4cm, $2\sqrt{3}$ cm

Questions 9-12
are similar
to example 5

Solve the Exercises

Solve the compound inequalities which include (and) graphically:

13 $-12 < x$ and $x \leq -7$

14 $2 \leq y + 4 < 6$

Solve the compound inequalities which include (and) algebraically, then represent the solution set on the line of numbers:

15 $14 \leq 3x + 7$ and $3x + 7 < 26$

16 $\frac{1}{15} \geq \frac{z+3}{5} \geq \frac{1}{25}$

Solve the compound inequalities which includes (or) graphically:

17 $z - 2 < -7$ or $z - 2 > 4$

18 $x - 6 \leq -1$ or $x - 6 > 4$

Solve the compound inequalities which include (or) algebraically, then represent the solution set on the line of numbers:

19 $\frac{y}{2} < 3\frac{1}{2}$ or $\frac{y}{2} > 7\frac{1}{2}$

20 $5x \leq -1$ or $5x \geq 4$

Write the compound inequality which shows the length of the third side in the triangle which has two known-length sides:

21 3cm, 10cm

22 6cm, 4cm

23 1cm, 3cm

Solve the problems

24 **Sound:** Human's ear can hear sound which its frequency is not less than 20 Hz and not more than 20000 Hz. Write a compound inequality represents the frequencies which human's ear can not hear them, then represent on line of numbers.



25 **Cars tyre:** The ideal air pressure which is recommended for tyres of saloon cars is not less than 28 pascal (kg/ing^2) and not more than 36 pascal. Write a compound inequality which represents the pressure, then represent on line of numbers.

Cars tyre: The ideal air pressure which is recommend

Note: Pascal is unit for measuring the pressure of air which is Kg ling^2 .



26 **Magnetic train:** Hanging magnetic train which operates in the magnetic lifting force, briefly it is called (Maglev). Different types of the magnetic trains were designed all over the world, the speed of those train is not less than 300 k/h and not more than 550 k/h. Write an equality represents the speed of train, then represent on line of numbers.

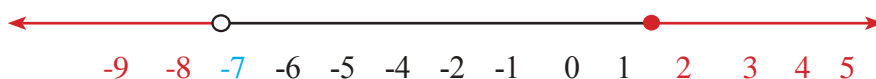


Think

27 **Challenge:** Write a compound inequality shows the range of the third side length in each triangle:

7 cm , 12 cm , x cm

28 **Correct the mistake:** Sawsen said that the compound inequality $-4 < x+3$ and $x+3 \leq 5$ represents the set of solution in the following line of numbers.



Show Sawsen's mistake, then correct it.

29 **Numerical sense:** Mention if the three lengths are for a triangle or not? Explain that.

i) 3.2cm, 5.2cm, 6.2cm

ii) 1cm, 1cm, $\sqrt{2}$ cm

Write

A compound inequality represents the minimum degree temperature of which is 18° and the maximum degree temperature of which is 27° .

Lesson [1-5]

Absolute Value Inequalities

Idea of the lesson:

*Solving inequalities which contain an absolute value.

Vocabulary:

* Absolute value

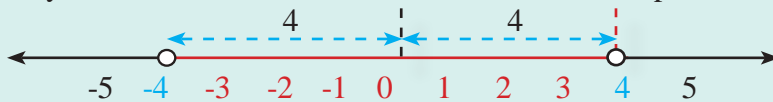
Learn

Babylon hotel is one of the tourist hotels in Baghdad. It locates in Al-jadriya area. The ideal temperature of water in the swimming pool is 25 celsius, with increas or decrease of one degree. Write an absolute value inequality represents the range of water temperature in the swimming pool.



[1-5-1] Absolute value Inequalities $|g(x)| \leq a$, $|g(x)| < a$, where $X \in \mathbb{R}$

You have previously learned about the compound inequalities which contain (and) and (or), and how to solve them graphically and algebraically, and how to represent the solution set in the line of numbers. Now, you will learn the absolute value inequality with from $|g(x)| \leq a$, $|g(x)| < a$, $a \in \mathbb{R}$ for example : $|x| < 4$ which means: What are the values of x which is far of zero in less than 4 units? they include all numbers between -4 and 4, then represent them in the line of numbers which is:



We note that the solution of this quality is: $\{-4 < x \text{ and } x < 4\}$

That means that the absolute value inequality was connect to relation of less than (less or equals to) represents a compound inequality which includes (and).

In general form : $|x| \leq a \Rightarrow -a \leq x \leq a$, $a > 0$

Example (1)

Write the absolute value inequality which represents the temperature of water in the swimming pool, then represent it graphically.

Assume that the temperature of water is (x) celsius, so the inequality which represents the temperature of pool when it is not more than 26 celsius is:

$$x \leq 25 + 1 \Rightarrow x - 25 \leq 1$$

and the inequality which represents the temperature of pool when it is not less than 24 celsius is:

$$x \geq 25 - 1 \Rightarrow x - 25 \geq -1$$

So the absolute value inequality is the compound inequality which represents the range of water temperature in the swimnting pool. $x - 25 \geq -1$ and $x - 25 \leq 1 \Rightarrow -1 \leq x - 25 \leq 1 \Rightarrow |x - 25| \leq 1$

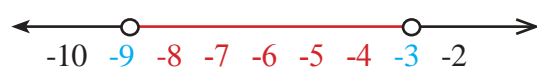
The representation of the solution set and the line of numbers is:



Example (2)

Solve the absolute value inequalities, then represent the solution in the line of numbers.

$$\begin{aligned} \text{i) } |x + 6| < 3 &\Rightarrow -3 < x + 6 < 3 \Rightarrow -3 - 6 < x < 3 - 6 \\ &\Rightarrow -9 < x < -3 \end{aligned}$$



$$\begin{aligned} \text{ii) } |y| - 5 \leq 1 &\Rightarrow |y| \leq 1 + 5 \Rightarrow |y| \leq 6 \\ &\Rightarrow -6 \leq y \leq 6 \end{aligned}$$



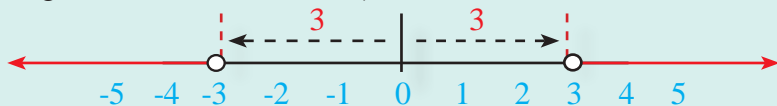
[1-5-2] Absolute value Inequalities which is in form of $|g(x)| \geq a$, $|g(x)| > a$ where $X \in \mathbb{R}$

After you have learned the absolute value inequality which contains the form of $|g(x)| \leq a$, $|g(x)| < a$ where $x \in \mathbb{R}$. Now, you will learn the absolute value inequality which contains is in the form of $|g(x)| \geq a$ $|g(x)| > a$ where $x \in \mathbb{R}$ for example: $|x| > 3$, which means: distance between x and zero greater than 3 that is

$x > 3$ or $x < -3$ and the inequality solution set is $\{x: x < -3\} \cup \{x: x > 3\}$

So the absolute value inequality with the relation greater than (greater or equal) is a compound relation which includes (or).

In general form : $|x| \geq a \Rightarrow x \geq a$ or $x \leq -a$, $a > 0$



Example (3) Solve the absolute value inequality, then represent the solution on the line of numbers.

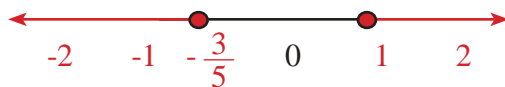
i) $|x + 4| > 2 \Rightarrow -2 > x + 4$ or $x + 4 > 2 \Rightarrow -6 > x$ or $x > -2$

$$\Rightarrow S = S_1 \cup S_2 = \{x: x < -6\} \cup \{x: x > -2\}$$



ii) $|5y - 1| \geq 4 \Rightarrow -4 \geq 5y - 1$ or $5y - 1 \geq 4 \Rightarrow -\frac{3}{5} \geq y$ or $y \geq 1$

$$\Rightarrow S = S_1 \cup S_2 = \{y: y \leq -\frac{3}{5}\} \cup \{y: y \geq 1\}$$



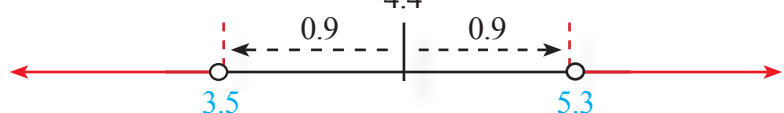
iii) In analysis of a blood for adult men, the natural range of potassium is (3.5 – 5.3) mmol/L. Write the absolute value inequality which represents the unnatural range of potassium in human blood.

The inequality which represents the unnatural quantity of potassium and less than the lowest value of average is: $x < 3.5$

The inequality which represents the unnatural quantity of potassium and more than the greatest average is: $x > 5.3$

The unnatural range of potassium is the compound inequality solution: $x > 5.3$ or $x < 3.5$

We find the absolute value inequality which represents the unnatural range of potassium:



$$3.5 > x \text{ or } x > 5.3 \Leftrightarrow 4.4 - 0.9 > x \text{ or } x > 4.4 + 0.9$$

$$\Leftrightarrow -0.9 > x - 4.4 \text{ or } x - 4.4 > 0.9 \Leftrightarrow |x - 4.4| > 0.9$$

we take the middle between two points , then we subtract and add the radius of the distance.

Example (4) Find the solution set for the following absolute value inequalities:

i) $|2x - 5| + 3 < 11 \Rightarrow |2x - 5| < 8 \Rightarrow -8 < 2x - 5 < 8 \Rightarrow -3 < 2x < 13$

$$\Rightarrow -\frac{3}{2} < x < \frac{13}{2} \Rightarrow \{x: x > -\frac{3}{2}\} \cap \{x: x < \frac{13}{2}\} \Rightarrow \{x: -\frac{3}{2} < x < \frac{13}{2}\}$$

ii) $|7 - y| < 8 \Rightarrow -8 < 7 - y < 8 \Rightarrow -15 < -y < 1 \Rightarrow -1 < y < 15 \Rightarrow \{y: y > -1\} \cap \{y: y < 15\}$

iii) $|\frac{2t-8}{4}| \geq 9 \Rightarrow |\frac{2(t-4)}{4}| \geq 9 \Rightarrow |\frac{t-4}{2}| \geq 9 \Rightarrow |t-4| \geq 18$

$$t - 4 \leq -18 \text{ or } t - 4 \geq 18 \Rightarrow t \leq -14 \text{ or } t \geq 22 \Rightarrow \{t: t \leq -14\} \cup \{t: t \geq 22\}$$

iv) $|\frac{5-3v}{2}| \geq 6 \Rightarrow |5-3v| \geq 12 \Rightarrow -12 \geq 5-3v \text{ or } 5-3v \geq 12 \Rightarrow -3v \geq -17 \text{ or } -3v \geq 7$

$$\Rightarrow v \geq \frac{17}{3} \text{ or } v \leq \frac{7}{3} \Rightarrow \{v: v \geq \frac{17}{3}\} \cup \{v: v \leq \frac{7}{3}\}$$

Make sure of your understanding

Write the absolute value inequality which represents the following problems:

- 1 The ideal temperature inside flats is 22° celsius with increase or decrease of 2° celsius.

Questions 1 are similar to examples 1,3

Solve the absolute value inequalities, then represent the solution on the line of numbers.

2 $|x + 1| < 5$

3 $|3z - 7| \leq 2$

Questions 2-5 are similar to example 2

4 $|x| + 8 < 9$

5 $|5y| - 2 \leq 8$

6 $|x + 4| > 6$

7 $|5z - 9| > 1$

Questions 6-9 are similar to example 3

8 $|2x| + 7 \geq 8$

9 $|4y| - 2 > 3$

10 $|5 - x| < 10$

11 $|4z - 14| > 2$

Questions 10-13 are similar to example 4

12 $|\frac{x - 12}{4}| \leq 9$

13 $|\frac{6 - 2y}{4}| \geq 9$

Solve the Exercises

Write the absolute value inequality which represents the following problems:

- 14 The temperature inside fridge should be 8° celsius with increasing or decreasing of 0.5° celsius.

Write the range of ideal temperature inside fridge.

- 15 The boiling degree of water is 100° celsius at the sea surface level. It increases or decreases in the mountainous areas and valleys in no more than 20° celsius. Write the range of vibration in the boiling degree of water.

Solve the following absolute value inequalities:

16 $|x + 3| < 6$

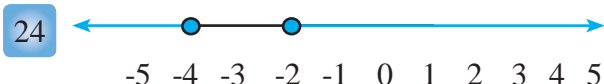
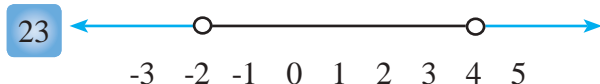
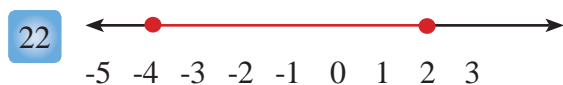
17 $|2z| - 5 < 2$

18 $2|x| - 7 \geq 1$

19 $|11z| - 2 \geq 9$

20 $|\frac{4}{5}z - 1| > \frac{4}{5}$

Write an inequality includes an absolute value for all of the following graphic inequalities:



Solve the problems

Write the absolute value inequality which represents each of the following problems:

- 25 **Badger:** The animal, Badger is one of mammals which belongs to the division of preanants. It has short legs. It lives in holes which the Badger itself made. The length of its body, from head to tail, is from 68cm to 76cm. Write the range of Badger length.



- 26 **Health:** The natural pulse rate (number of heart (beats) for adult men is from 60 to 90 beats in minute. Write the range of unnatural heart beats of human.



- 27 **Transportation:** The civilian plane flies in height from 8 km to 10 km where it is considered a moderate area. Write the range of the civilian aviation area.



Think

- 28 **Challenge:** Solve the absolute value inequalities and represent the solution on the line of numbers .

i) $\left| \frac{\sqrt{3}(x+1)}{\sqrt{2}} \right| \leq \sqrt{6}$

ii) $\left| \frac{\sqrt{12} - \sqrt{3}y}{\sqrt{5}} \right| \geq \sqrt{15}$

- 29 **Correct the mistake:** Khulood said that the absolute value inequality $|6 - 3y| \geq 7$ represents a compound inequality with a relation (and). and with its solution : $\left\{ y : -\frac{1}{3} \leq y \leq \frac{13}{2} \right\}$ Show the mistake of Khulood, then correct it.

- 30 **Numerical sense:** Write the solution set for the following absolute value inequalities in the real numbers set:

i) $|z| - 1 < 0$

ii) $|x - 1| > 0$

Write

An absolute value inequality represents a situation from life, then represent the solution set on the straight line of numbers.

Chapter Test

Simplify the following numerical sentences by using the ordering of operations in the real numbers:

1 $(\sqrt{3} + \sqrt{5}) (\sqrt{3} + \sqrt{5}) = \dots$

2 $\frac{\sqrt{3} - \sqrt{6}}{\sqrt{3}} - \frac{\sqrt{8} - 5}{3\sqrt{2}} = \dots$

Use the ordering of operations and the calculator to write each of the following which should be written to the nearest tenth.

3 $(\frac{1}{125})^{\frac{1}{3}} - (-\frac{1}{2})^0 + (121)^{\frac{1}{2}} \times (\frac{1}{9})^{\frac{1}{2}} = \dots$

4 If $f: \mathbb{Z} \rightarrow \mathbb{R}$, where $f(x) = x^2$. Draw an arrowy diagram for the mapping, then show if the mapping is injective, surjective or bijective?

5 If the mapping $f: \mathbb{N} \rightarrow \mathbb{N}$, where $f(x) = 3x + 1$, $g: \mathbb{N} \rightarrow \mathbb{N}$, where, $g(x) = x^2$

Find: $(g \circ f)(5)$, $(f \circ g)(5)$, $(g \circ f)(2)$, $(f \circ g)(2)$.

6 If the mapping $f: \mathbb{R} \rightarrow \mathbb{R}$ where $f(x) = 3x + 1$ and the mapping $g: \mathbb{R} \rightarrow \mathbb{R}$, since $g(x) = 2x + 5$.

Does $(f \circ g)(x) = (g \circ f)(x)$? find the value of x if $(f \circ g)(x) = 28$

Write the terms of the following sequences:

7 Find the terms between u_3 and u_8 for an arithmetic sequence which its second term is $\frac{-3}{2}$ and $d = 2$.

8 Find the terms between u_4 and u_9 for an arithmetic sequence which its third term is 6 and $d = \frac{-5}{2}$.

Determine the type of the following sequences (increasing, decreasing, constant):

9 $u_n = 9 - 3n$

10 $u_n = n^2 - 2$

11 $u_n = \frac{1}{3n+1}$

Write the first five terms for each of the following sequences

12 $\{\frac{n}{n+2}\} = \dots$

13 $\{4\sqrt{2}\} = \dots$

14 $\{\frac{-n}{n+5}\} = \dots$

Solve the compound inequalities, then represent the solution set on the line of numbers.

15 $12 \leq x + 6$, $x + 6 < 20$

16 $\frac{1}{8} \geq \frac{z+2}{2} > \frac{1}{16}$

17 $x - 3 \leq -5$ or $x - 3 > 5$

18 $7t - 5 > -1$ or $7t - 5 \leq -14$

19 $y \leq 0$ or $y + 7 \geq 16$

20 $\frac{y}{3} < 1\frac{1}{3}$ or $\frac{y}{3} > 9\frac{1}{3}$

Write the compound inequality shows the range of the third side in each triangle:

21 4cm , 9cm

22 5cm , 12cm

23 7cm , 15cm

Solve the following absolute value inequalities:

24 $|x - 6| \leq 3$

25 $|3z - 5| < 4$

26 $|x + 1| > \frac{1}{2}$

27 $6|x| - 8 \geq 3$

28 $|3y| - 2 > 9$

29 $|8z| - 1 > 7$

30 $|4 - 3y| \geq 14$

31 $|\frac{6-3y}{9}| \geq 5$

Algebraic Expressions

- lesson 2-1 Multiplying Algebraic Expressions.
- lesson 2-2 Factoring Algebraic Expressions by using Greater Common Factor.
- lesson 2-3 Factoring Algebraic Expressions by using Special Identities.
- lesson 2-4 Factoring the Algebraic Expression of three terms by Probe and Error (Experiment).
- lesson 2-5 Factoring Algebraic Expressions Contains Sum of Two Cubes or difference Between Two Cubes.
- lesson 2-6 Simplify Rational Algebraic Expressions .

Al-Mustansiriya school is an ancient one .It was established in the reign of Abbasiyn in Baghdad in 1233 .It was an important cultural and scientific center. It locates in AL- Risafa side of Baghdad . There is a rectangular area in the middle of school which has a great fountain and o'clock of school .If we assume that the length of the internal area of school is $(x+14)$ meters and its width is $(x+2)$ meters, then we can calculate the area by multiplying the tow algebraic expressions $(x+14) (x+2)$.

Prestest

Find the result of adding or subtracting the following algebraic expressions:

1 $(3x^2 + 4x - 12) + (2x^2 - 6x + 10)$

2 $(\frac{1}{2}zy + 5z - 7y) - (\frac{1}{4}zy - 3z + 2y)$

Find the result of multiplying the following algebraic terms:

3 $7x^2 \times \frac{1}{14x}$

4 $\sqrt{2}yz \times \sqrt{2}yz^2$

5 $\frac{3}{4}v^2t \times \sqrt{12}t^{-1}$

6 $3h(\frac{1}{6}v - \frac{1}{3}h^{-2})$

Find the result of multiplying two algebraic expressions:

7 $(x+2)(x-2)$

8 $(5-2z)(3+3z)$

9 $(\frac{1}{2}x^2 + 6)(\frac{4}{3}x^2 + 12)$

10 $(2\sqrt{3}t - 4)^2$

11 $(x+3)(x^2 - 3x + 9)$

12 $(xy + 1)(x^{-1}y - xy^{-1} - 1)$

Find the result of multiplying by using the vertical method:

13 $(y-1)(y+1)$

14 $(2x+3)(4x^2 - x - 5)$

15 $(3-z)(3+5z-z^2)$

Find the result of dividing the following algebraic expressions:

16 $\frac{3xy^2}{15x^2y}$

17 $\frac{-47z^{-2}}{7z^2}$

18 $\frac{8x^3 + 4x^2 - 2x}{2x}$

19 $\frac{21 - 14a + 7a^2}{7a}$

Factoring the algebraic expressions by using the greater common factor :

20 $3y^3 + 6y^2 - 9y$

21 $\frac{1}{2}zx^2 - 2z^2x + 4zx$

Lesson [2-1]

Multiplying Algebraic Expressions

Learn

Idea of the lesson:

* Multiplying an algebraic expression by other algebraic expression which represents special cases.

Vocabulary:

- * Square of sum
- * Square of difference
- * Cubic of sum
- * Cubic of difference

A square-shaped home garden was surrounded by a fence. The length of its side is h meter with an aisle of one meter width. What is the area of aisle according to h ?



[2-1-1] Multiplying two Algebraic Expressions each one Contains two terms.

You have previously learned how to multiply two algebraic terms with each other and how to multiply an algebraic expression by another one. Now, you will learn how to multiply two algebraic expressions which each one of them has two terms with each other, and they represent a square of sum or a square of difference or sum multiply by difference, by using the properties that you previously studied which are distributing, substituting and ordering.

Example (1) Find the area of the aisle which surrounds the square-shaped garden.

The area of the aisle is the difference between the two area of the big square (garden with aisle) and the small square (the garden)

$$(h+2)^2 = (h+2)(h+2) = h^2 + 2h + 2h + 4 = h^2 + 4h + 4$$

$$h \times h = h^2$$

$$(h^2 + 4h + 4) - h^2 = h^2 - h^2 + 4h + 4 = 4h + 4$$

The area of garden with aisle

The area of garden

The area of aisle

Example (2) Find the result of multiplying an algebraic expressions by another algebraic expression where each one has two terms :

i) $(x + y)^2 = (x + y)(x + y) = x^2 + xy + yx + y^2 = x^2 + 2xy + y^2$ *Square of sum for two terms*

ii) $(x - y)^2 = (x - y)(x - y) = x^2 - xy - yx + y^2 = x^2 - 2xy + y^2$ *Square of difference between two terms*

iii) $(x + y)(x - y) = x^2 - xy + yx - y^2 = x^2 - y^2$ *Sum of two terms \times the difference between them.*

iv) $(x + 3)(x + 5) = x^2 + 5x + 3x + 15 = x^2 + 8x + 15$ *Sum of two terms \times sum of two terms.*

v) $(x + 2)(x - 6) = x^2 - 6x + 2x - 12 = x^2 - 4x - 12$ *Sum of two terms \times the difference between two terms.*

vi) $(x - 1)(x - 4) = x^2 - 4x - x + 4 = x^2 - 5x + 4$ *difference between two terms \times difference between two terms.*

Example (3) Find the result of multiplying the following algebraic expressions:

i) $(z + 3)^2 = z^2 + 6z + 9$

ii) $(h - 5)^2 = h^2 - 10h + 25$

iii) $(2x - 7)(2x + 7) = 4x^2 - 49$

iv) $(3y + 1)(y + 2) = 3y^2 + 7y + 2$

v) $(v + \sqrt{2})(v - \sqrt{2}) = v^2 - 2$

vi) $(n - \sqrt{3})(5n - \sqrt{3}) = 5n^2 - 6\sqrt{3}n + 3$

[2-1-2] Multiplying algebraic expressions from two terms by another three terms

You have previously learned the multiplying of algebraic expression which have many terms. Now, you will learn special cases of multiplying an algebraic expression which consists of two terms by another algebraic expression which consists of three terms by using the properties that you studied in distributing, substituting and ordering.

Example (4) Find the result of multiplying an algebraic expression which consists of two terms by an algebraic expression which consists of three terms:

i) $(x+2)(x^2-2x+4) = x^3-2x^2+4x+2x^2-4x+8 = x^3+8 = x^3+2^3$ *The result of multiplying is the sum of two cubes*

ii) $(y-3)(y^2+3y+9) = y^3+3y^2+9y-3y^2-9y-27 = y^3-27 = y^3-3^3$ *The result of multiplying is the difference between two cubes*

iii) $(y+2)^3 = (y+2)(y+2)^2 = (y+2)(y^2+4y+4)$ *Cube of two terms sum*

$$= y^3+4y^2+4y+2y^2+8y+8 = y^3+6y^2+12y+8$$

iv) $(z-3)^3 = (z-3)(z-3)^2 = (z-3)(z^2-6z+9)$ *Cube of the difference between two terms*

$$= z^3-6z^2+9z-3z^2+18z-27 = z^3-9z^2+27z-27$$

Example (5) Find the result of multiplying the following algebraic expressions :

i) $(2v+5)(4v^2-10v+25) = 8v^3-20v^2+50v+20v^2-50v+125 = 8v^3+125 = (2v)^3+5^3$

ii) $(\frac{1}{3}-z)(\frac{1}{9}+\frac{1}{3}z+z^2) = \frac{1}{27}+\frac{1}{9}z+\frac{1}{3}z^2-\frac{1}{9}z-\frac{1}{3}z^2-z^3 = \frac{1}{27}-z^3 = (\frac{1}{3})^3-z^3$

iii) $(x-\sqrt[3]{2})(x^2+\sqrt[3]{2}x+\sqrt[3]{4}) = x^3+\sqrt[3]{2}x^2+\sqrt[3]{4}x-\sqrt[3]{2}x^2-\sqrt[3]{4}x-\sqrt[3]{8} = x^3-2$

$$= x^3+\sqrt[3]{2}x^2-\sqrt[3]{2}x^2+\sqrt[3]{4}x-\sqrt[3]{4}x-2 = x^3-2$$

iv) $(x+\frac{1}{2})^3 = (x+\frac{1}{2})(x+\frac{1}{2})^2 = (x+\frac{1}{2})(x^2+x+\frac{1}{4}) = x^3+x^2+\frac{1}{4}x+\frac{1}{2}x^2+\frac{1}{2}x+\frac{1}{8}$

$$= x^3+x^2+\frac{1}{2}x^2+\frac{1}{4}x+\frac{1}{2}x+\frac{1}{8} = x^3+\frac{3}{2}x^2+\frac{3}{4}x+\frac{1}{8}$$

v) $(y-5)^3 = (y-5)(y-5)^2 = (y-5)(y^2-10y+25)$

$$= y^3-10y^2+25y-5y^2+50y-125 = y^3-15y^2+75y-125$$

Make sure of your understanding

Find the result of multiplying an algebraic expression by another algebraic expression where both of them have two terms:

1 $(x + 3)(x - 3)$

2 $(\sqrt{7} - h)^2$

3 $(z + \sqrt{5})(z - \sqrt{5})$

4 $(v + 5)(v + 1)$

5 $(x - 3)(x - 2)$

6 $(3x - 4)(x + 5)$

7 $(\frac{1}{3}y + 3)(\frac{1}{3}y + 2)$

Questions 1-7
are similar
to examples 2-3

Find the result of multiplying an algebraic expression which consists of two terms by another algebraic expression which consists of three terms:

8 $(y+2)(y^2 - 2y+4)$

9 $(2z + 4)(4z^2 - 8z + 16)$

10 $(v - \sqrt[3]{3})(v^2 + \sqrt[3]{3}v + \sqrt[3]{9})$

11 $(\sqrt[3]{\frac{2}{7}} + m)(\sqrt[3]{\frac{4}{49}} - \sqrt[3]{\frac{2}{7}}m + m^2)$

12 $(x + 5)^3$

13 $(y - 4)^3$

Questions 8-13
are similar
to examples 4-5

Solve the Exercises

Find the result of multiplying an algebraic expression by another algebraic expression where both of them have two terms:

14 $(n - 6)^2$

15 $(2x - 3)(x + 9)$

16 $(y + \sqrt{6})(y - \sqrt{6})$

17 $(4 - y)(5 - y)$

Find the result of multiplying an algebraic expression which consists of two terms by another algebraic expression which consists of three terms:

18 $(x+6)(x^2 - 6x+36)$

19 $(z - 3)^3$

20 $(x - \sqrt[3]{4})(x^2 + \sqrt[3]{4}x + \sqrt[3]{16})$

21 $(\sqrt[3]{\frac{1}{5}} + n)(\sqrt[3]{\frac{1}{25}} - \sqrt[3]{\frac{1}{5}}n + n^2)$

Solve the problems

22 Swimming pool: Baghdad hotel is one of the important tourist hotels in Baghdad, the capital of Iraq. The length of the swimming pool is $(x+9)$ meter and the width is $(x+1)$ meter. It is surrounded by an aisle which its width is 1 meter. Write the area of the swimming pool with the aisle in simplest form.



23 History: Babylon city locates to the north of Al-Hila city in Iraq. Babylonians lived there since about 3000 years BC. In 575 , they built the gate of Ishtar which considers the eighth gate of Babylon wall. Wael drew a painting represents the gate of Ishtar. The dimensions of the painting was $(y+7)$, $(y-4)$ cm. Write the painting area which was drawn by Wael in simplest form by y .



24 Ornament at fish : A cubic-shaped aquarium, the length of its side is $(v+3)$ cm. Write the volume of the aquarium in simplest form.



Think

25 Challenge: Find the result of the following in simplest form:

$$(x + 1)^2 - (x - 2)^2$$

26 Correct the mistake: Nisreen wrote the result of multiplying the two algebraic expressions, as follow:

$$(\sqrt{5} h - 4) (h - 6) = 5 h^2 + 10 h - 24$$

Determine Nisreen's mistake , then correct it.

27 Numerical sense: Which of the following two numbers is greater.

$$(\sqrt{3} - \sqrt{2})^2 \text{ or the number } (\sqrt{3} + \sqrt{2})^2 \text{ ? Clarify your answer.}$$

Write

The result of multiplying the two algebraic expressions :

$$(2z + \frac{1}{2}) (2z - \frac{1}{2})$$

Lesson [2-2] Factoring the Algebraic Expression by using a Greater Common Factor

Idea of the lesson:

*Factoring the algebraic expression by using the greater common factor.

Vocabulary:

*Factoring the algebraic expression
*The greater common factor
*Binomial terms
*Inverse
*Checking the correction of solution.

Learn

The monument of Kahrmana square in the middle of Baghdad is one of the distinctive civilizational landmarks in Iraq . It locates in the center of the square in Al-Karada. The radius of the circular statue is (r) meter. It is surrounded by a basin which is like a circular aisle. If the radius and the basin of the statue is $r + 2$ meter, find the basin area.



[2-2-1] Factoring the algebraic expression by using a greater common factor

You have previously learned how to find the greater common factor for numbers . You have also learned how to factor the algebraic expression by using the greater common factor (GCF) . Now, you will increase your skills by learning the factoring of algebraic expression which consist of two or three terms by using the greater common factor, then checking the correction of solution.

Example (1) The radius of the base of kahrmana statue is r meter, and the radius of the statue with the basin is $r + 2$ meter. Find the basin area.

$$A_1 = r^2 \pi$$

$$A_2 = (r + 2)^2 \pi = (r^2 + 4r + 4) \pi = r^2 \pi + 4r \pi + 4 \pi$$

$$A = A_2 - A_1 = r^2 \pi + 4r \pi + 4 \pi - r^2 \pi \\ = 4r \pi + 4 \pi = 4 \pi (r + 1)$$

the area of statue .

the area of statue with basin

the area of basin

(4 π) the greater common factor

The area of basin which surrounds the statue is $4 \pi (r + 1)$ square meters

Example (2) Factoring each expression by using the greater common factor (GCF), then checking the correction of solution :

i) $6x^3 + 9x^2 - 18x = 3x (2x^2 + 3x - 6)$

$$3x (2x^2 + 3x - 6) = 3x (2x^2) + 3x (3x) - 6(3x) \\ = 6x^3 + 9x^2 - 18x$$

The greater common factor is 3x

Checking :

To check by using the multiplication of algebraic expressions.

Open the bracket with simplify the numerical roots.

The greater common factor is $2\sqrt{3} yz$

ii) $\sqrt{12} y^2 z + \sqrt{2} (\sqrt{6} yz^2 - \sqrt{24} yz)$

$$= 2\sqrt{3} y^2 z + 2\sqrt{3} yz^2 - 4\sqrt{3} yz$$

$$= 2\sqrt{3} yz (y + z - 2)$$

$$2\sqrt{3} yz (y + z - 2) = 2\sqrt{3} y^2 z + 2\sqrt{3} yz^2 - 4\sqrt{3} yz$$

We see that the variables are equaled in terms with the original. expression and it is also with numerical factors because:

Checking :

To check , we use the multiplication of the algebraic expression .

$$2\sqrt{3} = \sqrt{12} , 2\sqrt{3} = \sqrt{2} \sqrt{6} , 4\sqrt{3} = \sqrt{2} \sqrt{24}$$

Example (3) Factoring each expression by using the binomial as a greater common factor :

i) $5x(x+3) - 7(x+3) = (x+3)(5x-7)$

The greater common factor is $(x+3)$

ii) $\frac{1}{2}(y-1) + \frac{1}{3}y^2(y-1) = (y-1)\left(\frac{1}{2} + \frac{1}{3}y^2\right)$

The greater common factor is $(y-1)$

iii) $\sqrt{3}v^2(z+2) - \sqrt{5}v(z+2) = v(z+2)(\sqrt{3}v - \sqrt{5})$

The greater common factor is $v(z+2)$

[2-2-2] Factoring an algebraic expression by using the property of grouping

You have previously learned in the previous how to factor the algebraic expression which consists of two or three terms by using the greater common factor. Now, you will learn how to factor an algebraic expression which consists of four terms or more by using the grouping of the terms, where the grouping terms have common factors.

Example (4) Factoring each algebraic expression by using the grouping, then check the correction of the solution:

i) $4x^3 - 8x^2 + 5x - 10 = (4x^3 - 8x^2) + (5x - 10)$

$$= 4x^2(x - 2) + 5(x - 2)$$

$$= (x-2)(4x^2 + 5)$$

Grouping terms which have common factors.

Factoring the grouping terms

The greater common factor is $(x-2)$

$$(x-2)(4x^2+5) = x(4x^2+5) - 2(4x^2+5)$$

$$= 4x^3 + 5x - 8x^2 - 10 = 4x^3 - 8x^2 + 5x - 10$$

ii) $\sqrt{2}h^2t + \sqrt{3}t^2v - \sqrt{8}h^2v - \sqrt{12}v^2t$

$$= (\sqrt{2}h^2t - \sqrt{8}h^2v) + (\sqrt{3}t^2v - \sqrt{12}v^2t)$$

$$= \sqrt{2}h^2(t-2v) + \sqrt{3}tv(t-2v)$$

$$= (t-2v)(\sqrt{2}h^2 + \sqrt{3}tv)$$

checking:

Using the property of distributing

Using multiplication and ordering

Grouping terms

Factoring the grouping terms

The greater common factor is $(t-2v)$

$$(t-2v)(\sqrt{2}h^2 + \sqrt{3}tv) = t(\sqrt{2}h^2 + \sqrt{3}tv) - 2v(\sqrt{2}h^2 + \sqrt{3}tv)$$

$$= \sqrt{2}h^2t + \sqrt{3}t^2v - \sqrt{8}h^2v - \sqrt{12}v^2t$$

checking:

Using the property of distributing

Using multiplication and ordering

Example (5) Factoring the algebraic expression by using the grouping with the inverse:

$$14x^3 - 7x^2 + 3 - 6x = (14x^3 - 7x^2) + (3 - 6x)$$

$$= 7x^2(2x - 1) + 3(1 - 2x)$$

$$= 7x^2(2x - 1) + 3(-1)(2x - 1)$$

$$= 7x^2(2x - 1) - 3(2x - 1)$$

$$= (2x - 1)(7x^2 - 3)$$

Grouping the terms

Factoring the grouping terms

Using the inverse

Writing $+3(-1)$ as -3

The greater common factor is $(2x-1)$

Make sure of your understanding

Factor each expression by using the greater common factor (GCF) , then check the correction of solution:

1 $9x^2 - 21x$

2 $10 - 15y + 5y^2$

Questions 1- 4
are similar
to example 2

3 $14z^4 - 21z^2 - 7z^3$

4 $\sqrt{8} \, t^2r + \sqrt{2} \, (tr^2 - \sqrt{3} \, tr)$

Factor each expression by using the binomial as a greater common factor:

5 $3y(y - 4) - 5(y - 4)$

6 $\frac{1}{4}(t+5) + \frac{1}{3}t^2(t+5)$

Questions 5-8
are similar
to example 3

7 $\sqrt{2} \, n(x+1) - \sqrt{3} \, m(x+1)$

8 $2x(x^2-3) + 7(x^2-3)$

Factor each expression by using the property of grouping , then check the correction of solution:

9 $3y^3 - 6y^2 + 7y - 14$

10 $21 - 3x + 35x^2 - 5x^3$

Questions 9 -12
are similar
to example 4

11 $2r^2k + 3k^2v - 4r^2v - 6v^2k$

12 $3z^3 - \sqrt{18} \, z^2 + z - \sqrt{2}$

Factor each expression by using the property of grouping with the inverse:

13 $21y^3 - 7y^2 + 3 - 9y$

14 $\frac{1}{2}x^4 - \frac{1}{4}x^3 + 5 - 10x$

Questions 13-16
are similar
to example 5

15 $6z^3 - 9z^2 + 12 - 8z$

16 $5t^3 - 15t^2 - 2t + 6$

Solve the Exercises

Factor each expression by using the greater common factor(GCF), then check the correction of solution

17 $12y^3 - 21y^2$

18 $6v^2(3v - 6) + 18v$

Factor each expression by using the binomial as a greater common factor:

19 $\frac{1}{7}(y+1) + \frac{1}{3}y^2(y+1)$

Factor each expression by using the property of grouping, then check the correction solution:

20 $5x^3 - 10x^2 + 10x - 20$

21 $3t^3k + 9k^2s - 6t^3s - 18s^2k$

Factor the expression by using the property of grouping with inverse:

22 $12x^3 - 4x^2 + 3 - 9x$

Solve the problems

23 Solar energy : The solar panels are the main component in the solar energy systems which generate electricity .The cells are manufactured from semiconducting materials such as silicon. They absorb the light of sun . What are the dimensions of the solar panel , if its area was $3x(x-4)-22(x-4)$ square meter.



24 Flamenco bird: Flamenco bird is one of the migratory birds which has beautiful shape . Its color is pink. These birds travel for long distances during the season of the annual migration passing by the marshes in the south of Iraq to get food from the water pools. If the area of the water pool which was covered by the flamenco birds in one of the Iraqi marshes is $4y^2 + 14y + 7(2y+7)$ square meter.



What is the shape of that pool, and what are its dimensions ?

25 Baghdad o'clock : It is a high building which has a Four_ Faces o'clock at the top of it.This building locates in the celebration park in Baghdad . It was established in 1994 .What is the radius of the internal circle of the o'clock if you know that its area is $z^2 \pi - 3z \pi + (3z - 9) \pi$.



Think

26 Challenge: Factor of the following expressions in a simplest form:

$$5x^5 y + 7y^3 z - 10x^5 z - 14z^2 y^2$$

27 Correct the mistake: Ibtisam had written the result of factoring the following expression, as follow:

$$\sqrt{2} t^4 - \sqrt{24} t^3 + t^2 - \sqrt{12} t = (t + 2\sqrt{3}) (\sqrt{2} t^2 - t)$$

Discover the mistake of Ibtisam and correct it.

28 Numerical sense: What is the unknown number in this expression.

$$x^2 + 3x + 5x + 15 = (x + 3) (x + \boxed{})$$

Write

The result of subtraction the expression $(x + y) (x - y)$ from the expression $(x + y) (x + y)$ in simplest form .

Lesson [2-3]

Factoring the Algebraic Expression by using Special Identities

Idea of the lesson:

*Factoring the algebraic expression as a difference between two squares and a complete square.

Vocabulary:

- *The difference between two squares
- *Perfect square
- *Completing square
- *The missing term

Learn

Al-Shaab international stadium in Baghdad is one of the important stadiums in Iraq. It was established in 1966. If the area of the playground, which was allocated for playing football



, is $x^2 - 400$ square meter, what are the dimensions of the playground?

[2-3-1] Factoring the algebraic expression by the difference between two squares.

You have previously learned how to find the result of multiplying an algebraic expression which represents the sum of two terms by another algebraic expression which represents the difference between them, and the result represents the difference between their two squares. Now, you will learn the inverse operation of multiplication which is factoring the algebraic expression which is as a difference between two squares $(x^2 - y^2) = (x + y)(x - y)$.

The expression $x^2 + y^2$ can not be factored in this stage.

Example (1)

Find the dimensions of football playground which its area is $x^2 - 400$ square meter?

$$\begin{aligned} x^2 - 400 &= (x)^2 - (20)^2 \\ &= (x + 20)(x - 20) \end{aligned}$$

Write each term as a perfect square

Write the factoring

The first bracket: the square root of the first term + the square root of the second term.

The second bracket: the square root of the first term - the square root of the second term.

So the length of the football playground is $x+20$ meter and its width is $x-20$ meter.

Example (2)

Factoring each of the following algebraic expressions as a difference between two squares

i) $x^2 - 9 = (x + 3)(x - 3)$

iii) $49 - v^2 = (7 + v)(7 - v)$

v) $5h^2 - 7v^2 = (\sqrt{5}h + \sqrt{7}v)(\sqrt{5}h - \sqrt{7}v)$

vii) $8x^3y - 2xy^3 = 2xy(4x^2 - y^2)$
 $= 2xy(2x + y)(2x - y)$

ii) $36y^2 - z^2 = (6y + z)(6y - z)$

iv) $2x^2 - z^2 = (\sqrt{2}x + z)(\sqrt{2}x - z)$

vi) $12 - t^2 = (2\sqrt{3} + t)(2\sqrt{3} - t)$

Factoring by using the common factor

Factoring by using the difference between two squares.

viii) $\frac{1}{16}z^4 - \frac{1}{81} = \left(\frac{1}{4}z^2 + \frac{1}{9}\right)\left(\frac{1}{4}z^2 - \frac{1}{9}\right) = \left(\frac{1}{4}z^2 + \frac{1}{9}\right)\left(\frac{1}{2}z + \frac{1}{3}\right)\left(\frac{1}{2}z - \frac{1}{3}\right)$

[2 -3-2] Factoring the Algebraic Expression by the Perfect square

you have previously learned how to find the result of multiplying a square of sum two terms and a square of difference between two terms , the result was consisted of three terms. Now , you will learn the inverse operation of multiplication which is factoring an expression consists of three terms in form of perfect square .

$x^2 + 2xy + y^2 = (x + y)^2$, $x^2 - 2xy + y^2 = (x - y)^2$ The algebraic expression $ax^2 \pm bx + c$ will be a perfect square if $bx = \pm 2\sqrt{(ax^2)(c)}$ where $a \neq 0$

Example (3) Factor each of the following algebraic expressions which are in a form of a perfect square.

i) $x^2 + 6x + 9 = (x)^2 + 2(x \times 3) + (3)^2$

$$= (x + 3)(x + 3)$$

$$= (x + 3)^2$$

ii) $y^2 - 4y + 4 = (y)^2 - 2(y \times 2) + (2)^2$

$$= (y - 2)^2$$

iii) $16z^2 - 8z + 1 = (4z)^2 - 2(4z \times 1) + (1)^2 = (4z - 1)^2$

*Write the first and last terms as a perfect square
Write the middle term as a double of the first term root multiplying by the root of the last term.
Write factoring of expression.*

The final factoring as $(\text{root of last term} \pm \text{first term root})$

Note the sign between the two numbers is the sign of the middle term.

Example (4) Determine which of the following algebraic expressions represents a perfect square and factor it

i) $x^2 + 10x + 25$

$$\begin{array}{ccc} (x)^2 & & (5)^2 \\ & \swarrow & \searrow \\ & 2(x)(5) = 10x & \end{array} \quad \text{perfect square}$$

$$x^2 + 10x + 25 = (x+5)^2$$

iii) $4 - 37v + 9v^2$

$$\begin{array}{ccc} (2)^2 & & (3v)^2 \\ & \swarrow & \searrow \\ & -2(2)(3v) = -12v \neq -37v & \end{array} \quad \text{Not perfect square}$$

ii) $y^2 + 14x + 36$

$$\begin{array}{ccc} (y)^2 & & (6)^2 \\ & \swarrow & \searrow \\ & 2(y)(6) = 12y \neq 14y & \end{array} \quad \text{Not perfect square}$$

iv) $9h^2 - 6h + 3$

$$\begin{array}{ccc} (3h)^2 & & (\sqrt{3})^2 \\ & \swarrow & \searrow \\ & -2(3h)(\sqrt{3}) = -6\sqrt{3}h \neq -6h & \end{array} \quad \text{Not perfect square}$$

Example (5) Write the missing term in the algebraic expression $ax^2 + bx + c$ to make it a perfect square , then factor it

i) $25x^2 - \dots + 49$ $bx \pm \sqrt{(ax^2)(c)} \cdot 2$

$$bx = 2\sqrt{(ax^2)(c)} \Rightarrow bx = 2\sqrt{(25x^2)(49)} \Rightarrow bx = 70x$$

$$\Rightarrow 25x^2 - 70x + 49 = (5x - 7)^2$$

To become a perfect square , we apply the law of the middle term.

ii) $\dots + 8x + 16$

$$bx = 2\sqrt{(ax^2)(c)} \Rightarrow 8x = 2\sqrt{(ax^2)(16)} \Rightarrow 64x^2 = 4 \times 16 \times ax^2 \Rightarrow ax^2 = x^2$$

$$\Rightarrow x^2 + 8x + 16 = (x + 4)^2$$

iii) $y^2 + 14y + \dots$

$$by = 2\sqrt{(ay^2)(c)} \Rightarrow 14y = 2\sqrt{(y^2)(c)} \Rightarrow 196y^2 = 4 \times y^2 \times c \Rightarrow c = 49$$

$$\Rightarrow y^2 + 14y + 49 = (y + 7)^2$$

Make sure of your understanding

Factor each of the following algebraic expressions as a difference between two squares:

1 $x^2 - 16$

2 $36 - 4x^2$

3 $h^2 - v^2$

4 $9m^2 - 4n^2$

5 $27x^3z - 3xz^3$

6 $\frac{1}{4}y^2 - \frac{1}{16}$

Questions 1-6
are similar
to example 2

Factor each of the following algebraic expressions as a perfect square:

7 $y^2 - 8y + 16$

8 $9z^2 - 6z + 1$

9 $v^2 + 2\sqrt{3}v + 3$

10 $4h^2 - 20h + 25$

Questions 7-10
are similar
to example 3

Determine which one of the following algebraic expressions represents a perfect square, then factor it.

11 $x^2 + 18x + 81$

12 $16 - 14v + v^2$

13 $64h^2 - 48h - 9$

14 $3 - 4\sqrt{3}t + 4t^2$

Questions 11-14
are similar
to example 4

Write the missing term in the algebraic expression ax^2+bx+c to become a perfect square, then factor it.

15 $\dots + 14y + 49$

16 $z^2 + 4z + \dots$

17 $3 - \dots + 9x^2$

18 $4x^2 + 2\sqrt{5}x + \dots$

Questions 15-18
are similar
to example 5

Solve the Exercises

Factor each of the following algebraic expressions in simplest form :

19 $25 - 4x^2$

20 $y^2 - 121$

21 $12 - 3t^2$

22 $8y^3x - 2x^3y$

23 $\frac{1}{3}z^5 - \frac{1}{12}z$

24 $4x^2 + 20x + 25$

25 $16n^2 + 8\sqrt{3}n + 3$

26 $4t^3 - 12t^2 + 9t$

Determine which of the following algebraic expressions represents a perfect square, then factor it:

27 $4x^2 + 18x + 16$

28 $y^2 + 10y + 25$

29 $2h^2 - 12h - 18$

30 $4v^2 + 4v + 4$

Write the missing term in the algebraic expression ax^2+bx+c to become a perfect square, then factor it:

31 $y^2 + \dots + 36$

32 $25 - 20x + \dots$

33 $5 - \dots + 16x^2$

34 $81 + 18z + \dots$

Solve the problems

35 Al-Malwiya minaret : It locates in Samara city, Iraq . It is one of the Iraqi distinctive landmark because of its unique shape. It is also one of the Iraqi famous ancient landmarks which belongs to the reign of Abbassiyyn . It based on a square base which its area is $x^2 - 8x + 16$ square meter What is the length of the base side which the minaret based on according to x ?



36 Farm for breeding cows : Saad has a square-shaped farm for breeding cows. The length of its side , is X meter. He extended his farm to became in a rectangular shaped farm According to that, the area of the farm became $x^2 - 81$ square meter, What are the length and width of the farm after extension according to x ?



37 Painting : Bashar drew a painting represents the marshes in the south of Iraq. The expression which represents the area of painting was $4x^2 - 8x + 9$ cm². Does the expression of the painting area represent a perfect square or not ?



Think

38 Challenge: Determine which the following algebraic expressions represent a perfect square and factor it:

$$\frac{1}{9}x^2 - \frac{1}{6}x + \frac{1}{16}$$

39 Correct the mistake: Muntaha said that the expression $(2x+1)(2x-1)$ is a factoring to the perfect square $4x^2 - 4x + 1$. Determine the mistake of Muntaha and correct it.

40 Numerical sense: Does the expression $9x^2 + 12x - 4$ represent a perfect square or not ? Clarify your answer.

Write

A factoring for the algebraic expression $4x^2 - 8x + 4$.

Lesson [2-4]

Factoring the Algebraic Expression of three terms by Probe and Error (Experiment).

Learn

Idea of the lesson:

- *Factoring the algebraic expression which consists of three terms by using
- *Probe and Error.

Vocabulary:

- *The two middles.
- *The two parties.
- *The middle term.

Assyrian winged bull (shido lamas). It is the way in which this name is written in the Assyrian writings. The origin of the word lamas is derived from the Summerian word Lammu.

There is a statue of it in AL-Moosal,s monument. What are the dimensions of the painting of winged bull which its area is $x^2 + 10x + 21$ centimeter square ?



[2-4-1] Factoring the algebraic expression x^2+bx+c

You have previously learned how to find the result of multiplying an algebraic expression by another algebraic expression which both of them consists of two terms:

i) $(x+2)(x+3) = x^2 + 5x + 6$, ii) $(x+3)(x-5) = x^2 - 2x - 15$, iii) $(x-1)(x-4) = x^2 - 5x + 4$

Now you will learn the inverse operation of multiplication which is factoring the algebraic expression which consists of three terms x^2+bx+c by using the probe and error (experiment) for factoring the algebraic expression, We find two real.

m, n, where $b = m + n, c = nm$ and write $x^2 + bx + c = (x + n)(x + m)$.

Example (1)

What are the dimensions of the painting of winged bull which its area is $x^2 + 10x + 21$ centimeter squar?

For factoring the algebraic expression, we follow the following steps:
Factoring the algebraic expression:

Factors of number 21	Sum of the two factors
(1) (21)	$1 + 21 = 22$
(3) (7)	$3 + 7 = 10$
(-3) (-7)	$(-3) + (-7) = -10$

Result of multiplying the two parties $+7x$
Result of multiplying the two middles $+3x$
 The middle term $+10x$

$$x^2 + 10x + 21 = \overbrace{(x+3)}^{\text{Two parties}} \overbrace{(x+7)}^{\text{Two middles}}$$

The width of the painting is $(x + 3)\text{cm}$
 The length of the painting is $(x + 7)\text{cm}$

Example (2)

Factoring the following algebraic expression $y^2 + y - 12$

Factors of number 12	Sum of two Factors
(1) (-12)	$1 - 12 = -11$
(12) (-1)	$12 - 1 = 11$
(2) (-6)	$2 - 6 = -4$
(6) (-2)	$6 - 2 = -4$
(3) (-4)	$3 - 4 = -1$
(4) (-3)	$4 - 3 = 1$

The result of multiplying the two parties $+4y$
The result of multiplying $-3x$
 The middle term $+y$

$$y^2 + y - 12 = (y - 3)(y + 4)$$

Example (3) Factoring the following algebraic expressions :

i) $z^2 - z - 6 = (z - 3)(z + 2)$	The middle term	$2z - 3z = -z$
ii) $x^2 - 9x + 18 = (x - 3)(x - 6)$	The middle term	$-6x - 3x = -9x$
iii) $y^2 + 6y - 27 = (y + 9)(y - 3)$	The middle term	$-3y + 9y = +6y$
iv) $x^2 - xy + 20 = (x - 5y)(x + 4y)$	The middle term	$+4xy - 5xy = -xy$
v) $15 - 8z + z^2 = (5 - z)(3 - z)$	The middle term	$-5z - 3z = -8z$

[2-4-2] Factoring the algebraic expression ax^2+bx+c where $a \neq 0$

Now, you will learn how to factor an algebraic expression which consists of three terms as in the form of $ax^2 + bx + c$ and that $a \neq 0$.

Example (4) Factoring each of the following algebraic expressions :i) $6x^2 + 17x + 7$

$$6 = \begin{cases} (1)(6) \\ (2)(3) \end{cases}, 7 = (1)(7)$$

We find the factors of 6, 7, as follow

$$(1)(6) \quad (1)(7) \Rightarrow (1)(1) + (6)(7) = 43$$

The result of multiplying the two parties $+14x$

$$(1)(6) + (7)(1) = 13$$

The result of multiplying the two middles $+3x$

$$(2)(3) \quad (1)(7) \Rightarrow (2)(1) + (3)(7) = 23$$

The middle term $+17x$

$$(2)(7) + (3)(1) = 17$$

$$6x^2 + 17x + 7 = (2x + 1)(3x + 7)$$

ii) $7y^2 - 26y - 8$

$$8 = \begin{cases} (1)(8) \\ (2)(4) \end{cases}, 7 = (1)(7)$$

We find the factors of 7, 8, as follow

$$(1)(1) - (8)(7) = -55$$

The result of multiplying the two parties $-28y$

$$(1)(7) - (8)(1) = -1$$

The result of multiplying the two middles $+2y$

$$(2)(1) - (4)(7) = -26$$

The middle term $-26y$

$$(2)(7) - (4)(1) = -10$$

$$7y^2 - 26y - 8 = (7y + 2)(y - 4)$$

Example (5) Factor each of the following algebraic expressions in a simplest form:

i) $3z^2 - 17z + 10 = (3z - 2)(z - 5)$	The middle term	$-15z - 2z = -17z$
ii) $4v^2 - v - 3 = (4v + 3)(v - 1)$	The middle term	$-4v + 3v = -v$
iii) $15 + 11h + 2h^2 = (5 + 2h)(3 + h)$	The middle term	$+5h + 6h = 11h$
iv) $6x^2 - 51x + 63 = 3(2x^2 - 17x + 21) = 3(x - 7)(2x - 3)$	The middle term	$-3x - 14x = -17x$
v) $3x^2 - 10xy - 3y^2 = (3x - y)(x + 3y)$	The middle term	$-9xy - xy = -10xy$

Make sure of your understanding

Factor each of the following algebraic expressions in simplest form:

1 $x^2 + 6x + 8$

2 $1 - 2z + z^2$

3 $x^2 - 13x + 12$

4 $3 + 2z - z^2$

5 $x^2 - 2x - 3$

6 $15 - 8z + z^2$

Questions 1-6
are similar
to examples 1,3

Factor each of the following algebraic expressions in simplest form:

7 $2x^2 + 5x + 3$

8 $3y^2 - 14y + 8$

9 $3x^2 - 10x + 8$

10 $8 - 25z + 3z^2$

11 $5y^2 - y - 6$

12 $6 + 29z - 5z^2$

13 $x^2 - 9xy + 20y^2$

14 $3y^2 - 19yx - 14x^2$

Questions 7-14
are similar
to examples 4-5

Put the signs between the terms in brackets to make the factoring of the algebraic expression correct:

15 $x^2 + 9x + 20 = (x \dots 4)(x \dots 5)$

16 $y^2 - 12y + 20 = (y \dots 2)(y \dots 10)$

Questions 15-18
are similar
to example 4

17 $6x^2 - 7x + 2 = (2x \dots 1)(3x \dots 2)$

18 $20 - 7y - 3y^2 = (5 \dots 3y)(4 \dots y)$

Solve the Exercises

Factor each of the following algebraic expressions in simplest form:

19 $x^2 + 9x + 14$

20 $y^2 - 5y + 6$

21 $3 + 2z - z^2$

22 $x^2 - 2x - 3$

Factor each of the following algebraic expressions in simplest form:

23 $2x^2 + 12x - 14$

24 $4y^2 - 6y + 2$

25 $10 + 9z - 9z^2$

26 $2x^2 + 3x + 1$

27 $13y^2 - 11y - 2$

28 $30x^2 - xy - y^2$

Put the signs between the terms in brackets to make the factoring of the algebraic expression correct:

29 $x^2 + x - 20 = (x \dots 4)(x \dots 5)$

30 $35 + 3y - 2y^2 = (5 \dots y)(7 \dots 2y)$

Solve the problems

31 Al-Ikhdher Castle: It is an ancient castle located in Karbala governorate in the middle of Iraq. The ruins of the castle are still existed nowadays. It represents a unique defensive fortress which surrounded by a great rectangular-shaped wall. What are the dimensions of external wall by x , if the castle area with wall represents by $6x^2 - 39x + 60$ square meter .



32 Entertaining games: Discovery swing considers one of the dangerous games in the fun city. The expression $5t^2 + 5t - 30$ represents the path of Discovery in the fun city , where (t) represents the time of movement , and the factoring of expression helps to know the time which the swinging takes in the first time. Factor the expression.



33 The subway: The subway considers a system of underground railway. It is one of the fast means for transportation in the big cities and in those cities which have a high density of population. Each train consists of several vehicles. If the expression $14y^2 - 23y + 3$ represents the ground area of the vehicle in square meter. What are the dimensions of the vehicle by y ?



Think

34 Challenge: Factor of the following algebraic expressions in simplest form:
 $4x^3 + 4x^2 - 9x - 9$

35 Correct the mistake: Sa,ad factored the expression $6z^2 - 16z - 6$, as follow:

$$6z^2 - 16z - 6 = (3z - 1)(2z + 6)$$

Discover Sa,ad's mistake, then correct it.

36 Numerical sense: Can we determine if the signs of the two bracket in factoring the expression $x^2 - 12x + 35$ are different or similar and without factoring the expression ?
 Clarify your answer.

Write

The signs between the terms inside brackets to make the factoring of the algebraic expression correct.

$$6z^2 + 5z - 56 = (3z \dots 8)(2z \dots 7)$$

Lesson [2-5]

Factoring the Algebraic Expression, sum of two cubes or difference between two cubes.

Idea of the lesson:

* Factoring the algebraic expression of three terms which represents sum (difference between) two cubes.

Vocabulary:

* Sum of two cubes
* Difference between two cubes

Learn

Rubik's cube, which was invented by the Hungarian sculptor and architect

Ernő Rubik in 1974, is a three – dimensions mechanical puzzle. What is the sum volume of two cubus of Rubik's , the length of the first cube side is 3 dcm, and the length of the side of the second cube is 4 dcm?



[2-5-1] Factoring the algebraic expression, sum of two cubes.

You have previously learned in the first lesson of this chapter the multiplication of two –terms algebraic expression by three-terms algebraic expression . The result of their multiplication represents an expression as a sum of two cubes, like $((x + 2) (x^2 - 2x + 4) = x^3 + 8 = x^3 + 2^3)$ Now, you will learn the inverse operation which is factoring the two- terms algebraic expression which represents the sum of two cubes: $x^3 + y^3 = (x + y) (x^2 - x y + y^2)$ where $x = \sqrt[3]{x^3}$, $y = \sqrt[3]{y^3}$

Example (1)

What is the sum of two volumes of two cubes of Rubik . The side length of the first cube is 3dcm, while the side length of the second one is 4dcm.

$$v_1 + v_2 = 3^3 + 4^3$$

$$= (3 + 4) (3^2 - 3 \times 4 + 4^2)$$

$$= 7 (9 - 12 + 16) = 7 \times 13 = 91 \text{dcm}^3$$

The cube volume = length x width x hight = (length of side)³

Law of factoring sum of two cubes.

Example (2)

Factor each of the following expressions in simplest form:

i) $x^3 + 5^3 = (x + 5) (x^2 - 5x + 5^2) = (x + 5) (x^2 - 5x + 25)$

ii) $y^3 + 8 = y^3 + 2^3 = (y + 2) (y^2 - 2y + 4)$

iii) $8z^3 + 27 = 2^3 z^3 + 3^3 = (2z)^3 + 3^3 = (2z + 3) (4z^2 - 6z + 9)$

iv) $\frac{1}{a^3} + \frac{1}{64} = \frac{1}{a^3} + \frac{1}{4^3} = (\frac{1}{a} + \frac{1}{4}) (\frac{1}{a^2} - \frac{1}{4a} + \frac{1}{16})$

v) $\frac{27}{x^3} + \frac{8}{125} = \frac{3^3}{x^3} + \frac{2^3}{5^3} = (\frac{3}{x})^3 + (\frac{2}{5})^3 = (\frac{3}{x} + \frac{2}{5}) (\frac{9}{x^2} - \frac{6}{5x} + \frac{4}{25})$

vi) $\frac{1}{2} t^3 + 4 = \frac{1}{2} (t^3 + 8) = \frac{1}{2} (t^3 + 2^3) = \frac{1}{2} (t + 2) (t^2 - 2t + 4)$

vii) $0.008 + v^3 = (0.2)^3 + v^3 = (0.2 + v) (0.04 - 0.2v + v^2)$

[2-5-2] Factoring the Algebraic Expression difference between two cubes

You have learned in the first lesson of this chapter the multiplying of an algebraic expression which consists of two terms by another one which consists of three terms

$(x - 3)(x^2 + 3x + 9) = x^3 - 27 = x^3 - 3^3$, and the result of their multiplying represents an expression as a difference between two cubes, like

Now, you will learn the inverse operation which is factoring the algebraic expression which consists of two terms and in the form of difference between two cubes $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

where $x = \sqrt[3]{x^3}$, $y = \sqrt[3]{y^3}$.

Example (3)

Cube-shaped basin filled with water, the length of its side is 1m.

The water was transferred to another cube-shaped basin which is bigger than the first one, the length of its side is 1.1 m.

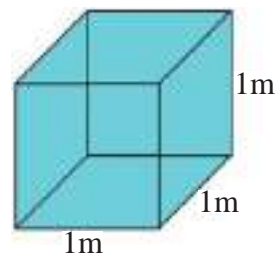
What is the additional quantity of water that we need to fill the big basin?

the additional quantity of water = the volume of big cube - the volume of small cube.

$$v_2 - v_1 = (1.1)^3 - 1^3$$

$$= (1.1 - 1)((1.1)^2 + 1.1 \times 1 + 1^2) \quad \text{Law of factoring the difference between two cubes}$$

$$= 0.1(1.21 + 1.1 + 1) = 0.1 \times 3.31 = 0.331 \text{ m}^3$$



Example (4)

Factor each of the following algebraic expressions in simplest form:

i) $x^3 - 3^3 = (x - 3)(x^2 + 3x + 3^2) = (x - 3)(x^2 + 3x + 9)$

ii) $y^3 - 64 = y^3 - 4^3 = (y - 4)(y^2 + 4y + 16)$

iii) $27z^3 - 8 = 3^3z^3 - 2^3 = (3z)^3 - 2^3 = (3z - 2)(9z^2 + 6z + 4)$

iv) $\frac{1}{b^3} - \frac{1}{125} = \frac{1}{b^3} - \frac{1}{5^3} = \left(\frac{1}{b} - \frac{1}{5}\right)\left(\frac{1}{b^2} + \frac{1}{5b} + \frac{1}{25}\right)$

v) $\frac{1}{3}t^3 - 9 = \frac{1}{3}(t^3 - 27) = \frac{1}{3}(t^3 - 3^3) = \frac{1}{3}(t - 3)(t^2 + 3t + 9)$

vi) $0.216 - n^3 = (0.6)^3 - n^3 = (0.6 - n)(0.36 + 0.6n + n^2)$

vii) $1 - 0.125z^3 = 1 - (0.5)^3z^3 = (1 - 0.5z)(1 + 0.05z + 0.25z^2)$

viii) $32 - \frac{1}{2}m^3 = \frac{1}{2}(64 - m^3) = \frac{1}{2}(4^3 - m^3) = \frac{1}{2}(4 - m)(16 + 4m + m^2)$

Make sure of your understanding

Factor each of the following algebraic expressions in simplest form :

1 $y^3 + 216$

2 $x^3 + z^3$

3 $125 + 8z^3$

4 $\frac{1}{27}x^3 + \frac{1}{8}$

5 $\frac{1}{a^3} + \frac{1}{64}$

6 $\frac{1}{3}t^2 + 9$

7 $0.125 + v^3$

8 $1 + 0.008z^3$

Questions 1- 8
are similar
to examples 1-2

Factor each of the following algebraic expressions in simplest form :

9 $a^3 - 8^3$

10 $8y^3 - 64$

11 $\frac{1}{c^3} - \frac{1}{8}$

12 $\frac{1}{2}v^3 - 4$

13 $0.125 - m^3$

14 $25 - \frac{1}{5}n^3$

15 $3b^3 - 81$

16 $0.216v^3 - 0.008t^3$

Questions 9-16
are similar
to examples 3-4

Solve the Exercises

Factor each of the following algebraic expressions in simplest form :

17 $6^3 + x^3$

18 $125y^3 + 1$

19 $\frac{1}{64} + \frac{8}{125}y^3$

20 $\frac{1}{5}v^3 + 25$

21 $0.125x^3 + 0.008y^3$

Factor each of the following algebraic expressions in simplest form :

22 $y^3 - 64$

23 $\frac{1}{x^3} - \frac{27}{8}$

24 $9 - \frac{1}{3}n^3$

25 $25c^3 - \frac{1}{5}$

26 $0.01x^3 - 0.0084y^3$

Solve the problems

27 Library: Shtotgart's library, in Germany, is one of the most beautiful libraries in the world. It is also one of the largest libraries in line with the requirements of the modern education in Germany the shape of the building was cubic. The side length is $\frac{1}{2}y^3 - 13\frac{1}{2}$ meter. Factor the expression which represents the side length.



28 Aquarium: The volume of aquarium of ornamental fish is $25x^3$ cubic meter.

A cube-shaped stone was put inside the aquarium. The size of stone was $\frac{1}{5}$ cubic meter. The aquarium was filled with water. write the expression which represents the volume of water then factor it.



29 Residence : The designs of new house – buildings start to take different shapes which are more complicated in architecture. These houses were designed in shape of cubes If the volume of the first house is $\frac{8}{a^3}$ cubic meter and the second house is $\frac{27}{b^3}$ cubic meter. Write is the volume of the two houses?



Think

30 Challenge: Factor of the following algebraic expressions in simplest form:
 $0.002z^3 - 0.016y^3$

31 Correct the mistake: Bushra factored the expression $8v^3 - 0.001$,as follow
 $8v^3 - 0.001 = (2v + 0.1)(4v^2 - 0.4v + 0.01)$
 Discover Bushra's mistake, then correct it..

32 Numerical sense:Is it possible to add 27 , 8 by using the method of factoring the sum of two cubes ? clarify your answer.

Write

The signs between the terms inside brackets to make the factoring of algebraic expression correct:

$$125 - x^3 = (5 \dots x)(25 \dots 5x \dots x^2)$$

Lesson [2-6]

Simplifying Rational Algebraic Expressions

Idea of the lesson:

- *Multiplying and dividing the rational algebraic expressions then write them in simplest form.
- *Adding and subtracting the rational algebraic expressions then write them in simplest form.

Vocabulary:

- * Ratio
- * Fraction

Learn

Hassan had bought a group of flowers bouquets in $x^2 - x - 6$ dinars.

The cost of one bouquet was $2x - 6$ dinars. Write the ratio of one bouquet cost to the total cost of all bouquets, then write it in simplest form.



[2-6-1] Simplifying the multiplying and dividing of rational algebraic expressions

You have previously learned the properties of the rational and real numbers, you have also learned how to simplify the numerical sentences by using the least common multiplier (L.C.M) and ordering operations. Now, you will learn how to simplify the rational algebraic expressions (Fractional) by dividing each of numerator and dominator by a common factor, and repeat it so that no way stay for that, and then, we can say that the expression is in a simplest form.

Example (1)

Write the cost ratio of one flower bouqued to the total cost of the bouquets in a simplest form.

$$\frac{\text{cost of one bouquet}}{\text{cost of total bouquet}} = \frac{2x - 6}{x^2 - x - 6} = \frac{2(x - 3)}{(x - 3)(x + 2)}$$

$$= \frac{\cancel{2(x - 3)}}{\cancel{(x - 3)}(x + 2)} = \frac{2}{x + 2}$$

Factor the numerator and dominator

By dividing the numerator and

dominator on the common factor

Example (2)

Write each of the following expressions in simplest form:

$$\text{i) } \frac{x^2 - 4}{(x^2 - 4x + 4)} = \frac{(x + 2)(x - 2)}{(x - 2)^2} = \frac{(x + 2)\cancel{(x - 2)}}{(x - 2)\cancel{(x - 2)}} = \frac{x + 2}{x - 2}$$

$$\text{ii) } \frac{5z + 10}{z - 3} \times \frac{z^3 - 27}{(z^2 + 6z + 8)} = \frac{5\cancel{(z + 2)}}{\cancel{z - 3}} \times \frac{\cancel{(z - 3)}(z^2 + 3z + 9)}{\cancel{(z + 2)}(z + 4)} = \frac{5(z^2 + 3z + 9)}{z + 4}$$

$$\text{iii) } \frac{16 - x^2}{3x + 5} \times \frac{(3x^2 + 2x - 5)}{(x^2 + 3x - 4)} = \frac{\cancel{(4 + x)}(4 - x)}{\cancel{(3x - 5)}} \times \frac{\cancel{(3x + 5)}\cancel{(x - 1)}}{\cancel{(x + 4)}\cancel{(x - 1)}} = 4 - x$$

$$\text{iv) } \frac{8 + t^3}{4 + 9t + t^2} \div \frac{(2 + t)^3}{t^2 + 9t + 14} = \frac{8 + t^3}{4 - 2t + t^2} \times \frac{t^2 + 9t + 14}{(2 + t)^3}$$

$$= \frac{\cancel{(2 + t)}(4 - 2t + t^2)}{\cancel{4 - 2t + t^2}} \times \frac{\cancel{(t + 2)}(t + 7)}{\cancel{(2 + t)}^3} = \frac{t + 7}{2 + t} = \frac{t + 7}{t + 2}$$

Multiply the first by the second which is inverted

Factoring the numerator and dominator and dividing by the common factor

[2-6-2] Simplifying adding and subtracting of the rational algebraic expressions.

You have previously learned how to factor the algebraic expressions and how to find the least common multiplier (L.C.M): represents the result of multiplying the common factors by the biggest power and unjoined factors), when simplifying fractional numerical sentences.

Now, you will learn how to simplify the adding and subtracting of the (fractional) rational algebraic expressions by factoring each of the numerator and dominator of the faction to simplest form, then adding and subtracting the fractional expressions by using the common multiplier and simplify the expression to simplest form.

Example (3) Write the rational algebraic expression in simplest form.

$$\frac{y^2}{(y+2)} - \frac{4}{(y+2)}$$

$$= \frac{y^2 - 4}{(y+2)} = \frac{\cancel{(y+2)}(y-2)}{\cancel{(y+2)}} = y - 2$$

The least common multiplier (x + 2)

Factoring the numenator as a form of a difference between two squares by dividing each of numerator and dominator by y + 2

Example (4) Write each of the following expressions in simplest form:

$$\text{i) } \frac{7x - 14}{x^2 - 4} + \frac{5}{(x+2)} = \frac{7(x-2)}{(x+2)(x-2)} + \frac{5}{x+2}$$

By factoring the numenator and dominator

$$= \frac{7}{x+2} + \frac{5}{x+2}$$

The least common multiplier (x + 2)

$$= \frac{7+5}{x+2} = \frac{12}{x+2}$$

$$\text{ii) } \frac{4z}{2z-5} - \frac{z}{z+3} = \frac{4z}{2z-5} \times \left(\frac{z+3}{z+3}\right) - \frac{z}{z+3} \times \left(\frac{2z-5}{2z-5}\right)$$

The least common multiplier

$$= \frac{4z(z+3) - z(2z-5)}{(2z-5)(z+3)} = \frac{2z^2 + 17z}{(2z-5)(z+3)} = \frac{z(2z+17)}{(2z-5)(z+3)}$$

$$\text{iii) } \frac{t^2 + 2t + 4}{t^3 - 8} + \frac{12}{3t - 6} = \frac{t^2 + 2t + 4}{(t-2)(t^2 + 2t + 4)} + \frac{12}{3(t-2)} = \frac{1}{(t-2)} + \frac{4}{(t-2)} = \frac{5}{(t-2)}$$

$$\text{iv) } \frac{8}{v+4} + \frac{2}{v-4} - \frac{2}{v^2-16} = \frac{8}{v+4} + \frac{2}{v-4} - \frac{1}{(v+4)(v-4)} = \frac{8(v-4) + 2(v+4) - 1}{(v+4)(v-4)}$$

$$= \frac{8v - 32 + 2v + 8 - 1}{(v+4)(v-4)} = \frac{10v - 25}{(v+4)(v-4)} = \frac{5(2v-5)}{(v+4)(v-4)}$$

Make sure of your understanding

Write each of the following expressions in simplest form:

$$1 \quad \frac{2z^2 - 4z + 2}{z^2 - 7z + 6}$$

$$2 \quad \frac{y^3 + 27}{y^3 - 3y^2 + 9y}$$

Questions 1-6
are similar
to examples 1-2

$$3 \quad \frac{5x + 3}{x + 3} \times \frac{x^2 + 5x + 6}{25x^2 - 9}$$

$$4 \quad \frac{z^2 + 7z - 8}{z - 1} \times \frac{z^2 - 4}{z^2 + 6z - 16}$$

$$5 \quad \frac{x^2 - 9}{x^3 + 4x + 4} \times \frac{x^2 - 4}{x^2 - x - 6}$$

$$6 \quad \frac{2y^2 - 2y}{y^2 - 9} \div \frac{y^2 + y - 2}{y^2 + 2y - 3}$$

Write each of the following expressions in simplest form:

$$7 \quad \frac{2}{x^2 - 9} + \frac{3}{x^2 - 4x + 3}$$

$$8 \quad \frac{2y^3 - 128}{y^3 + 4y^2 + 16y} - \frac{y - 1}{y}$$

Questions 7-12
are similar
to examples 3-4

$$9 \quad \frac{z^2 + z + 1}{z^4 - z} - \frac{z + 3}{z^2 + 2z - 3}$$

$$10 \quad \frac{x^2 - 1}{x^2 - 2x + 1} - 1$$

$$11 \quad \frac{3}{z - 1} + \frac{2}{z + 3} + \frac{8}{z^2 + 2z - 3}$$

$$12 \quad \frac{y - 3}{y - 1} + \frac{5y - 15}{(y - 3)^2} - \frac{3y + 1}{y^2 - 4y + 3}$$

Solve the Exercises

Write each of the following expressions in simplest form:

$$13 \quad \frac{x + 5}{12x} \times \frac{6x - 30}{x^2 - 25}$$

$$14 \quad \frac{3 - x}{4 - 2x} \times \frac{x^2 + x - 6}{9 - x^2}$$

$$15 \quad \frac{y^2 - 7y}{y^3 - 27} \div \frac{y^2 - 49}{y^2 + 3y + 9}$$

Write each of the following expressions in simplest form:

$$16 \quad \frac{5}{x^2 - 36} - \frac{2}{x^2 - 12x + 36}$$

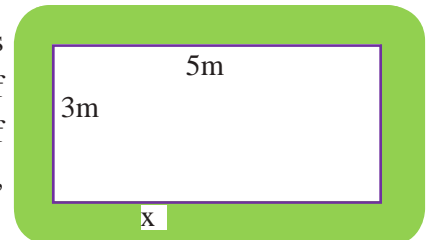
$$17 \quad \frac{3}{x - 2} - \frac{2}{x - 2} - \frac{4 + 2x + x^2}{x^3 - 8}$$

Solve the problems

18 Library: If the algebraic expression $x^2 - 4$ represents the number of scientific books in the library, and the algebraic expression $x^2 + x - 6$ represents the number of literary books in the library. Write the ratio of the scientific books to the literary books in a simplest form.



19 Geometry: Dimensions of a rectangle are 3,5 meters. It was extended to bigger one by surrounding it by an aisle with width of x meter. Write the algebraic expression which represents the sum of the two ratios of the rectangle length before and after the extension, in a simplest form.



20 Fireworks: The algebraic expression $20 + 15t - 5t^2$ represents the height, in meters, of a fireworks shell which was shot from a 20 meter-high building roof, where t represents the time of reaching the shell to the target, in seconds. And the algebraic expression $4 + 19t - 5t^2$ represents the height of another shell which was shot from a roof of a 4 meter-high building. Write the ratio of the first shell height to the height of the second shell, in simplest form.



Think

21 Challenge: Factoring of the following algebraic expression:

$$\frac{y^2 - 5}{2y^3 - 16} \div \frac{y - \sqrt{5}}{(2y^2 + 4y + 8)}$$

22 Correct the mistake: Samah simplified the algebraic expression and wrote it in a simplest form, as follow

$$\frac{z^2 - z - 30}{5 + z} \times \frac{2z + 12}{z^2 - 36} = 1$$

Discover samah's mistake and correct it.

23 Numerical sense: What is the result of adding the two algebraic expressions without using paper and pen ? clarify your answer.

$$\frac{5}{x^2 - 49} \times \frac{-4}{(x - 7)(x + 7)}$$

Write

The value of the algebraic expression in a simplest form:

$$\frac{z^2 - 4}{2z + 8} \div \frac{z^2 - z - 12}{2z^2 + 2z - 12}$$

Chapter Test

Find the result of multiplying an algebraic expression by another algebraic expression, each one of them consists of two terms:

1 $(x + 5)^2$

2 $(v - \sqrt{2})(v + \sqrt{2})$

3 $(2 - x)(5 - x)$

4 $(2y - 3)(y + 9)$

Find the result of multiplying a two terms algebraic expression by another algebraic expression, which consists of three terms:

5 $(x + 11)(x^2 - 11x + 121)$

6 $(\frac{1}{3} - y)(\frac{1}{9} + \frac{1}{3}y + y^2)$

7 $(y - 1)^3$

8 $(z + \frac{1}{4})^3$

Factor the expression by using the greater common factor (GCF), then check the correction of solution:

9 $8x^2 - 12x$

10 $7y^3 + 14y^2 - 21y$

11 $\sqrt{18}z^3r + \sqrt{2}(zr^2 - zr)$

Factor the expression by using the binomial as a greater common factor:

12 $\frac{2}{3}(y + 5) + \frac{1}{3}y(y + 5)$

13 $\sqrt{5}z(z^2 - 1) - \sqrt{2}z^2(z^2 - 1)$

Factor the expression by using the property of grouping:

14 $6x^4 - 18x^3 + 10x - 30$

15 $56 - 8y + 14y^2 - 2y^3$

Factor the expression by grouping with inverse:

16 $9x^3 - 6x^2 + 8 - 12x$

17 $\sqrt{11}z^3 - \sqrt{44}z^2 + 5(2 - z)$

Factor each of the following algebraic expressions:

18 $16 - x^2$

19 $\frac{1}{3}z^2 - \frac{1}{27}$

20 $\frac{1}{16}v - \frac{1}{2}v^4$

21 $8x^3 - \frac{1}{125}$

22 $81 - 18y + y^2$

23 $7z^3 - 36z + 5$

Determine which of the following algebraic expressions represent a perfect square, then factor it:

24 $25x^2 + 30x + 9$

25 $49 - 14y + y^2$

26 $4v^2 + 4\sqrt{5}v + 5$

Write the missing term in the algebraic expression $ax^2 + bx + c$ to become a perfect square, then factor it:

27 $x^2 + \dots + 81$

28 $36 - 12y + \dots$

29 $7 - \dots + 4z^2$

Factor each of the following algebraic expressions:

30 $x^2 + 7x + 10$

31 $x^2 - 5\sqrt{3}x + 18$

32 $2v^2 + 9v + 7$

33 $32 - 16x + 2x^2$

34 $\frac{1}{4}y^2 - 2y + 3$

35 $12 - 7\sqrt{2}v + 2v^2$

36 $8 + 27x^3$

37 $125y^3 - 1$

38 $\frac{1}{v^3} - \frac{8}{27}$

39 $1 + 0.125y^3$

40 $z^3 - 0.027$

41 $3 - \frac{1}{9}v^3$

Write each of the following algebraic expressions in a simplest form:

42 $\frac{27 - 8z^3}{4z^2 - 9} \div \frac{9 + 6z + 4z^2}{9 + 6z}$

43 $\frac{7}{x^2 - 25} - \frac{6}{x^2 + 10x + 25}$

44 $\frac{y^2 - 1}{1 - y^3} + \frac{1 + y}{1 + 2y + y^2}$

45 $\frac{z + 3}{z + 5} - \frac{z - 5}{z - 3} + \frac{1}{z^2 + 2z - 15}$

Equations

- lesson 3-1 Solving the system of two linear Equations with two variables.
- lesson 3-2 Solving the Quadratic Equations with one variable.
- lesson 3-3 Using Prope and Error to Solve the Quadratic Equations.
- lesson 3-4 Solving the Quadratic Equations by the Perfect square.
- lesson 3-5 Using General Law to Solve Equations.
- lesson 3-6 Solving the Fractional Equations.
- lesson 3-7 Problem solving Plan (Writing Equations).

Basil and Sa'ad had travelled in tourist tours by Baghdad International airport. Basil's group was less than sa'ad group in 22 persons. If the number of all travelers is 122 persons. We can calculate the number of each group by solving the two linear equations of first degree $x + y = 122$, $x - y = 22$, where the variable x represents the number of persons in Sa'ad-s group and the variable y represents the number of persons in Basil-s group.

Pretest

Find the result of multiplying an algebraic expression by another algebraic expression, each one consists of two terms:

1 $(y - 5)^2$

2 $(z + 2)(z - 2)$

3 $(x - \sqrt{5})(x + \sqrt{5})$

4 $(4 - y)(6 - y)$

5 $(3z - 2)(z + 8)$

Find the result of multiplying an algebraic expression of two terms by another algebraic expression of three terms:

6 $(x + 3)(x^2 - 3x + 9)$

7 $(\frac{1}{2} - y)(\frac{1}{4} + \frac{1}{2}y + y^2)$

Factor the expression by using the greater common factor (GCF), then check the correction of solution:

8 $5x^2 - 10x$

9 $9y^3 + 6y^2 - 3y$

10 $\sqrt{12}z^2 + \sqrt{3}z$

Factor the expression by using the binomial as a greater common factor:

11 $x(5 - x) - 3(5 - x)$

12 $\frac{1}{2}(y + 1) + \frac{1}{2}y(y + 1)$

13 $\sqrt{3}z(z - 1) - \sqrt{2}(z - 1)$

Factor the expression by grouping:

14 $6x^3 - 12x^2 + 5x - 10$

15 $9 - 18y + 7y^2 - 14y^3$

16 $\sqrt{2}z^4 - \sqrt{6}z^3 + z - \sqrt{3}$

Factor the expression by grouping with the inverse:

17 $4x^3 - 2x^2 + 9 - 6x$

18 $\frac{3}{4}y^3 - \frac{1}{4}y^2 + 4 - 12y$

19 $\sqrt{4}z^3 - \sqrt{25}z^2 + 3(5 - 4z)$

Factor each of the following algebraic expressions:

20 $y^2 - 25$

21 $\frac{1}{2}z^2 - \frac{1}{8}$

22 $36 - 12x + x^2$

23 $y^2 - 2y - 15$

Determine which of the following algebraic expressions represents a perfect square, then factor it:

24 $16x^2 + 40x + 25$

25 $64 - 16y + y^2$

26 $z^2 - 6z - 9$

Write the missing term in the algebraic expression $ax^2 + bx + c$ to become a perfect square, then factor it.

27 $x^2 + \dots + 64$

28 $9 - 24y + \dots$

29 $5 - \dots + 4z^2$

Factor each of the following algebraic expressions:

30 $18 - 3y - y^2$

31 $z^2 - 2\sqrt{3}z + 3$

32 $4 - 21x + 5x^2$

33 $1 + 27z^3$

34 $y^3 - 125$

35 $y^3 - \frac{1}{8}$

36 $\frac{1}{x^3} - \frac{1}{64}$

37 $1 - 0.125z^3$

Lesson [3-1]

Solving the System of two Linear Equations with two variables.

Learn

Idea of the lesson:

*Solve the system of two linear equations graphically and by substitution and elimination.

Vocabulary:

- *Linear equation.
- *Linear Equations system.
- *Solving the system.

Ahmed has a factory for canning dates. The cost of the empty cans is 100000 dinars. The process of filling one can with dates costs 500 dinars, then it sells in 1000 dinars. Ahmed would like to know the number of cans that he should sell to get profit.



[3-1-1] Solving a System of two Linear Equations graphically.

Assume $\vec{L}_1 = a_1x + b_1y = c_1$, $\vec{L}_2 = a_2x + b_2y = c_2$, represent two equations of first degree (linear) with two variables x, y . To solve this system graphically we follow:

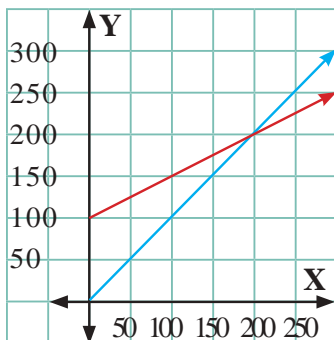
- 1) Representing each of the two lines in the coordinate plane.
- 2) Finding a point of intersection the two lines by drawing two columns from the point on the two axis X - axis and Y - axis then the intersection point will represent the solution set.

Example (1)

Finding the number of cans that Ahmed should sell to get profit.

$$y = 500x + 100000 \quad \dots\dots (1)$$

$$y = 1000x \quad \dots\dots (2)$$



Assuming that the costs of the production is the variable (y), and the number of cans which were sold is the variable (x), accordingly :

An equation represents the total production costs.

An equation represents the total value of sales.

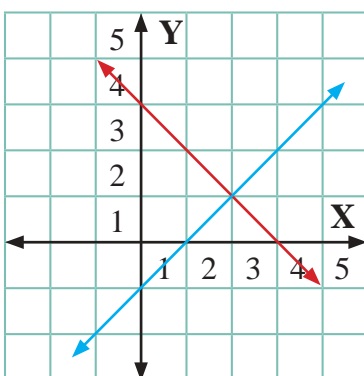
The scale of axis (y) is by thousands of dinars.

We represent the two equations graphically and determine the intersection point of the two lines (200,200) which represents selling 200 cans. The profit starts when more than 200 cans have being sold.

The ordered pair (200,200) which represents a solution for two equations is called a solution of the system.

Example (2)

Find a solution set for the system graphically in R .



$$x - y = 1 \quad \dots\dots (1)$$

$$x + y = 3 \quad \dots\dots (2)$$

We represent the two equations graphically and determine the intersection point of the two lines (2, 1).

The solution set of the system is $S = \{ (2, 1) \}$

To check the correction of solution, we substitute the value of the two variables x, y in both equations to get two correct statements.

$$x - y = 1 \rightarrow 2 - 1 = 1 \rightarrow 1 = 1 \quad \text{Substituting by the equations...}(1).$$

$$x + y = 3 \rightarrow 2 + 1 = 3 \rightarrow 3 = 3 \quad \text{Substituting by the equations....}(2).$$

[3-1-2] Solving the system of two Linear Equations by Substitution

We can summarize this method to solve a system of two linear with two variables x, y by transforming one of the two equations to an equation with only one variable by finding a relation between x, y from one of the two equations, then substituting it in the other equation.

Example (3) Find the solution set of the system by using the substitution:

$$\begin{aligned} \text{i) } y &= 4x \quad \dots\dots (1) \\ y &= x + 6 \quad \dots\dots (2) \end{aligned} \Rightarrow 4x = x + 6$$

$$\Rightarrow 4x - x = 6 \Rightarrow x = 2$$

$$y = x + 6 \Rightarrow y = 2 + 6 \Rightarrow y = 8$$

we Substitute the value y from the equation...(1) in the equation(2) solving the equation, then find the variable value x .

Substituting the value of (x) by the equation...(2) to find the variable value (y) .

So the solution set for the system is $\{(2, 8)\}$

$$\begin{aligned} \text{ii) } x + 8y &= 10 \quad \dots\dots (1) \\ x - 4y &= 2 \quad \dots\dots (2) \end{aligned} \Rightarrow x = 2 + 4y$$

$$2 + 4y + 8y = 10 \Rightarrow 12y = 8 \Rightarrow y = \frac{2}{3}$$

$$x + 8y = 10 \Rightarrow x + 8 \times \frac{2}{3} = 10 \Rightarrow x = 10 - \frac{16}{3} \Rightarrow x = \frac{14}{3}$$

We find the value of x from the equation...(2), then

Substituting in the equation...(1).

Substituting the value of y by the equation...(1).

So the solution set of the system is $\{(\frac{2}{3}, \frac{14}{3})\}$

[3-1-3] Solving the system of two linear Equations by Elimination method.

This method can be summarized to solve a system of two equations with two variables x, y by eliminating one of the two variables through making the coefficient of one of them equal in value and different in sign in both equations.

Example (4) Find the solution set for the system by using the elimination method:

$$\begin{aligned} \text{i) } x + 2y &= 5 \quad \dots\dots(1) \\ 3x - y &= 1 \quad \dots\dots (2) \end{aligned}$$

$$\Rightarrow \begin{cases} x + 2y = 5 & \dots\dots(1) \\ 6x - 2y = 2 & \dots\dots(2) \end{cases}$$

$$7x = 7 \Rightarrow x = 1$$

by adding

Multiplying the equation...

(2) by the number 2 then adding them to the equation....(1).

Substituting the value x in one of two equations (simplest equation)

Substituting in the equation ...(1)

So the solution set of the system is $\{(1, 2)\}$

$$x + 2y = 5 \Rightarrow 1 + 2y = 5 \Rightarrow 2y = 4 \Rightarrow y = 2$$

Multiplying the equation...(1) by the number 2 and the equation...(2) by the number 3, then we subtract the two equations.

$$\begin{aligned} \text{ii) } 3x + 4y &= 10 \quad \dots\dots (1) \\ 2x + 3y &= 7 \quad \dots\dots (2) \end{aligned}$$

$$\Rightarrow \begin{cases} 6x + 8y = 20 & \dots\dots(2) \\ 6x \pm 9y = \pm 21 & \dots\dots(1) \end{cases}$$

$$y = 1$$

by subtracting

We substitute the value x in the one of the two equations (before changing the sign) substitute in the equation...(2)

$$2x + (3 \times 1) = 7 \Rightarrow 2x = 4 \Rightarrow x = 2$$

So the solution set of the system is $\{(2, 1)\}$

Make sure of your understanding

Find a solution set of the system in \mathbb{R} graphically:

$$\begin{cases} 1 & 3x - y = 6 \\ & x - y = 3 \end{cases}$$

$$\begin{cases} 2 & y - x = 3 \\ & y + x = 0 \end{cases}$$

$$\begin{cases} 3 & y = x - 2 \\ & y = 3 - x \end{cases}$$

Questions 1-3
are similar
to examples 1-2

Find the solution set of the system by using the method of substitution, for each of the following :

$$\begin{cases} 4 & 2x + 3y = 1 \\ & 3x - 2y = 0 \end{cases}$$

$$\begin{cases} 5 & x - 2y = 11 \\ & 2x - 3y = 18 \end{cases}$$

$$\begin{cases} 6 & y - 5x = 10 \\ & y - 3x = 8 \end{cases}$$

Questions 4-6
are similar
to example 3

Find the solution set of the system by using the elimination method, for each of the following :

$$\begin{cases} 7 & 3x - 4y = 12 \\ & 5x + 2y = -6 \end{cases}$$

$$\begin{cases} 8 & x - 3y = 6 \\ & 2x - 4y = 24 \end{cases}$$

$$\begin{cases} 9 & 3y - 2x - 7 = 0 \\ & y + 3x + 5 = 0 \end{cases}$$

Questions 7-9
are similar
to example 4

Find the solution set of the system, then check the e correction of the solution:

$$\begin{cases} 10 & \frac{2x}{3} - \frac{y}{2} = 1 \\ & \frac{3y}{3} - \frac{x}{3} = 4 \end{cases}$$

$$\begin{cases} 11 & 0.2x - 6y = 4 \\ & 0.1x - 7y = -2 \end{cases}$$

$$\begin{cases} 12 & \frac{1}{2}x + \frac{2}{3}y = 2\frac{3}{4} \\ & \frac{1}{4}x - \frac{2}{3}y = 6\frac{1}{4} \end{cases}$$

Solve the Exercises

Find the solution set of the system graphically:

$$\begin{cases} 13 & x - y = -4 \\ & y + x = 6 \end{cases}$$

$$\begin{cases} 14 & y = x - 4 \\ & x = 2 - y \end{cases}$$

Find the solution set of the system by using the method of substitution, for each of the following:

$$\begin{cases} 15 & 3x + 2y = 2 \\ & x - y = 8 \end{cases}$$

$$\begin{cases} 16 & 2x - y = -4 \\ & 3x - y = 3 \end{cases}$$

Find the solution set of the system by using the method of elimination, for each of the following:

$$\begin{cases} 17 & 3x = 22 - 4y \\ & 4y = 3x - 14 \end{cases}$$

$$\begin{cases} 18 & 5x - 3y = 6 \\ & 2x + 5y = -10 \end{cases}$$

Solve the problems

19 Weather : During January, the number of days (x) in which the temperature goes down in Baghdad decreases about 10 Celsius , is nearly less in 9 days than the number of days (y) in which the temperature goes up about 10 Celsius. Write two equations represent this situation, then find their solution by using the method of elimination to find the number of days in each situation.



20 Trade : A commercial shop had sold 25 fridges and washing machines. The price of one fridge was million dinars while price of one washer was 500000 dinars. If the total cost of fridges and washers was 20millions dinars, then how many appliances did the seller sell from each type? Write two equations represent the problem, then solve them by using the substitution method.



21 Graduation party : Sajad and Anwer held a party on the occasion of their graduation from college. The number of friends who were invited by Sajad is more in three from the friends who were invited by Anwer. The total number of friends who came to the party is 23 persons. How many persons did each of Sajad and Anwer invite? write two equations represent the problem , then solve them to find the required.



Think

22 Challenge: Find the solution set for the system:

$$\left. \begin{array}{l} \frac{2}{6}x - \frac{1}{3}y = 1 \\ \frac{1}{2}x + \frac{1}{2}y = 3 \end{array} \right\}$$

23 Correct the mistake: Ahmed said that the solution set of the system:

Is the set $\{(\frac{5}{16}, \frac{5}{9})\}$

$$\left. \begin{array}{l} 2x + 3y = 6 \\ 3x + 2y = 1 \end{array} \right\}$$

Discover Ahmed's mistake, then correct it.

Write

A solution set for the system : $\begin{array}{l} 5x - 6y = 0 \\ x + 2y = 4 \end{array}$

Lesson [3-2]

Solving Quadratic Equations with one variable

Idea of the lesson:

*Solving the equation which consists of two terms by factoring the difference between two squares.

Vocabulary:

- *Equation
- *Second degree
- *One variable
- *Difference between two squares

Learn

Zaqlara is one of the Iraqi civilized landmarks. It sits in south of Iraq. Basil drew a square-shaped picture its area is 9 m^2 . Find the side length of the picture.



[3-2-1] Using difference between two squares to solve equations

The general equation of second degree with one variable $ax^2 + bx + c = 0$ where, $a \neq 0$ and $a, b, c \in \mathbb{R}$ and solving it means finding a values set of the variable (x) which satisfies the equation, by making it correct statement. We will study in this item solving the equations consist of two terms by using the greater common factor, difference between two squares and the property of zero-product.

Example (1)

Write an equation represents the area of picture, then solve it to find the side length of the picture.

Assume that the side length of picture is the variable (x) and the equation which represents the area of picture is: $x^2 = 9$

$$\begin{aligned} x^2 - 9 = 0 &\Rightarrow (x + 3)(x - 3) = 0 \\ &\Rightarrow x + 3 = 0 \quad \text{or} \quad x - 3 = 0 \\ &\Rightarrow x = -3, \text{ neglect} \end{aligned}$$

*Factoring by using the difference between two squares
property of zero-product
The wall picture length is 3 m*

Example (2)

solve the following equation by using the difference between two squares, then check the correct of solution.

$$\begin{aligned} 16 - y^2 = 0 &\Rightarrow (4 + y)(4 - y) = 0 && \text{Factoring by using the difference between two squares} \\ 4 + y = 0 &\Rightarrow \text{or } 4 - y = 0 \Rightarrow y = -4 \quad \text{or} \quad y = 4 \Rightarrow S = \{-4, 4\} && \text{solution set.} \end{aligned}$$

Check: each value in the solution set for the variable (y) must satisfies the equation

$$\text{L.S} = 16 - y^2 = 16 - (-4)^2 = 16 - 16 = 0 = \text{R.S}$$

By substitution $y = -4$

$$\text{L.S} = 16 - y^2 = 16 - 4^2 = 16 - 16 = 0 = \text{R.S}$$

By substitution $y = 4$

Example (3)

Solve the following equations by using the difference between two squares:

$$\text{i) } 4x^2 - 25 = 0 \Rightarrow (2x + 5)(2x - 5) = 0 \Rightarrow 2x + 5 = 0 \quad \text{or} \quad 2x - 5 = 0$$

$$\Rightarrow x = -\frac{5}{2} \quad \text{or} \quad x = \frac{5}{2} \Rightarrow s = \left\{ -\frac{5}{2}, \frac{5}{2} \right\}$$

$$\text{ii) } 3z^2 - 12 = 0 \Rightarrow 3(z^2 - 4) = 0 \Rightarrow (z + 2)(z - 2) = 0 \quad \text{By dividing the two parties on 3, then by factoring.}$$

$$\Rightarrow z + 2 = 0 \quad \text{or} \quad z - 2 = 0 \Rightarrow s = \{-2, 2\}$$

$$\text{iii) } 2y^2 - 6 = 0 \Rightarrow y^2 - 3 = 0 \Rightarrow (y + \sqrt{3})(y - \sqrt{3}) = 0 \Rightarrow y = -\sqrt{3} \quad \text{or} \quad y = \sqrt{3} \Rightarrow s = \{-\sqrt{3}, \sqrt{3}\}$$

$$\text{iv) } x^2 - 5 = 0 \Rightarrow (x + \sqrt{5})(x - \sqrt{5}) = 0 \Rightarrow x = -\sqrt{5} \quad \text{or} \quad x = \sqrt{5} \Rightarrow s = \{-\sqrt{5}, \sqrt{5}\}$$

$$\text{v) } (z + 1)^2 - 36 = 0 \Rightarrow (z + 1 + 6)(z + 1 - 6) = 0 \Rightarrow (z + 7)(z - 5) = 0 \Rightarrow s = \{-7, 5\}$$

[3-2-2] Using Square root Property to Solve the equations root

You have learned in the previous item how to solve the equation of second degree with one variable by the factoring method using the difference between two squares. Now, we will find the solution set for the second –degree equation with one variable by using the method of square root property .

$$\sqrt{x^2} = |x| \geq 0$$

$$25 = 5^2 \Rightarrow \sqrt{25} = \sqrt{5^2} = |5| = 5$$

$$25 = (-5)^2 \Rightarrow -\sqrt{(-5)^2} = |-5| = 5$$

In general form : If a is positive real number then $x^2 = a \Rightarrow x = \pm \sqrt{a}$

Example (4)

Solve the following equation by using the property of square root, then check the correction of solution:

$$x^2 = 9 \Rightarrow x = \pm \sqrt{9} \Rightarrow x = \pm 3$$

By using the property of the square root

$$\Rightarrow S = \{3, -3\}$$

Solution set of the equation

Check: Each value in the solution set of the variable (x) must satisfy the equation

$$L.S = x^2 = 3^2 = 9 = R.S$$

By substitution $x = 3$

$$L.S = x^2 = (-3)^2 = -3 \times -3 = 9 = R.S$$

By substitution $x = -3$

Example (5)

Solve the following equation by using the square root rule:

$$i) y^2 = 36 \Rightarrow y = \pm \sqrt{36} \Rightarrow x = \pm 6 \Rightarrow S = \{6, -6\}$$

$$ii) z^2 = \frac{9}{25} \Rightarrow z = \pm \sqrt{\frac{9}{25}} \Rightarrow z = \pm \frac{3}{5} \Rightarrow S = \{\frac{3}{5}, -\frac{3}{5}\}$$

$$iii) x^2 + 81 = 0 \Rightarrow x^2 = -81 \text{ There is no solution for this equation in the real numbers}$$

(there is no a real number has a negative square)

$$iv) 3y^2 = 7 \Rightarrow y^2 = \frac{7}{3} \Rightarrow y = \pm \sqrt{\frac{7}{3}} \Rightarrow y = \pm \frac{\sqrt{7}}{\sqrt{3}} \Rightarrow S = \{\frac{\sqrt{7}}{\sqrt{3}}, -\frac{\sqrt{7}}{\sqrt{3}}\}$$

$$v) 4x^2 - 5 = 0 \Rightarrow 4x^2 = 5 \Rightarrow x^2 = \frac{5}{4} \Rightarrow x = \pm \sqrt{\frac{5}{4}} \Rightarrow x = \pm \frac{\sqrt{5}}{2} \Rightarrow S = \{\frac{\sqrt{5}}{2}, -\frac{\sqrt{5}}{2}\}$$

Notice: If the two sides of a correct equation were quadrtd, then the resulted equation stills correct ($y = x \Rightarrow y^2 = x^2$) example $\sqrt{x} = 5 \Rightarrow (\sqrt{x})^2 = 5^2 \Rightarrow x = 25$

The inverted is not correct $x^2 = y^2 \nRightarrow y = x$

Example (6)

Solve the following equations :

$$i) 3\sqrt{x} = 18 \Rightarrow \sqrt{x} = 6 \Rightarrow (\sqrt{x})^2 = 6^2 \Rightarrow x = 36 \Rightarrow S = \{36\} \text{ By squared the both sides of the equation}$$

$$ii) \sqrt{y+8} = 3 \Rightarrow (\sqrt{y+8})^2 = 3^2 \Rightarrow y+8 = 9 \Rightarrow y = 9-8 \Rightarrow y = 1 \Rightarrow S = \{1\}$$

$$iii) \sqrt{5z} = 7 \Rightarrow (\sqrt{5z})^2 = 7^2 \Rightarrow 5z = 49 \Rightarrow z = \frac{49}{5} \Rightarrow S = \{\frac{49}{5}\}$$

$$iv) \sqrt{\frac{x}{13}} - 1 = 0 \Rightarrow \sqrt{\frac{x}{13}} = 1 \Rightarrow (\sqrt{\frac{x}{13}})^2 = 1^2 \Rightarrow \frac{x}{13} = 1 \Rightarrow x = 13 \quad S = \{13\}$$

Make sure of your understanding

Solve the following equations by using the difference between two squares, then check the correction of solution:

1 $x^2 - 16 = 0$

2 $81 - y^2 = 0$

3 $2z^2 - 8 = 0$

Questions 1-3
are similar
to example 2

Solve the following equations by using the difference between two squares:

4 $4x^2 - 9 = 0$

5 $5y^2 - 20 = 0$

6 $(y + 2)^2 - 49 = 0$

7 $(3 - z)^2 - 1 = 0$

8 $x^2 - 3 = 0$

9 $y^2 - \frac{1}{6} = 0$

Questions 4-9
are similar
to example 3

Solve the following equations by using the rule of the square root:

10 $x^2 = 64$

11 $z^2 = 7$

12 $2y^2 = \frac{49}{8}$

13 $6z^2 - 5 = 0$

14 $4(x^2 - 12) = 33$

15 $z^2 + \frac{2}{3} = \frac{5}{6}$

Questions 10-15
are similar
to example 4

Solve the following equations:

16 $3\sqrt{x} = 15$

17 $\sqrt{y - 5} = 2$

18 $\sqrt{2z} = 6$

Questions 16-18
are similar
to example 5

Solve the Exercises

Solve the following equations, then check the correction of solution:

19 $x^2 = 49$

20 $5y^2 - 10 = 0$

Solve the following equations in R by using the difference between two squares:

21 $9x^2 - 36 = 0$

22 $9(x^2 - 1) - 7 = 0$

23 $y^2 - \frac{1}{36} = 0$

Solve the following equations by using the rule of the square root:

24 $x^2 = 121$

25 $50 - 2y^2 = 0$

26 $x^2 = \frac{1}{64}$

27 $7(x^2 - 2) = 50$

Solve the following equations:

28 $6\sqrt{x} = 30$

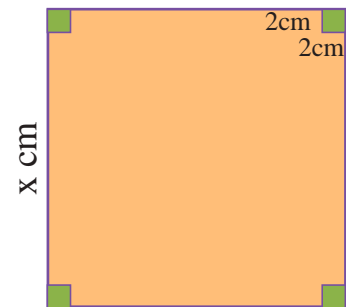
29 $\sqrt{4z} = 8$

Solve the problems

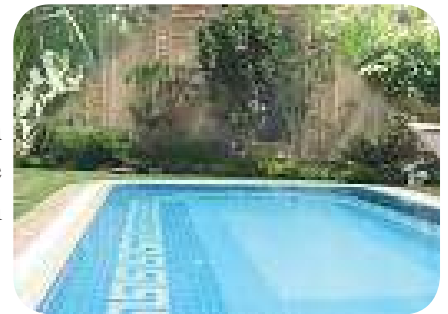
30 Carpets: A rectangular- shaped carpet, its length is 12 m and width 3m. It was divided into parts to cover the floor of a square- shaped room. write the equation which represent the problem and find the side length of the room?



31 Geometry: A piece of cardboard which was square-shaped, its side length is x cm. four equal pieces were cut from its four corners. The side length of each square is 2cm. It was folded to form a box without cover. which was a rectangular parallelepiped-shaped box, its volume 32 cm^3 write the equation which represent the problem and. Find the side length of the origin cardboard.



32 Fountain: A square- shaped swimming pool was designed in the center of a square-shaped garden. It's side length is 3m. The remained area of the garden which was surrounded the pool was 40 m^2 . write the equation which represent the problem and the side length of the garden?



Think

33 Challenge: Solve the following equations:

i) $9(x^2 + 1) = 34$

ii) $4x^2 - 3 = 0$

34 Does the given set represent the solution set for the equation or not?

i) $(2y + 1)^2 = 16$, $\left\{ \frac{3}{\sqrt{2}} , -\frac{3}{\sqrt{2}} \right\}$

ii) $3x^2 - 7 = 0$, $\left\{ \frac{7}{\sqrt{3}} , -\frac{7}{\sqrt{3}} \right\}$

35 Correct the mistake: Salah said that the set $\left(\left\{ \frac{4}{\sqrt{5}} , -\frac{4}{\sqrt{5}} \right\} \right)$ represents the solution set for the equation $5x^2 = 4$. Discover Salah's mistake and correct it.

36 Numerical sense: A positive integer consists of one digit, If one was subtracted from its square, the result would be a number from the multiplying of ten. What is the number?

Write

The solution set for the equation:

$$(8 - 3y)^2 - 1 = 0$$

Lesson [3-3] Using Probe and Error to Solve the Quadratic Equations(Experiment)

Idea of the lesson:

*Solving the equations of the second degree which consist of three terms by factoring in experiment.

Vocabulary:

*Quadratic equation
*Experiment

Learn

If the length of the basketball court increases in about 2m more than the double of its width, and its area is 480m^2 . Find the two dimensions of the court.



[3-3-1] Solving the equation $x^2 + bx + c = 0$.

You have previously learned how to find the factoring of an algebraic expression which consists of three terms by experiment. Now, you will use the factoring to solve the equations of second degree which consist of three terms $x^2 + bx + c = 0$ where b, c are real numbers (factoring the expression to two brackets with two different signs or two similar signs according to the sign of the absolute term and the middle term).

Example (1) Finding the two dimensions of a basketball court.

Assume that the width of the court is the variable x , so the length of the court will be $2x+2$.

Court area = length \times width

$$x(2x + 2) = 480 \Rightarrow x^2 + 2x - 480 = 0 \Rightarrow x^2 + x - 240 = 0$$

$$\Rightarrow (x + 16)(x - 15) = 0$$

$$\Rightarrow \begin{cases} x + 16 = 0 & \Rightarrow x = -16 \\ \text{or } x - 15 = 0 & \Rightarrow x = 15 \end{cases}$$

$$\text{or } x - 15 = 0 \Rightarrow x = 15$$

*The middle term $-15x + 16x = x$.
We neglect it because there is no negative length.*

So the width of the court is 15m and the length is $2 \times 15 + 2 = 32\text{m}$

Example (2) Solve the following equations by factoring in experiment:

i) $x^2 - 7x + 12 = 0 \Rightarrow (x - 3)(x - 4) = 0 \Rightarrow x = 3 \text{ or } x = 4 \Rightarrow S = \{3, 4\}$

ii) $y^2 + 8y + 15 = 0 \Rightarrow (y + 3)(y + 5) = 0 \Rightarrow y = -3 \text{ or } y = -5 \Rightarrow S = \{-3, -5\}$

iii) $z^2 + z - 30 = 0 \Rightarrow (z + 6)(z - 5) = 0 \Rightarrow z = -6 \text{ or } z = 5 \Rightarrow S = \{-6, 5\}$

iv) $x^2 - 2x - 63 = 0 \Rightarrow (x - 9)(x + 7) = 0 \Rightarrow x = 9 \text{ or } x = -7 \Rightarrow S = \{9, -7\}$

v) What is the number which its square increases in 12?

Assume that the number is x , then the square of the number will be x^2 , and the numerical sentence which represents the problem is

$$x^2 - x = 12 \Rightarrow x^2 - x - 12 = 0 \Rightarrow (x - 4)(x + 3) = 0 \Rightarrow x = 4 \text{ or } x = -3$$

So, the number is either 4 or -3

[3 -3-2] Solving the equation $ax^2 + bx + c = 0$, $a \neq 0$

You have previously learned how to solve an equation by experiment method and the variable x^2 without coefficients. Now, you will learn how to solve the same equation but with existence of coefficients for the variable x^2 .

Example (3) A swimming pool which its length is less in three times of its width in 1 m. If the area of the swimming pool is $140m^2$, find its dimensions.

Assume that the width of the swimming pool is the variable x so the length of the pool is $3x - 1$

The equation which represents the problem is

We solve the equation $x(3x-1)=140$:

$$x(3x-1) = 140 \Rightarrow 3x^2 - x - 140 = 0$$

$$\Rightarrow \begin{cases} (3x + 20)(x - 7) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} 3x + 20 = 0 \Rightarrow x = -\frac{20}{3} \end{cases}$$

$$\text{or } x - 7 = 0 \Rightarrow x = 7$$

The middle term $-21x + 20x = -x$

We neglect it because there is no negative length

So the width of the swimming pool is 7m and its length is 20m.



Example (4) Solve the following equations by factoring in experiment:

i) $4y^2 - 14y + 6 = 0 \Rightarrow (4y - 2)(y - 3) = 0$

$-12y - 2y = -14y$ The middle term

$$\Rightarrow \begin{cases} 4y - 2 = 0 \Rightarrow y = \frac{1}{2} \\ \text{or } y - 3 = 0 \Rightarrow y = 3 \Rightarrow S = \{\frac{1}{2}, 3\} \end{cases}$$

ii) $3x^2 - x - 21 = 0 \Rightarrow (3x - 3)(x + 7) = 0$

$21x - 3x = 18x$ The middle term

$$\Rightarrow \begin{cases} 3x - 3 = 0 \Rightarrow x = 1 \\ \text{or } x + 7 = 0 \Rightarrow x = -7 \Rightarrow S = \{1, -7\} \end{cases}$$

iii) $20 + 13z + 2z^2 = 0 \Rightarrow (4 + z)(5 + 2z) = 0$

$8z + 5z = 13z$ The middle term

$$\Rightarrow \begin{cases} 4 + z = 0 \Rightarrow z = -4 \\ \text{or } 5 + 2z = 0 \Rightarrow z = -\frac{5}{2} \Rightarrow S = \{-4, -\frac{5}{2}\} \end{cases}$$

iv) $9x^2 - 69x - 24 = 0 \Rightarrow 3(3x^2 - 23x - 21) = 0 \Rightarrow 3x^2 - 23x - 21 = 0$

$$\Rightarrow (3x + 1)(x - 8) = 0$$

$-24x - x = -23x$ The middle term

$$\Rightarrow \begin{cases} 3x + 1 = 0 \Rightarrow x = -\frac{1}{3} \\ \text{or } x - 8 = 0 \Rightarrow x = 8 \Rightarrow S = \{-\frac{1}{3}, 8\} \end{cases}$$

Make sure of your understanding

Solve the following equations by factoring in experiment:

1 $x^2 - 9x + 18 = 0$

2 $x^2 - 4x - 32 = 0$

3 $y^2 + 48y - 49 = 0$

4 $y^2 + 9y - 36 = 0$

5 $x^2 - 3x + 2 = 0$

6 $y^2 - 8y - 33 = 0$

7 What is the number which its square is more greater than its double in 35?

8 What is the number that if we add its fourfold to it's square, the result will be 45?

9 A carpet ,its length is more than its width in about 2m and its area is 48m^2 , what are the dimensions of the carpet?

10 $15x^2 - 11x - 14 = 0$

11 $6 + 7x - 5x^2 = 0$

12 $42 + 64y + 24y^2 = 0$

13 $36 - 75x + 6x^2 = 0$

14 $70 - 33y + 2y^2 = 0$

15 A rectangular- shaped land, its length is more than its width in 4m, what are the two dimensions of the land if its area is 60m^2 Rectangular Carpat ?

Questions 1-6
are similar
to example 2

Questions 7-9
are similar
to example 1

Questions 10-14
are similar
to example 4

Questions 15 is
similar
to example 3

Solve the Exercises

Solve the following equations by factoring in experiment:

16 $x^2 - 15x + 56 = 0$

17 $y^2 + 16y + 63 = 0$

18 $x^2 + 15x - 16 = 0$

19 $y^2 - y - 42 = 0$

20 A rectangular- shaped metal, its width decreases in 2m from its length. What are the two dimensions of the piece of metal? If its area is 24m^2 ?

21 A dining hall, its length less from the twice of its width in 3m and its area is 54m^2 , what are the dimensions of the hall?

Find the solution set for the following equations, then check the correction of solution:

22 $x^2 - 4x + 3 = 0$

23 $y^2 - 9y - 36 = 0$

24 $4 - 26x + 12x^2 = 0$

Solve the problems

25 Sport: If the length of a picture of football stadium increases in 4m more than the twice of its width, its area was 160m^2 .

What are the two dimensions of the picture?



26 Field of ostriches: If the length of a field for breeding ostriches decreases in 4m than the twice of its width. If its area was 96m^2 , will a 44m length fence be enough to surround the field?



27 Picture frame: Samir bought a picture frame, its length is twice of its width.

Samir needs to shorten the frame in 2cm from both length and width to make it suitable for picture. What are the dimensions of the frame which Samir bought if the picture area is 40 cm^2 ?



Think

28 Challenge: Solve the following equations by factoring in experiment.

i) $(x - 3)(x + 2) = 14$

ii) $3y^2 - 11y + 10 = 80$

29 Clarify: Does the given set represents a solution set for the equation or not?

i) $4x^2 + 2x = 30$, $\{-\frac{2}{5}, 3\}$

ii) $42 - 33y + 6y^2 = 0$, $\{2, \frac{7}{2}\}$

30 Correct the mistake: Rana said that the solution set for the equation $2x^2 - 34x + 60 = 0$ is $\{3, 15\}$.

Determine Rana's mistake, then correct it.

Write

An equation represents the following problem, then find its solution:

What is the integer number which its square is less than its twice in 35?

Lesson [3-4]

Solving the Quadratic Equations by the Perfect square.

Learn

Idea of the lesson:

*Solving the quadratic equations by method of perfect square.

Vocabulary:

- *First term
- *Last term
- *Perfect square
- *Completing the square

Jaguar (Panthera onca) is a kind of tigers.

The equation $x^2 - 20x + 100 = 0$ represents a square region area in square meters which is allocated to the tiger inside a zoo.

What is the expression which represented the side length of the squared area ?



[3-4-1] Solving the equations by the Perfect square.

You have previously learned how to factor the algebraic expression which is a perfect square. Now, we will use the factoring in solving equations by factoring the complete square to find the solution set of the equation.

Example (1)

What is the expression which represented by the side length of the square area?

$$x^2 - 20x + 100 = 0$$

To factor the left side of the equation, we should be sure that the expression represents a perfect square.

A perfect square because:

the middle term $= 2 \times$ (the first term root \times the last term root).

$$x^2 - 20x + 100 = 0 \Rightarrow (x - 10)^2 = 0 \Rightarrow (x - 10)(x - 10) = 0 \quad \text{Factoring of the expression}$$

$$\Rightarrow x - 10 = 0 \Rightarrow x = 10$$

$$\text{or } x - 10 = 0 \Rightarrow x = 10$$

So the side length of the square region area which is allocated to tiger is 10m .

Example (2)

Solving the following equations by the complete square:

i) $4x^2 + 20x + 25 = 0$

The middle term $2 \times (2x \times 5) = 20x$

$$\Rightarrow (2x + 5)^2 = 0 \Rightarrow 2x + 5 = 0 \Rightarrow 2x = -5 \Rightarrow x = -\frac{5}{2}$$

We take one of the repeated factors

ii) $y^2 - y + \frac{1}{4} = 0$

The middle term $2 \times (y \times \frac{1}{2}) = y$

$$\Rightarrow (y - \frac{1}{2})^2 = 0 \Rightarrow y - \frac{1}{2} = 0 \Rightarrow y = \frac{1}{2}$$

We take one of the repeated factors

iii) $3 - 6\sqrt{3}z + 9z^2 = 0$

The middle term $2 \times (\sqrt{3} \times 3z) = 6\sqrt{3}z$

$$\Rightarrow (\sqrt{3} - 3z)^2 = 0 \Rightarrow \sqrt{3} - 3z = 0 \Rightarrow 3z = \sqrt{3}$$

We take one of the repeated factors

$$z = \frac{1}{\sqrt{3}}$$

[3-4-2] Solving Equations by Completing the square

Now, you will learn how to solve an equation of second degree by completing the square:

- 1- We put the quadratic equation as follow: $ax^2 + bx = -c$, where $a \neq 0$
- 2- If $1 \neq a$, the equation will be divided by a .
- 3- We add the expression (quadrature of the half factor x) to the two sides of the equation.
- 4- We factor the left side which becomes a perfect square after step 3 , then we simplify the right part.
- 5- We take the square root for the two sides, then we find the values of x .

Example (3) Solve the following equations by completing the square:

i) $x^2 - 4x - 12 = 0 \Rightarrow x^2 - 4x = 12$

we write the equation as in the first step

$$\Rightarrow x^2 - 4x + 4 = 12 + 4$$

Adding the expression $(\frac{1}{2} \times -4) = 4$ to the two sides of the equation.

$$\Rightarrow x^2 - 4x + 4 = 16 \Rightarrow (x - 2)^2 = 16$$

$$\Rightarrow x - 2 = \pm 4 \Rightarrow \begin{cases} x - 2 = 4 \Rightarrow x = 6 \\ \text{or } x - 2 = -4 \Rightarrow x = -2 \end{cases} \Rightarrow S = \{6, -2\}$$

We take the square root of the two sides of the equation

ii) $2y^2 - 3 = 3y \Rightarrow 2y^2 - 3y = 3$

We write the equation as in the first step

$$\Rightarrow y^2 - \frac{3}{2}y = \frac{3}{2}$$

By dividing the two sides of the equation by 2

$$\Rightarrow y^2 - \frac{3}{2}y + \frac{9}{16} = 3 + \frac{9}{16}$$

Adding the expression $(\frac{1}{2} \times -\frac{3}{2})^2 = \frac{9}{16}$ to the two sides of the equation by factoring the left side and simplify the right side of the equation.

$$\Rightarrow (y - \frac{3}{4})^2 = \frac{33}{16}$$

$$\Rightarrow y - \frac{3}{4} = \pm \frac{\sqrt{33}}{4}$$

We take the square root of the two sides of the equation.

$$\Rightarrow \begin{cases} y - \frac{3}{4} = \frac{\sqrt{33}}{4} \Rightarrow y = \frac{3 + \sqrt{33}}{4} \\ \text{or} \\ y - \frac{3}{4} = -\frac{\sqrt{33}}{4} \Rightarrow y = \frac{3 - \sqrt{33}}{4} \end{cases}$$

$$\Rightarrow S = \left\{ \frac{3 - \sqrt{33}}{4}, \frac{3 + \sqrt{33}}{4} \right\}$$

Example (4) The length of a rectangle is greater than its width in 2cm estimate, the length and width of the rectangle by nearing to the nearest integer n if its area was 36cm^2 .

$x(x + 2) = 36$ Assume that the rectangular width is the variable x , then the rectangle length is $x+2$.

The equation which represent the problem:

$$\Rightarrow x^2 + 2x = 36$$

We solve the equation by completing the square

$$\Rightarrow x^2 + 2x + 1 = 36 + 1 \quad \text{We add } (\frac{1}{2} \times 2)^2 = 1 \text{ to the two sides of the equation . neglect } x \approx -1$$

$$\Rightarrow (x + 1)^2 = 37 \Rightarrow x + 1 = \pm \sqrt{37} \Rightarrow x + 1 \approx \pm 6 \Rightarrow x + 1 \approx 6 \Rightarrow x \approx 5$$

$$\text{Note : } \sqrt{37} \approx \sqrt{36} = 6$$

$$\Rightarrow x + 1 \approx -6 \Rightarrow \text{neglect } x \approx -7$$

So the approximate width of the rectangle is 5 cm and its length is 7 cm.

Make sure of your understanding

Solve the following equations by the perfect square:

1 $x^2 + 12x + 36 = 0$

3 $4x^2 - 4x + 1 = 0$

5 $x^2 + 16x = -64$

2 $y^2 - 10y + 25 = 0$

4 $y^2 + 2\sqrt{7}y + 7 = 0$

6 $\frac{1}{16} - \frac{1}{2}x + x^2 = 0$

Questions 1-6
are similar
to example 2

Solve the following equations by complete square :

7 $x^2 - 10x - 24 = 0$

9 $4x^2 - 3x - 16 = 0$

11 $x^2 - \frac{6}{5}x = \frac{1}{5}$

8 $y^2 - 3 = 2y$

10 $3y^2 + 2y = 1$

12 $5y^2 + 15y - 30 = 0$

Questions 7-12
are similar
to example 3

Solve the Exercises

Solve the following equations by the perfect square:

13 $x^2 + 12x + 144 = 0$

14 $y^2 + 4\sqrt{2}y + 8 = 0$

15 $3y^2 + 36 - 12\sqrt{3}y = 0$

Solve the following equations by complete square

16 $y^2 + 2\sqrt{3}y = 3$

17 $x^2 - 2x = 0$

18 $x^2 - \frac{2}{3}x = 4$

Solve the following equations by complete square, then find the result by nearing to the nearest integer:

19 $x^2 - 6x = 15$

20 $y(2y + 28) = 28$

Solve the problems

21 Babylon city: It is Babylon in Latin. It is an Iraqi city which sites nearby the river of Euphrates. It was the capital of Babylonians during the reign of Hamoraby (1750-1792) BC. Find the from the equation $x^2 - 28x + 196 = 0$ which represents the lenght of the side of one of the square – shaped hall find the value of x.



22 Panda bear: The area which was allocated to the Panda bear in a zoo is a rectangular- shaped area. which is 126 m^2 . its width is less than the length in 8 m.find the dimensions of the allocated area for panda by nearing to the nearest integer .



23 Whales: Some whales are swimming in groups towards the beach and no one know why because there is no scientific illustration to this phenomena. Those who interested in protecting the environment try to return them to the sea. Solve the equation $x^2 + 20x = 525$ by the method of completing the square to find the value of x which represents the number of whales which swam toward the beach in Australia.



Think

24 Challenge: Solve the following equations by completing the square, then find the result by nearing to the nearest integer :

i) $4x(x - 6) = 27$

ii) $6y^2 - 48y = 6$

25 Correct the mistake: Sawsen solved the equation $4x^2 - 4\sqrt{3}x + 3 = 0$ by the method of completing the square, then wrote the solution set for the equation as follows $S = \{\frac{\sqrt{3}}{4}, -\frac{\sqrt{3}}{4}\}$. Discover Sawsen's mistake and correct it.

26 Numerical sense: Does the solution set of the equation $y^2 - 4y + 4 = 0$ contains two equaled values in the expression which one of them is positive and the other is negative? Clarify your answer.

Write

The solution set for the equation:

$$\frac{1}{81} - \frac{2}{9}z + z^2 = 0$$

Lesson [3-5]

Using General Law to Solve the Equations.

Idea of the lesson:

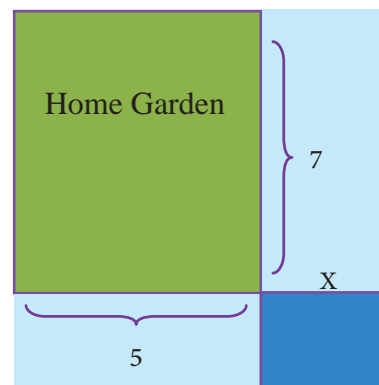
- * Solving the linear equations from the second degree by using the general law.

Vocabulary:

- * Coefficient
- * Absolute term
- * General law

Learn

An aisle was required to be paved on the two sides of a home garden. It was paved with ceramics. The length of the garden is 7m and its width is 5m. And the area of paving is 45 m² find the width of the aisle which was required to be paved with ceramics.



[3-5-1] Solve the Equation by Using the Law $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and $a \neq 0$.

You have previously learned how to solve an equation of second degree by many methods, but there are equations which cannot be solved by the previous methods, so we will solve them by the general law (constitution) by finding the real roots for the quadratic equation, as follow:

- 1) We put the quadratic equation in the general form (standard) $ax^2 + bx + c = 0$.
- 2) We write the values of coefficients : a coefficient x^2 , b coefficient x with its sign, c represents the absolute term with its sign.

Supstitutiong by the general law to find the two values of the variable.

Example (1)

From learn paragraph , what is the width of the aisle which needs to be paved on the two sides of the garden?

Assume that the width of the aisle is x, then the area of the right part of the aisle equals 7x, the area of the front part = 5x , the area of the aisle angle = x^2 and the sum of the two area of the paving is 45m².

$$x^2 + 7x + 5x = 45 \Rightarrow x^2 + 12x = 45 \quad \text{The equation which represents the problem}$$

$$x^2 + 12x - 45 = 0 \quad \text{Put the equation in the general form}$$

$$a = 1, b = 12, c = -45 \quad \text{Determine the coefficients and substituting, in the general law}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-12 \pm \sqrt{144 - 4 \times 1 \times (-45)}}{2 \times 1} \Rightarrow x = \frac{-12 \pm \sqrt{144 + 180}}{2}$$

$$\Rightarrow x = \frac{-12 \pm \sqrt{324}}{2} \Rightarrow x = \frac{-12 \pm 18}{2} \Rightarrow \begin{cases} x = \frac{-12 + 18}{2} \Rightarrow x = 3 \\ \text{or } x = \frac{-12 - 18}{2} \Rightarrow x = -15 \end{cases} \quad \begin{array}{l} \text{The width of the aisle is 3m} \\ \text{Neglect because it is impossible} \end{array}$$

Example (2)

Find the solution set for the following equations by using the general law:

$$x^2 - 3x - 5 = 0, a = 1, b = -3, c = -5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{3 \pm \sqrt{9 + 20}}{2} \Rightarrow x = \frac{3 \pm \sqrt{29}}{2} \Rightarrow \text{or } \begin{cases} x = \frac{3 + \sqrt{29}}{2} \\ x = \frac{3 - \sqrt{29}}{2} \end{cases} \Rightarrow S = \left\{ \frac{3 + \sqrt{29}}{2}, \frac{3 - \sqrt{29}}{2} \right\}$$

[3-5-2] The Discriminative ($\Delta = b^2 - 4ac$)

In the first part of this lesson, you have learned how to solve the equation by the general law to find the real roots of the equation. Now, we will talk about the discriminant of the quadratic equation

$$ax^2 + bx + c = 0 \text{ which is } \Delta = b^2 - 4ac$$

and the type of the two roots of the equation determines as follow:

Roots type :

1- Two rational real roots.

2- Two irrational real roots.

3- Two equal real roots ($\frac{-b}{2a}$).

4- Two irrational real roots. (the solution set in $R = \emptyset$)

$$\Delta = b^2 - 4ac .$$

1) positive and perfect square (parameter rational number

2) Positive and not a complete square

3) Zero

4) Negative

Example (3) Determine the equation roots, firstly, then find the solution set if it is possible:

i) $2x^2 + 3x - 2 = 0$, $a = 2$, $b = 3$, $c = -2$

$$\Delta = b^2 - 4ac \Rightarrow \Delta = 9 - 4 \times 2 \times (-2) = 25$$

The discriminative expression is a perfect square that means the equation has two rational roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-3 \pm \sqrt{9 + 16}}{4} \Rightarrow x = \frac{-3 \pm \sqrt{25}}{4} \Rightarrow x = \frac{-3 + 5}{4} = \frac{1}{2} \text{ or } x = \frac{-3 - 5}{4} = -2$$

ii) $y^2 - 4y - 9 = 0$, $a = 1$, $b = -4$, $c = -9$

$$\Delta = b^2 - 4ac \Rightarrow \Delta = 16 - 4 \times 1 \times (-9) = 52$$

The discriminative expression is not a perfect square, so the equation has two irrational roots.

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{4 \pm \sqrt{16 + 36}}{2} \Rightarrow x = \frac{4 \pm \sqrt{52}}{2} \Rightarrow \{x = 2 + \sqrt{13} \text{ or } x = 2 - \sqrt{13}\}$$

iii) $z^2 + 8z = -16 \Rightarrow z^2 + 8z + 16 = 0$, $a = 1$, $b = 8$, $c = 16$

$$\Delta = b^2 - 4ac \Rightarrow \Delta = 64 - 4 \times 1 \times 16 = 0$$

The discriminative expression is zero , that means the equation has two equal real roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-8 \pm \sqrt{64 - 64}}{2} = -4$$

iv) $x^2 - 2x + 10 = 0$, $a = 1$, $b = -2$, $c = 10$

The discriminative expression is negative, therefore the equation doesn't have a solution in R .

$$\Delta = b^2 - 4ac \Rightarrow \Delta = 4 - 4 \times 1 \times 10 = -36$$

Example (4) What is the value of the constant (K) which makes the two roots of the equation $x^2 - (k+1)x + 4 = 0$ equal? Check your answer.

The two roots of the equation will be equal when the value of the discriminative expression

Δ equals zero.

$$a = 1 , b = -(k+1) , c = 4$$

Determine the values of the coefficients

$$\Delta = b^2 - 4ac \Rightarrow \Delta = (k+1)^2 - 4 \times 1 \times 4 \Rightarrow \Delta = (k+1)^2 - 16$$

$$\Delta = 0 \Rightarrow (k+1)^2 - 16 = 0$$

We substitute the value of the discriminant by zero because the two roots of the equation are equal to the root of the two sides of the equation.

$$\Rightarrow (k+1)^2 - 16 = 0 \Rightarrow (k+1)^2 = 16$$

$$\Rightarrow k+1 = \pm 4 \Rightarrow \begin{cases} k+1 = +4 \Rightarrow k = 3 \\ \text{or } k+1 = -4 \Rightarrow k = -5 \end{cases}$$

Check:

We substitute by the value $K = 3$ in the original equation, then we find the roots of the equation:

$$x^2 - (k+1)x + 4 = 0 \Rightarrow x^2 - 4x + 4 = 0 \Rightarrow (x-2)^2 = 0 \Rightarrow x = 2$$

$$x^2 - (k+1)x + 4 = 0 \Rightarrow x^2 + 4x + 4 = 0 \Rightarrow (x+2)^2 = 0 \Rightarrow x = -2$$

We substitute by the value $K = -5$ in the original equation, then we find the equation roots..

Make sure of your understanding

Find the solution set for the following equations by using the general law:

1 $x^2 - 4x - 5 = 0$

2 $y^2 + 5y - 1 = 0$

3 $3x^2 - 9x = -2$

4 $4y^2 + 8y = 6$

5 $4x^2 - 12x + 9 = 0$

6 $2y^2 - 3 = -5y$

Questions 1-6
are similar
to examples 1-2

Determine the roots of equation at firstly, then find the solution set if it is possible:

7 $2x^2 + 3x = 5$

9 $y^2 - 8y - 12 = 0$

8 $3x^2 - 7x + 6 = 0$

10 $y^2 + 12 = -9y$

Questions 7-10
are similar
to example 3

11 What is the value of the constant (k) which makes the two roots of the equation $x^2 - (k + 2)x + 36 = 0$ equaled? Check your answer.

Questions 11-14
are similar
to examples 3-4

12 What is the value of the constant (K)) which makes the two roots of the equation $4y^2 + 25 = (k - 5)y$ equaled? Check your answer.

13 What is the value of the constant (K)) which makes the two roots of the equation $z^2 + 16 = (k + 4)z$ equaled? Check your answer.

14 Show that the equation $z^2 - 6z + 28 = 0$ doesn't have a solution set in real numer.

Solve the Exercises

Find the solution set for the following equations by using the general law:

15 $x^2 - 7x - 14 = 0$

16 $y^2 + 3y - 9 = 0$

17 $2x^2 - 8(3x + 2) = 0$

18 $2y^2 - 2 = -10y$

Determine the equation roots at firstly, then find the solution set in R if it is possible:

19 $y^2 - 2y - 10 = 0$

20 $y^2 - 14y + 49 = 0$

21 What is the value of the constant (K) which makes the two roots of the equation $x^2 - (k + 6)x + 49 = 0$ equaled? Check your answer.

22 Show that the equation $2z^2 - 3z + 10 = 0$ doesn't have a solution set in real numer.

Solve the problems

23 Fireworks: In one of the occasions, a group of fireworks was shot vertically, they reached a certain height of 140m. Calculate the time (t) second in which the fireworks reached up to that height, if the following equation $5t^2 + 60t = 140$.



24 Trade: Samir calculates the cost of one men's suit, then he adds amount of profit and sell it in 120,000 dinars. If (P) in the equation $p^2 - 30p + 225 = 0$ represents the amount of Samir's profit in one suit which is in thousands of dinars. What is the cost of one suit?



Think

25 Challenge: Determine the roots of the equation at firstly, then find the solution set if it is possible:

i) $x^2 + 8x = 10$

ii) $3y^2 - 6y - 42 = 0$

26 Correct the mistake: Sa'ad said that the equation $2x^2 - 3x - 9 = 0$ doesn't have a solution in the set of the real numbers.

Discover sa'ad's mistake and correct it.

27 Numerical sense: Marwa used the discriminative expression for writing the two roots of the equation $z^2 - 8z + 16 = 0$ without factoring. illustrate how Marrwa was able to write the two roots of the equation.

Write

The type of the two roots of the equation $x^2 + 100 = 20x$ by using the discriminative expression without solving it.

Lesson [3-6]

Solving the Fractional Equations

Idea of the lesson:

*Solving the fractional equations of second degree.

Vocabulary:

* Numerator

*Denominator

*Fractional equation

Learn

If the price of a masterwork is $2x + 3$ thousands dinars, and the price of buying 6 pieces of masterworks is $x^2 + 3x - 1$ thousands dinars. So if the ratio of one masterwork price to the price of three masterworks is $\frac{1}{3}$, what is the price of buying one masterwork?



You have previously learned how to simplify the fractional algebraic expressions by dividing both the numerator and denominator by a common factor. Now you will use the factoring of algebraic expressions to solve fractional equations which have a variable in its denominator by get rid of fractions, then solve them by using one of the methods that you have previously learned.

Example (1)

Write the price of buying one masterwork.

$$\frac{\text{one master work price}}{\text{three master work price}} = \frac{1}{3} \Rightarrow \frac{2x + 3}{(x^2 + 3x - 1)} = \frac{1}{3}$$

Simplify the fraction by multiplying the two sides by the two middles.

$$\Rightarrow x^2 + 3x - 1 = 6x + 9 \Rightarrow x^2 - 3x - 10 = 0$$

Simplify the equation for factoring.

$$\Rightarrow (x - 5)(x + 2) = 0 \Rightarrow \begin{cases} x - 5 = 0 \Rightarrow x = 5 \\ \text{or } x + 2 = 0 \Rightarrow x = -2 \end{cases}$$

Neglect because there is no price in negative

Then the price of buying one masterwork is $(2x + 3 = 13)$ 13000 dinars.

Example (2)

Find the solution set for the following equation, then check the correction of the solution.

$$5x + \frac{x - 2}{3x} = \frac{2}{3}$$

We multiply the two sides of the equation by the least common multiple (LCM) to get rid of fractions

$$3x(5x) + 3x\left(\frac{x - 2}{3x}\right) = 3x\left(\frac{2}{3}\right) \Rightarrow 15x^2 + x - 2 = 2x \Rightarrow 15x^2 - x - 2 = 0$$

Simplify the equation

$$\Rightarrow (3x + 1)(5x - 2) = 0$$

Factoring by experiment

$$\Rightarrow \begin{cases} 3x + 1 = 0 \Rightarrow x = -\frac{1}{3} \\ \text{or } 5x - 2 = 0 \Rightarrow x = \frac{2}{5} \end{cases} \Rightarrow S = \left\{-\frac{1}{3}, \frac{2}{5}\right\}$$

Solution set

Check: Substituting by the original equation when $x = -\frac{1}{3}$

$$\text{LHS} = 5\left(-\frac{1}{3}\right) + \frac{-\frac{1}{3} - 2}{3 \times -\frac{1}{3}} = \frac{-5}{3} + \frac{-1}{-3} + 2 = \frac{-5}{3} + \frac{1}{3} + 2 = \frac{-5 + 1 + 6}{3} = \frac{2}{3} = \text{RHS}$$

it is also easy to check when $x = \frac{2}{5}$ (leave it to students)

You have previously learned how to simplify the adding of the (fractional) relative algebraic expressions and subtract them by factoring each of the numerator and denominator of the fraction to simplest form, then doing the operation of adding and subtracting the fractional expressions by using the least common multiple and simplify the expressions to the simplest form. Now, you will use that to solve the fractional equations to find the solutions set of the fractional equation.

Example (3) Find the solution set for the equation:

$$\begin{aligned}\frac{x}{x-3} + \frac{4x}{x+3} &= \frac{18}{x^2-9} \\ \Rightarrow \frac{x}{x-3} + \frac{4x}{x+3} &= \frac{18}{(x-3)(x+3)} \\ \Rightarrow x(x+3) + 4x(x-3) &= 18 \\ \Rightarrow x^2 + 3x + 4x^2 - 12x - 18 &= 0 \Rightarrow 5x^2 - 9x - 18 = 0\end{aligned}$$

Factoring the denominators to the possible

simplest form by multiplying the two sides of the equation by LCM $(x-3)(x+3)$

Simplifying the equation and solve it to find the values of the variable.

$$\Rightarrow (5x+6)(x-3) = 0 \Rightarrow x = -\frac{6}{5} \text{ or } x = 3$$

Note:

We have to exclude the values which make the denominator of any fractional term from the original equation terms, zero because it leads to divide by zero and that considers impossible.

So we exclude $x=3$ from the solution because $(\frac{x}{x-3} = \frac{3}{0})$, and the solution will be only $x = -\frac{6}{5}$

Check: Substituting by the original equation $x = -\frac{6}{5}$ to see if the two sides of the equation are equaled or not ?

$$\begin{aligned}\text{LHS} &= \frac{x}{x-3} + \frac{4x}{x+3} = \frac{-\frac{6}{5}}{-\frac{6}{5}-3} + \frac{4 \times -\frac{6}{5}}{-\frac{6}{5}+3} = \frac{6}{21} - \frac{8}{3} = -\frac{50}{21} \\ \text{RHS} &= \frac{18}{x^2-9} = \frac{18}{(-\frac{6}{5})^2-9} = \frac{18}{(\frac{36}{25})-9} = -\frac{450}{189} = -\frac{50}{21}\end{aligned}$$

$$\text{LHS} = \text{RHS}$$

So the value $x = -\frac{6}{5}$ satisfy the equation

Example (4) Find the solution set for the equation:

$$\begin{aligned}\frac{2}{x+2} - \frac{x}{2-x} &= \frac{x^2+4}{x^2-4} \\ \Rightarrow \frac{2}{x+2} + \frac{x}{x-2} &= \frac{x^2+4}{(x+2)(x-2)}\end{aligned}$$

Before multiplying the two sides of the equation by LCM for the denominators we try to

factor the denominator of the fraction for the right side and change $2-x = -(x-2)$

by using the information $a-b = -(b-a)$

$$\Rightarrow 2(x-2) + x(x+2) = x^2+4 \quad \text{By multiplying the two sides of the equation by LCM } (x+2)(x-2)$$

$$\Rightarrow 2x - 4 + x^2 + 2x - x^2 - 4 = 0 \Rightarrow 4x - 8 = 0 \Rightarrow x = 2$$

When we substitute $x=2$ in the original equation, we get a process of dividing by zero and it is impossible $(\frac{x}{2-x} = \frac{2}{0})$, so the equation doesn't have a solution in the real number set (\mathbb{R}) , that means the solution set in \mathbb{R} is an empty set (\emptyset) .

Make sure of your understanding

Find the solution set for each of the following equations and check the correct of solution:

1 $\frac{1}{x} + \frac{1}{2} = \frac{6}{4x^2}$

2 $\frac{y}{2} - \frac{7}{5} = \frac{3}{10y}$

Questions 1-6
are similar
to examples 1-2

3 $\frac{x+4}{2} = \frac{-3}{2x}$

4 $\frac{y+1}{y^2} = \frac{3}{4}$

5 $\frac{9x-14}{x-5} = \frac{x^2}{x-5}$

6 $\frac{1}{y^2-6} = \frac{2}{y+3}$

Find the solution set for each of the following equations:

7 $\frac{y-4}{y+2} - \frac{2}{y-2} = \frac{17}{y^2-4}$

8 $\frac{9}{x^2-x-6} - \frac{5}{x-3} = 1$

Questions 7-10
are similar
to examples 3-4

9 $\frac{12}{y^2-16} + \frac{6}{y+4} = 2$

10 $\frac{3y}{y-1} + \frac{2y}{y-6} = \frac{2x^2-15y+20}{y^2-7y+6}$

Solve the Exercises

Find the solution set for each of the following equations and check the correct of solution:

11 $\frac{4}{6x^2} + \frac{1}{3} = \frac{1}{x}$

12 $\frac{3y}{4} - \frac{6}{12y} + \frac{1}{4} = 0$

13 $\frac{9x+22}{x^2} = 1$

13 $\frac{9}{(y+2)^2} = \frac{3y}{y+2}$

Find the solution set for each of the following equations:

15 $\frac{3}{x-4} - \frac{2}{x-3} = 1$

16 $\frac{y-5}{y+5} - \frac{y+5}{y-5} = \frac{4y^2-24}{y^2-25}$

17 $\frac{6-x}{x^2+x-12} - \frac{2}{x+4} = 1$

18 $\frac{4+8y}{y^2-9} + \frac{6}{y-3} = 3$

Solve the problems

19 **Sports:** If a cyclist wanted to cut a distance of 60 km between the two cities A and B in certain speed. If his speed increases in about 10 km/h, then he will be able to cut this distance in an hour less than the first time. Find his speed at first.



20 **Transporting passengers:** One of the Iraqi airlines planes cuts a distance of 350 km from Baghdad to Erbil in a certain speed. If the speed of the plane increases in 100 km/h, then the plane will be able to cut the distance in time which will be less in 12 minutes from the first time. Find approximate speed of the plane at first.



21 **Racing:** Nawfel participated in triple race which includes swimming, riding bicycles and running, and he took two hours to finish the race, as shown in the nearby table, considering x represents Nawfel's speed average in swimming. Find the average of his approximate speed in swimming racing .

	Distance km	Speed h/km	Time
Swimming	$d_s = 1$	x	t_s
Riding bicycles	$d_b = 20$	$5x$	t_b
running	$d_r = 4$	$x + 4$	t_r

Note: use the equation of total time which Nawfel spent in the race, in term of his speed in swimming is $\tau(x) = t_s + t_b + t_r$

Think

22 **Challenge:** Find the solution set for each of the following equations:

$$\frac{3}{x+5} + \frac{4}{5-x} = \frac{x^2 - 15x + 14}{x^2 - 25}$$

23 **Correct the mistake:** Nammeer used the discriminative expression to show the roots of equation,

$$\frac{2}{x-7} \times \frac{1}{x-1} = 1$$

he said that the equation has two relative real roots. Discover Nammeer's mistake and correct it.

Write

The solution set in real numer .

$$\frac{1}{x+6} - \frac{5}{x-6} = 2$$

Lesson [3-7]

problem solving plan (Writing Equation)

Learn

Idea of the lesson:

*Using the strategy of writing an equation to solve problem.

A ship cuts a distance of 240 km between the port A and the port B in certain speed. If its speed increases in 10 km/h, It will be able to cut the distance in time which is less in 2 hours from the first time.

Find the speed of ship at first.



UNDERSTAND

What is the data in the problem: A ship cuts a distance of 240 km between the two cities A and B in certain speed. It cuts the same distance in time which is less in two hours from the first time if the speed of the ship increases in 10 km/h.

What is wanted in the problem: Finding the speed of ship at first.

PLAN

How can you solve the problem? Write an equation represents the problem, then solve it to find the speed of liner at first.

SOLVE

Assume that the first speed of ship = v , The first time = $\frac{240}{v}$

So its second speed = $v + 10$, the second time = $\frac{240}{v + 10}$

$$\frac{240}{v} - \frac{240}{v + 10} = 2$$

$$240v + 2400 - 240v = 2v(v + 10)$$

$$2400 = 2v^2 + 20v$$

$$v^2 + 10v - 1200 = 0 \Rightarrow (v + 40)(v - 30) = 0$$

$$\Rightarrow \begin{cases} v + 40 = 0 \Rightarrow v = -40 & \text{Neglect} \\ \text{or } v - 30 = 0 \Rightarrow v = 30 \text{ km/h} & \text{The speed of ship at first} \end{cases}$$

The first time – the second time equals 2
By multiplying the two sides of the equation by LCM
 $v(v + 10)$

CHECK

$$\text{The first time of the ship} = \frac{240}{v} = \frac{240}{30} = 8 \text{ h}$$

$$\text{The second time of the ship} = \frac{240}{v + 10} = \frac{240}{40} = 6 \text{ h}$$

The second time of ship is less than its first time in about two hours ($8 - 6 = 2\text{h}$), so the solution is correct.

Problems

Solve the following problems by the strategy of (writing an equation)

1 **Fountain:** A square-shaped area was planted with flowers in the middle of a square-shaped garden of a hotel, its side length is 4m. The remaining area which surrounds it is 84 m^2 . What is the side length of the garden?



2 **Babylon lion:** It is a statue which was found in Babylonian archeological city in Iraq in 1776. It was made of the solid black basalt stone. It locates on a base in the middle of a rectangular-shaped area which its length is greater than its width in 2m and its area is 15 m^2 . What are its dimensions?



3 **Lion:** Lion is one of the strongest animals on earth. It is the king of forest according to its strength. If the equation $x^2 - 30x$ represents the area which is under the control of the lion. What is the side length of the area? which represented by x , if the area 175 km^2 ?



4 **Fireworks:** In one occasion, a group of fireworks was shot vertically. They reached up to 200m height. Calculate the time in which the fireworks reached up to that height if the following equation $2t^2 + 30t = h$ represents the relation between height, in meters (h), in which the fireworks reach after t second.



Chapter Test

Find the solution set for the two equations graphically:

1 $\begin{cases} y = 1 + x \\ y = 2 - x \end{cases}$

2 $\begin{cases} y + x = 0 \\ y - x = 0 \end{cases}$

3 $\begin{cases} y - x - 5 = 0 \\ y + x - 1 = 0 \end{cases}$

Find the solution set for the two equations by using the substitution or elimination for each of the following:

4 $\begin{cases} 2x + y = 1 \\ x - y = 8 \end{cases}$

5 $\begin{cases} 4x - 2y = -4 \\ x + y = 6 \end{cases}$

6 $\begin{cases} \frac{x}{3} + \frac{y}{2} = 1 \\ x + y = 2 \end{cases}$

Solve the following equations by using the greater common factor and the difference between two squares:

7 $9x^2 - 25 = 0$

8 $3y^2 - 12 = 0$

9 $(7 - z)^2 - 1 = 0$

Solve the following equations by using the rule of square root:

10 $x^2 = 49$

11 $81 - y^2 = 0$

12 $z^2 = \frac{36}{9}$

Solve the following equation by factoring in experiment:

13 $x^2 + 9x + 18 = 0$

14 $z^2 - 2z - 48 = 0$

15 $3x^2 - x - 10 = 0$

16 $7z^2 - 18z - 9 = 0$

17 What is the number which its square decreases from its four times in 3 ?

18 A swimming pool, its length is greater than the twice of its width in 4 m and its area is 48m^2 .

What are the dimensions of the pool?

Solve the following equations by the perfect square:

19 $x^2 - 16x + 64 = 0$

20 $\frac{1}{9} - \frac{1}{3}z + \frac{1}{4}z^2 = 0$

Solve the following equations by completing the square:

21 $x^2 - 14x = 32$

22 $4y^2 + 20y - 11 = 0$

23 $z^2 - \frac{2}{3}z = 1$

Find the solution set for the following equations by using the general law:

24 $x^2 - 3x - 7 = 0$

25 $3y^2 - 12y = -3$

26 $5z^2 + 6z = 9$

Determine the equation roots and solution set if it is possible:

27 $2x^2 + 8x + 8 = 0$

28 $y^2 - 6y - 9 = 0$

29 $4z^2 - 3z + 7 = 0$

30 What is the value of constant (K) which makes the two roots of the equation $x^2 - (k + 6)x + 9 = 0$

Equal? Check your answer.

Find the solution set for each of the following equations and check the correction of solution:

31 $\frac{6x}{5} = \frac{5}{6x}$

32 $\frac{1}{6y^2} + \frac{1}{2} = \frac{1}{y}$

33 $\frac{z + 4}{z^2} = \frac{1}{2}$

Find the solution set for each of the following equations :

34 $\frac{4}{x - 5} - \frac{3}{x - 2} = 1$

35 $\frac{2y}{y + 2} + \frac{y}{2 - y} = \frac{7}{y^2 - 4}$

Coordinate Geometry

lesson 4-1 Graphical Representation of the Equations in the Coordinate Plane

lesson 4-2 Slope of the Line

lesson 4-3 The Equation of the Line

lesson 4-4 Parallel and Perpendicular Lines

lesson 4-5 Distance Between Two Points

lesson 4-6 Trigonometric Ratios

Skiing is one of the enjoyable sports in the world, where the mountain declines represent a good example for slope. Higher slope requires greater skills of skiers

Pretest

Determine the points on the coordinate plane and then determine its location in the quadrants or in axes for each of the following :-

1 A (3,6)

2 B (-3,-5)

3 C (0,2)

4 D (-3,0)

5 E (-4,2)

6 F (3,-2)

Determine the points on the coordinate plane , then figure out the resulting shape for each of the following:-

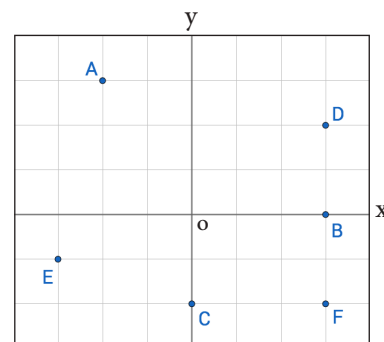
7 A (0,3),B(3,0) C(-3,0).

8 A (1,4),B(2,4) C(4,4)D(6,4).

9 A (-2,4),B(-2,-3) C(1,4),D(1,-3).

10 A (0,3),B(3,0) C(0,-3),D(-3,0).

11 Write the coordinate of the points which are indicated in the nearby coordinate plane :



Represent the following tables in the coordinate plane :

12

x	y
1	3
2	4
5	7

13

x	y
5	2
-2	-5
0	3

Find the value of (y) in each of the following:

14 $y = 2x - 5, x = 0$

15 $y = -x + 7, x = -1$

16 $y = x^2 + x + 2, x = 1$

17 $3y - x^2 = 9, x = -2$

If A (x_1, y_1), B(x_2, y_2) find the numerical value of the expression $(\frac{y_2 - y_1}{x_2 - x_1})$ for each of the following .

18 A(3,-5), B(-2,1)

19 A(-1,5), B(4,5)

Lesson [4-1]

Graphical Representation of The Equations In The Coordinate Plane.

Idea of the lesson

- Representing the linear equation in the Coordinate plane.
- Representing the quadratic equation in the Coordinate plane

Vocabulary:

- Ordered Pair
- Coordinate plane
- Linear equation
- Quadratic equation

Learn

In a study to determine the quantity of milk which the new born of Anteater needs in liters for some days, the researcher reached to the following equation:

$2y - x = 0$ where x represents number of days, y represent the quantity of milk in liters.

How I can represent the relation in the coordinate plane.



[4- 1- 1] Graphical representation of the linear equation in the coordinate plane

Linear Equation : The general formula of linear equation is $ax + by + c = 0, a, b, c \in \mathbb{R}$
Where a, b are not equal zero together. Its representation in the coordinate plane is line.

Example (1) To represent the equation $2y - x = 0$ in the coordinate plane we following :-

First step :- We make the equation in the form $y = f(x)$ (y denoted by x)

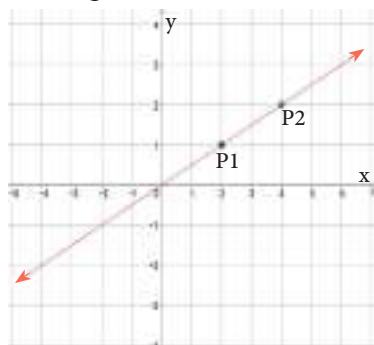
$$2y - x = 0 \Rightarrow 2y = x \Rightarrow y = \frac{1}{2}x$$

Second step :- Choose ,at least ,two values to the variable x , let $x=4, x=2$, then we substitute them in the equation to get ordered pairs .

$$x=2 \Rightarrow y = \frac{1}{2}(2) \Rightarrow y=1 \Rightarrow P_1(2,1)$$

$$x=4 \Rightarrow y = \frac{1}{2}(4) \Rightarrow y=2 \Rightarrow P_2(4,2)$$

Third step:- We make a table with the resulting values and represent the ordered pairs in the coordinate plane and connect the two points ,the figure that we get represents a line.



x	y	(x,y)
2	1	$P_1(2,1)$
4	2	$P_2(4,2)$

Note:- The equation of the line which passes through the origin is without the absolute term.

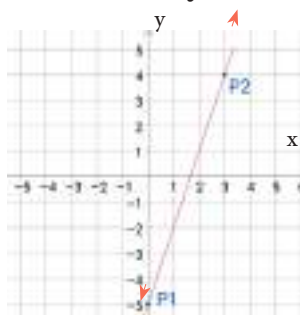
Example (2): Represent the following equations in the coordinate plane. What do you notice?

i) $y-3x+5=0$

ii) $y = 4$

iii) $x = -3$

i) $y-3x+5=0 \Rightarrow y=3x-5$



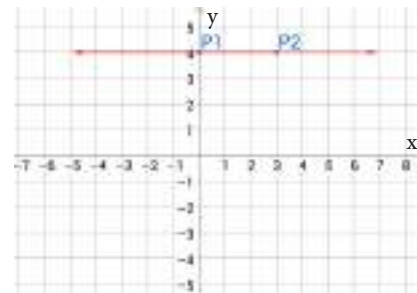
x	y=3x-5	(x,y)
0	3(0)-5=-5	P ₁ (0,-5)
3	3(3)-5=4	P ₂ (3,4)

The line is intersect X- axis , and Y- axis and it does not through the origin.

ii) $y = 4$

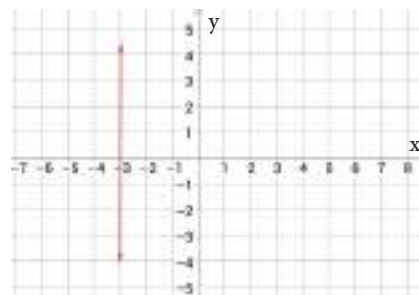
x	y=4	(x,y)
0	4	P ₁ (0,4)
3	4	P ₂ (3,4)

The line $y = 4$ parallel to the X- axis
and perpendicular to
Y-axis at the point (0,4)



iii) $x = -3$

The line $x = -3$ parallel to the Y-axis
and perpendicular to
X-axis at (-3,0)



It can put mentioned above in the following table:

Equation	the relation with the two axes
$ax+by+c = 0$	The line intersects the two axes and it does not passes through the origin
$ax+by = 0$	The line intersects the axes and passes through the origin
$y = k, k \in \mathbb{R}$	The line parallel to the X-axis and perpendicular to the Y- axis at the point (0,k)
$x = h, h \in \mathbb{R}$	The line parallel to the Y-axis and perpendicular to the X-axis at the point (h,0)

[4 - 1 - 2] The Graphical representation of the quadratic equation in the coordinate plane .

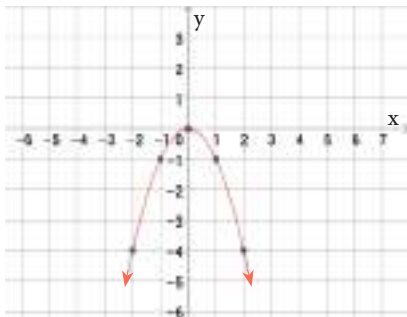
The general formula of the quadratic equation is : $y = ax^2 + bx + c$, $a \neq 0, a, b, c \in \mathbb{R}$

- In this item we will deal with the quadratic equation in the formula of $y = ax^2 + c$ where $a \neq 0, a, c \in \mathbb{R}$ and the way its representation .

-To represent the equation $y = ax^2 + c$ we do the nearby table and the graphical representation of the equation will be \cup or \cap

x	$y = ax^2 + c$	y	(x,y)
Supposed Values	Substitute the value of X	The Result	Ordered Pairs
-2			
-1			
0			
1			
2			

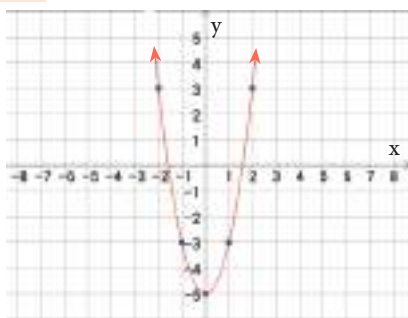
Example (3): Represent the equation $y = -x^2$



x	$y = -x^2$	y	(x,y)
-2	$-(-2)^2$	-4	(-2,-4)
-1	$-(-1)^2$	-1	(-1,-1)
0	$-(0)^2$	0	(0,0)
1	$-(1)^2$	-1	(1,-1)
2	$-(2)^2$	-4	(2,-4)

Example (4): Represent the equation

$$y = 2x^2 - 5$$



x	$y = 2x^2 - 5$	y	(x,y)
-2	$2(-2)^2 - 5$	3	(-2,3)
-1	$2(-1)^2 - 5$	-3	(-1,-3)
0	$2(0)^2 - 5$	-5	(0,-5)
1	$2(1)^2 - 5$	-3	(1,-3)
2	$2(2)^2 - 5$	3	(2,3)

Make sure of your understanding

Represent the following linear equations in the coordinate plane, then explain their relation with the two axes:

1 $y = 3x + 1$

2 $y = -4x$

3 $y + 3x - 2 = 0$

Questions 1 -6 are similar to examples (1, 2)

4 $y = 1 - 3x$

5 $y + 5 = 0$

6 $x - 5 = 0$

Represent the following quadratic equations in the coordinate plane:

7 $y = x^2 + 4$

8 $y = x^2$

9 $y = 1 - 3x^2$

Questions 7 -9 are similar to examples (3,4)

Solve the Exercises

Represent the following linear equations in the coordinate plane then explain their relation with the two axes:

10 $y = -x + 4$

11 $y = x$

12 $x = -\frac{5}{2}$

13 $y = 0$

Represent the following quadratic equations in the coordinate plane:

14 $y = x^2 - 1$

15 $y = 2x^2 + 3$

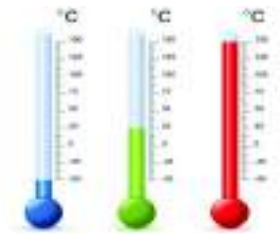
16 $y = -3x^2$

17 $y = 2x^2$

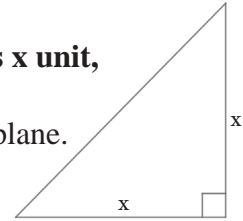
18 $4y = x^2$

Solve the problems

- 19 **Temperatures:** The equation $F^0 = \frac{9}{5}C^0 + 32$ shows the relation between the Celsius temperatures and Fahrenheit temperatures. Represent the equation graphically.



- 20 **Geometry:** A right – angled and isosceles triangle, its length right side is x unit, $f(x)$ represents its area. i) Write the relation $f(x)$ in terms of x .
ii) Represent the relation $f(x)$ in the coordinate plane.



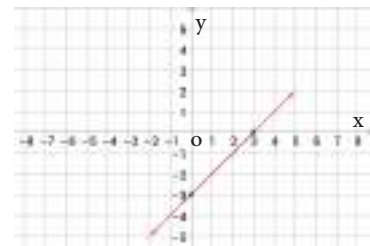
- 21 **Physics :** The law $F=9.8m$ represents the power of the earth gravity on a body, where F is the power in Newton, m is the body mass in Kg. Represent the law in the coordinate plane.

- 22 **Works :** A company for construction equipment takes 10,000 dinars as insurance, as well 5,000 dinars for every hour. Write the equation that expresses the problem, and then represent it graphically in the coordinate plane.



Think

- 23 **Discover the mistake:** Mohammed represented the following linear equation ($y=-3x+9$) in the nearby graphic figure. Discover Mohammed's mistake, then correct it.



- 24 **Open problem:** Give an example of linear equation as $ax+by+c=0$ for each case :
i) $a=0$ ii) $b=0$ iii) $c=0$

- 25 **Challenge :** The following ordered pairs $(0,4)$, $(1,6)$, $(-1,2)$ had formed a line, what is the intersection point of this line with the X – axis ?

- 26 **Justification:** Show if the following ordered pairs $(2,4)$, $(1,1)$, $(0,0)$, $(-1,1)$, $(-2,4)$ represent a linear function or quadratic one .

- 27 **Numerical sense :** $y = x^2 + 1$, $y = x + 1$. Which one represents a quadratic function ? clarify that.

Write

The steps which show that $y=4x+3$ is a linear equation?

Lesson [4-2]

Slope of the line

Idea of the lesson:

- Finding the slope of the line.
- Finding the Y- Intercept
- Finding the X- Intercept

Vocabulary:

- The vertical change
- The horizontal change
- X-Intercept
- Y-Intercept
- Slope

Learn

The mountain declines are good example for slope, whenever the higher the mount high, the slope will be increased. How can we determine the slope of the declines?



[4- 2- 1] Finding the slope of the line

The slope: The slope of the non- vertical line can be defined as the ratio between the vertical change and horizontal change.

Vertical change : Is the Y- change which equals $y_2 - y_1$

Horizontal change : Is the X- change which equals $x_2 - x_1$

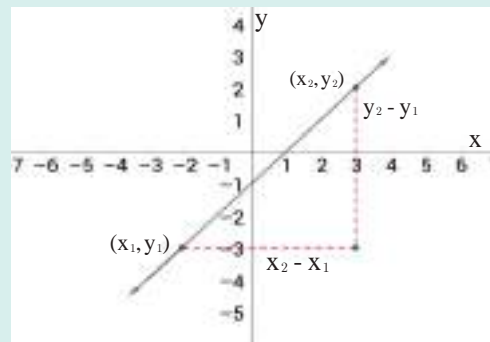
The slope = $\frac{\text{vertical change}}{\text{horizontal change}}$

i.e. $m = \frac{y_2 - y_1}{x_2 - x_1}$ where $x_2 - x_1 \neq 0$

m: is the slope of the line which passes through the

two points $(x_1, y_1), (x_2, y_2)$

The slope of the line can be either positive or negative if it is not horizontal or vertical It also may be zero (**horizontal**) or undefined (**vertical**) .



Example(1)

Find the slope of the line which passes through two points in each of the following:

i) A (5, 7), B (-2, 1)

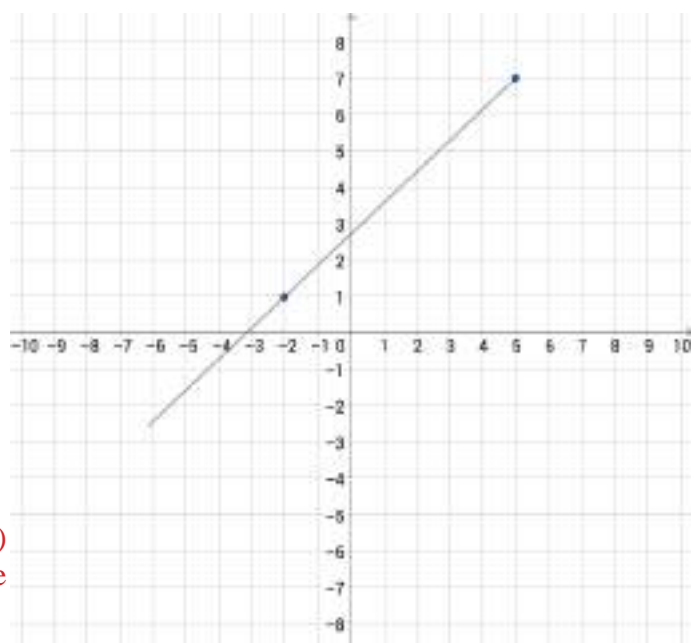
$m = \frac{y_2 - y_1}{x_2 - x_1}$ The slope of the line which passes through two points

$m = \frac{1 - 7}{-2 - 5}$ Substitute by the two points

$m = \frac{-6}{-7}$ By simplify

$m = \frac{6}{7}$ So the slope of \overleftrightarrow{AB} is $(\frac{6}{7})$ (positive).

The slope is positive (the line is to upward) when we move from left to right , values of y are increasing.



ii) A (-1, 5), B(4, 2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2 - 5}{4 - (-1)}$$

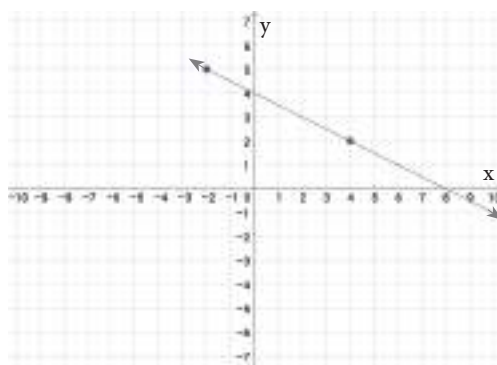
$$= \frac{-3}{5}$$

The slope of the line which passes through two points

Substitute by the two points.

So the slope of (\overrightarrow{AB}) is ($\frac{-3}{5}$) negative

The slope is negative (the line is to down ward) when we move from left to right, values of y are decreasing.



iii) A (1, - 2), B(4, - 2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-2 - (-2)}{4 - 1}$$

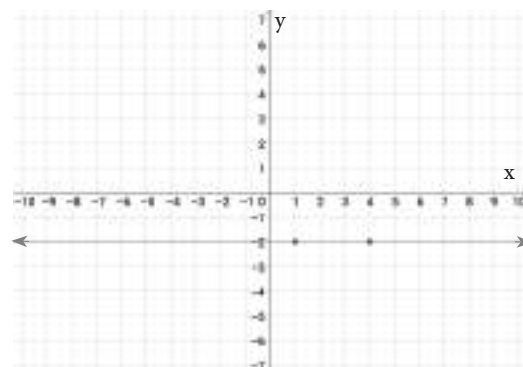
$$= \frac{0}{3} = 0$$

The slope of the line which passes through two points

Substitute by the two points.

So the slope of (\overrightarrow{AB}) is (0)

The slope is zero (the line is horizontal), it parallel to the X-axis ,the values of y are constant.



iv) A (-2, 3), B(-2, - 3)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-3 - 3}{-(-2) - (-2)}$$

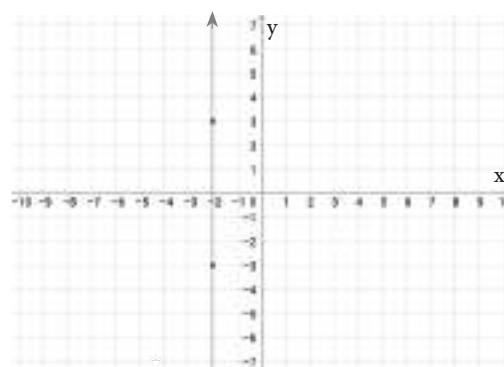
$$= \frac{-6}{0}$$

The slope of the line which passes through two points

Substitute by the two points

It is impossible to divide by 0, so the slope of (\overrightarrow{AB}) is undefined

The slope is undefined (the line is vertical), it parallel to the Y-axis, the values of x are constant.



Example (2)

The nearby table represents the change of temperatures by time (hours). Find the slope of line and explain what it means?

Time (hours)	Temperatures
1	-2
2	1
3	4
5	10

$$(x_1, y_1) = (1, -2)$$

$$(x_2, y_2) = (3, 4)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4 - (-2)}{3 - 1} = \frac{6}{2} = 3$$

Choose any two points from the table, let it be

The slope of the line which passes through two points.

The substitution and simplification

Since the slope of the line is 3 , then the temperatures are increasing 3 Celsius degrees every hour

[4-2-2] The Intersection of the line with axes in Coordinate plane

You can easily represent the equation of the line by finding the two points of line with the two axes.
X – intercept: is the value of x from the intersection of the line with the X – axis by substituting $y = 0$.
The intersection point is (x,0).

Y – intercept : is the value of y from the intersection of the line with the Y– axis by substituting $x = 0$ and the intersection point is (0,y).

Example (3)Find the X-intercept and Y- intercept for the line $3x + 5y = 15$ **Y- Intercept**

$$\begin{aligned}
 3x + 5y &= 15 \\
 3(0) + 5y &= 15 \\
 5y &= 15 \\
 y &= \frac{15}{5} \\
 y &= 3
 \end{aligned}$$

The equation
 Substituting $x=0$
 Simplify
 By dividing the two sides
 of equation by 5
 So the Y-intercept is 3

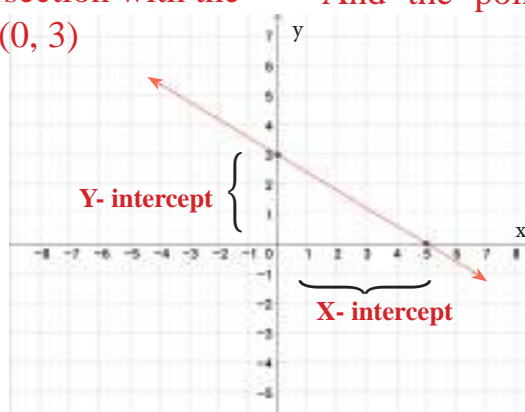
X-Intercept

$$\begin{aligned}
 3x + 5y &= 15 \\
 3x + 5(0) &= 15 \\
 3x &= 15 \\
 x &= \frac{15}{3} \\
 x &= 5
 \end{aligned}$$

The equation
 Substituting $y=0$
 Simplify
 By dividing the two sides of
 equation by 3
 So the X-intercept is 5

And the point of intersection with the
 Y- axis is (0, 3)

And the point of intersection with the
 X-axis is (5, 0)

**Example 4**

Find the X- intercept and Y- intercept if it is for each of the following:

i) $x = -2$,

ii) $y = 4$

$x = -2$, represents X- intercept and the point of intersection is (-2,0) The line // Y-axis

$y = 4$, represents Y- intercept and the point of intersection is (0,4) The line // X-axis

Make sure of your understanding

Find the slope of the line which through the two points. Is the slope positive or negative or zero or undefined? Then determine the direction of its movement for each of the following:

1 (-2, -2), (-4, 1)

2 (0,0), (3,2)

3 (-4,4), (2, -5)

4 (5,0), (0,2)

5 (4,3), (4, -3)

6 (-6, -1), (-2, -1)

Questions 1- 6 are
 Similar to
 Examples (1, 2)

Find the X-intercept and the Y- intercept for each the following:

7 $3x + 6y = 18$

8 $y + 2 = 5x - 4$

9 $y = -4x$

10 $y = -x + 8$

11 $5x = y - 8$

12 $y = -\frac{3}{4}x - 5$

13 $2x + 6y = 12$

14 $y + 4 = 2x - 4$

15 $y = -5x$

16 $x = 4$

17 $3y = -6$

18 $y = -\frac{1}{2}x + 4$

Questions 7- 18 are
 Similar to
 Examples (3, 4)

Solve the Exercises

Find the slope of the line which passes through the two points .Is the slope positive or negative or zero or undefined, then determine the direction of its movement for each of the following:

19 (6,2), (0,2)

20 (-2,4), (5,5)

21 (-2, - 3), (2,4)

22 (3, - 5), (0,0)

Find the X-intercept and the Y-intercept for the each of the following:

23 $3y - 7x = 9$

24 $y = -\frac{3}{2}x$

25 $x = -4$

26 $0 = y+3$

Solve the Problems

- 27 **Physics:** The nearby table represents the amount of liquid which flows from a basin during certain time, find the slope of the line which represented by the table, then illustrate what it means.

- 28 **Plants:** If the length of a plant is 30 cm, during each two months, it grows in constant distance which is 4cm.

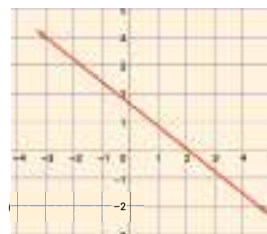
- Complete the table.
- What is the slope of the line which represented by the relation between the length of the plant and the time.
- Write the linear function which represented by the table.
- Represent the function in the coordinate plane.

The amount of leaky liquid	
Time second	Volume of liquid m ³
10	40
13	52
16	64
19	76

Time	0	2	4
length of plant			

Think

- 29 **Challenge:** Find the value of (a) which makes the slope of the line which passes through the two points (1, 6), (-5, a) equals ($\frac{1}{2}$).
- 30 **Critical thinking :-** Can you determine a slope of a line passes through the two points (3,7),(7,-3)?
- 31 **Discover the mistake:** The slope of the line which passes through the two points (0, 3), (3, - 1) is ($\frac{3-0}{3-(-1)} = \frac{3}{4}$), Discover the mistake and correct it.
- 32 **Open problem:** Mention two points on a line which its slope = $-\frac{1}{3}$
- 33 **Critical thinking:** From the nearby graphical figure, determine the direction



write

In your style the meaning of the slope equals zero and the slope is undefined.

Lesson [4-3]

The Equations of The line.

Idea of the lesson:

-Finding equation of a line Which its following data are given:

- Two points .
- Slope - point .
- Slope - intercept .

Vocabulary:

- The slope.
- The intercept.

Learn

A cyclist cuts a distance 20 Km in two hours and 50 Km in five hours, what is the linear equation which connects between the distance and the time?



[4- 3- 1] Writing an Equation of line with two points of it

The equation which passes through the two points $B(x_2, y_2)$, $A(x_1, y_1)$

You have previously learned finding a slope of a line passes through the two points A,B

$$\text{where } m = \frac{y_2 - y_1}{x_2 - x_1}$$

Assuming that the point $C(x, y)$ lie on the line , then the slope of the line which passes through the two points A,C is $m = \frac{y - y_1}{x - x_1}$

It is known that the slope of the line is constant in all its points , so : $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

This equation represents the equation of the line AB

Example (1)

We find the linear equation in the paragraph of (Learn)

Assume that $C(x, y) \in \overrightarrow{AB}$,

$B(5, 50)$, $A(2, 20)$

\downarrow \downarrow
 $x_2 = 5, y_2 = 50$ $x_1 = 2, y_1 = 20$

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{y - 20}{x - 2} = \frac{50 - 20}{5 - 2}$$

$$\frac{y - 20}{x - 2} = \frac{30}{3}$$

$$y - 20 = 10x - 20$$

Writing the equation of the line which passes through two points.

Substituting by $(x_2, y_2), (x_1, y_1)$

By simplify

Commutative multiplication

Then the line equation is

$$y - 10x = 0$$

[4- 3- 2] Writing Equation of line with the slope and one point of it

The equation of the line which its slope m and passes through the point (x_1, y_1)

You have previously learned an equation of a line which passes through two points which is

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

You have also learned that the slope of a line passes through the two points $(x_2, y_2), (x_1, y_1)$

$$\text{is } m = \frac{y_2 - y_1}{x_2 - x_1}$$

So the above equation can be written as $\frac{y - y_1}{x - x_1} = m$

And by commutative multiplication we get the required equation $y - y_1 = m(x - x_1)$

Example (2)

Use the equation of the slope and the point for each line to determine its slope and the point in which it passes.

i) $y - 3 = -5(x - 2)$

$$y - 3 = -5(x - 2)$$

$$y - y_1 = m(x - x_1)$$

$$m = -5, (x_1, y_1) = (2, 3)$$

Equation of the
slope –the point
By comparison

ii) $y + 7 = \frac{2}{5}x$

$$y - (-7) = \frac{2}{5}(x - 0)$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{2}{5}, (x_1, y_1) = (0, -7)$$

Equation of the
slope –the point
By comparison

Example (3) Find the equation of the line which its slope is $\frac{1}{2}$ and its X-intercept equals -1

$$y - y_1 = m(x - x_1)$$

$$m = \frac{1}{2}, \quad x_1 = -1, \quad y_1 = 0 \Rightarrow p(-1, 0)$$

$$y - 0 = \frac{1}{2}(x - (-1))$$

$$y = \frac{1}{2}(x + 1)$$

$$2y = x + 1$$

Equation of the slope- the point

The slope, the point

By substituting the slope and the point

Multiplying the two sides of equation by two

The equation of the required line. $2y - x = 1$

[4 -3- 3] Writing Equation of the line with the slope of it and one intercept with axes

The equation of the line according to its slope m and its Y-intercept K is $y = mx + k$

Example (4)

Use the equation of the slope and the intercept for each line to determine its slope and intercept:

i) $2x + 3y = 6$

ii) $5x = 7y + 8$

iii) $y = x$

iv) $y = 2$

v) $y = 0$

vi) $y + x = 5$

i) $2x + 3y = 6 \Rightarrow 3y = -2x + 6$

$$y = \frac{-2}{3}x + 2$$

$$y = mx + k$$

$$\therefore m = \frac{-2}{3}, \quad k = 2$$

By dividing the
two sides of the
equation by 3
The comparison
with the equation of
the slope-intercept

iii) $y = x \Rightarrow y = 1x + 0$

$$y = 1x + 0$$

$$y = mx + k$$

$$\therefore m = 1, \quad k = 0$$

The comparison
with the equation of
the slope-intercept

ii) $5x = 7y + 8 \Rightarrow 7y = 5x - 8$

$$y = \frac{5}{7}x - \frac{8}{7}$$

$$y = mx + k$$

$$\therefore m = \frac{5}{7}, \quad k = \frac{-8}{7}$$

By dividing the
equation by 7
The comparison
with equation of
the slope-intercept
section

iv) $y = 0x + 1$

$$y = mx + k$$

$$\therefore m = 0, \quad k = 1$$

The comparison
with equation of
the slope-intercept

v) $y = 0x + 0$

$$y = mx + k$$

$$\therefore m = 0, \quad k = 0$$

The comparison
with the equation of
the slope intercept

vi) $y = -1x + 5$

$$y = mx + k$$

$$\therefore m = -1, \quad k = 5$$

The comparison
with equation of
the slope-intercept

Example (5)

A line passes through the point (5, - 1) and its slope $-\frac{2}{5}$, find its intercept and equation.

The first method	The second method
$y = mx + k$ Equation of slope-intercept $m = -\frac{2}{5}$ Given $y = -\frac{2}{5}x + k$ By substituting the slope $-1 = -\frac{2}{5}(5) + k$ By simplifying $-1 = -2 + k$ $k = 1$ $y = -\frac{2}{5}x + 1$ the equation of the line	$y - y_1 = m(x - x_1)$ Equation slope - point $m = -\frac{2}{5}, p(5, - 1)$ Given $y - (-1) = -\frac{2}{5}(x - 5)$ By substituting the point and the slope $5y + 5 = -2x + 10$ By Multiplying the two sides by 5 by dividing sides by 5 after simplifying $5y + 2x = 5$ $y = -\frac{2}{5}(x) + 1$ The equation of the line

Make sure of your understanding

Find the line equation which each one of them passes through two points in each of the following:

1 (-3, 1), (2, - 1)

2 (0, 2), (2, - 4)

Questions 1 -2 are
Similar to example 1

Use the equation of slope and point for each line to determine its slope and the point which it passes through.

3 $y - 1 = 2(x - 3)$

4 $y + 1 = -x + 4$

Questions 3 -4 are
Similar to example 2

Find the line equation for each of the following , then find its intercept:

5 (4, 6), $-\frac{2}{5}$

6 (-1, - 3), $\frac{1}{3}$

Questions 5 -6 are Similar
to examples (3,5)

Use the equation of the slope and point for each line to determine its slope and intercept:

7 $5y = -2x - 1$

8 $-y = 7x$

Questions 7 -8 are
Similar to example 4

Solve the Exercises

Find the line equation which each one of them passes through two points in each of the following:

9 (0, 0), (-3, 7)

10 (0, 7), (-5, 0)

Use the equation of the slope and intercept for each to determine its slope and intercept:

11 $y + \frac{3}{2} = -5(x - 8)$

12 $y - x = 8$

Find the line equation for each of the following then find its intercept:

13 (-3, 7) -3 The slope

14 (1, - 4) $-\frac{1}{2}$ The slope

Use the equation of the slope and intercept for each line to determine its slope and intercept:

15 $y + 7 = 3x + 5$

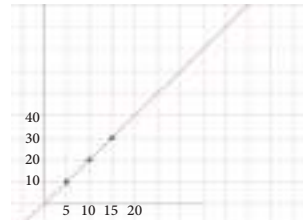
16 $\frac{1}{3}y = -5x - 1$

Solve the problems

- 17 **Biology:** Elephant's ivory is continuously grows at a rate of 1cm monthly. Assuming that you start monitoring an elephant which its ivory length was 100cm. Write in the form of slope-point, an equation represents the growing of the elephant ivory after (n) months of the monitoring.



- 18 **Physics:** The nearby graphic representation represents the amount of leaky water from a tank during certain time. Write in the form of two points, an equation represents the leakage of the water after (n) seconds.



- 19 **Money:** A person wants to pay 30 million dinars as monthly installments which will not exceed 1.5 million dinars, the following linear equation $y = -1.5x + 30$, where y represents the remaining value from the money, x represents the number of months. Use the equation of slope and intercept to determine its slope and intercept.



- 20 **Health:** In a new study, a man loses two hours of his age when he smoke a pack of cigarettes. Write the equation which represents that and represent it graphically.

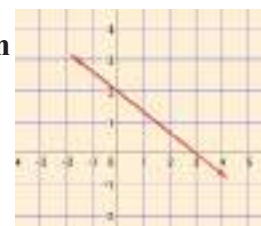


- 21 **Geometry:** Use the data in the nearby figure and then find the line equation in the following cases:

i) Two points

ii) Slope-point

iii) Slope-Y-intercept



Think

- 22 **Critical thinking:** Is it possible to find a line which its slope is 4 and passes through the two points (5, 7), (8, -2) ? If it is possible to find a line like this, then write an equation, otherwise illustrate your answer.
- 23 **Challenge:** A line, its horizontal intercept represents the additive inverse of its vertical intercept. It passes through the point (2, 3), write the equation of the slope-point for this line.
- 24 **Which one is correct :** An equation of line ,its slope is $\frac{3}{5}$ and passes through the point (-1, 7)
 Ahmed wrote the equation in the form $y - 7 = \frac{5}{3} (x + 1)$
 Mohammed wrote the equation in the form $y - 7 = \frac{3}{5} (x + 1)$, which answer is correct?

Write

A problem from life that can be represented by the equation of the line.

Lesson [4- 4]

Parallel And Perpendicular lines

Idea of the lesson:

- Distinguishing between the parallel lines.
- Distinguishing between the perpendicular lines.

Vocabulary:

- Parallel lines.
- Perpendicular lines.

Learn

In the nearby figure, number of lines are shown, some of them are parallel and others are perpendicular. How can we distinguish between the parallel and perpendicular lines?



[4- 4- 1] parallel lines

You have previously learned the parallel of lines and the required conditions for that.

The two parallel lines lie in one plane and there is not joint point between them.

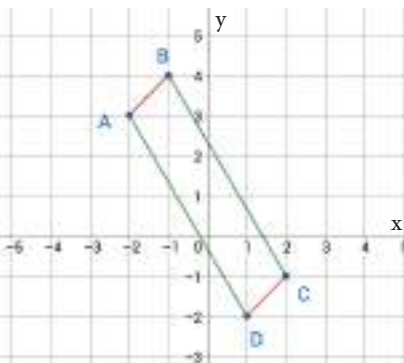
In this lesson we will distinguish the two parallel lines by their slope:

Any two lines will be paralleled when they have two equaled slopes but they must not be perpendicular.

The mathematical formula: ($\vec{L}_1 \parallel \vec{L}_2 \Leftrightarrow m_1 = m_2$)

Example(1)

Show if the points A(-2,3), B(-1,4), C(2, - 1), D(1, - 2) are vertices of the parallelogram ABCD by using the slope.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{\vec{AB}} = \frac{4 - 3}{-1 - (-2)}$$

$$m_{\vec{AB}} = \frac{1}{1}$$

$$m_{\vec{AB}} = 1$$

By the same way

$$m_{\vec{AD}} = \frac{-5}{3}$$

\therefore The figure ABCD is parallelogram (definition of parallelogram)

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{The slope formula between two points}$$

$$m_{\vec{CD}} = \frac{-2 - (-1)}{1 - 2} \quad \text{Substituting}$$

$$m_{\vec{CD}} = \frac{-1}{-1} \quad \text{Simplify}$$

$$m_{\vec{CD}} = 1$$

$$\therefore m_{\vec{AB}} = m_{\vec{CD}}$$

$$\therefore \vec{AB} \parallel \vec{CD}$$

$$m_{\vec{BC}} = \frac{-5}{3} \quad \therefore \vec{AD} \parallel \vec{BC}$$

Example(2) Prove that the points A(-2, - 1), B(-1,0), C(2,3) are collinear.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{The slope formula between two points}$$

$$m_{\vec{AB}} = \frac{0 - (-1)}{-1 - (-2)} \quad \text{Substituting}$$

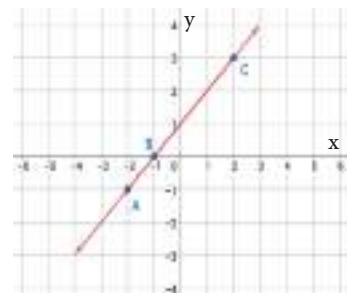
$$m_{\vec{AB}} = \frac{1}{1} = 1 \quad \therefore m_{\vec{AB}} = m_{\vec{BC}} \quad \text{Simplify}$$

\therefore The points A,B,C are collinear. (it's lie on a line)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{\vec{BC}} = \frac{3 - 0}{2 - (-1)}$$

$$m_{\vec{BC}} = \frac{3}{3} = 1$$



Example(3)

Find the equation of the line which passes through the point C (5,3) and parallel to line which passes through the two points A (4,5), B(2,-3).

-We find the slope of the line which passes through the two points A,B.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{-The slope of the line which passes through two points.}$$

$$m_{\vec{L}_1} = \frac{-3 - 5}{2 - 4} = \frac{-8}{-2} = 4 \quad \because \text{The two lines are parallel.}$$

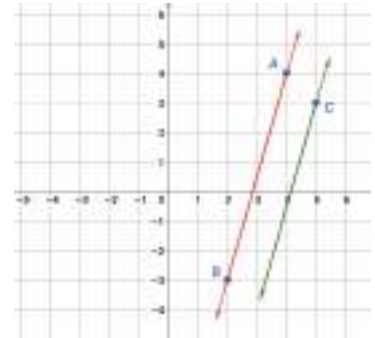
\therefore The slope of the required line $m_{\vec{L}_2} = 4$ (the same slope).

We find the equation of the required line.

$$y - y_1 = m(x - x_1) \quad \text{An equation of line slope - point}$$

$$y - 3 = 4(x - 5) \quad \begin{array}{l} \text{Substituting} \\ \text{Simplify} \end{array}$$

The equation of the required line is $y = 4x - 17$



Example (4)

IF $\vec{L}_1: y = \frac{-5}{3}x + 4$, $\vec{L}_2: y = \frac{5}{3}x + 4$, $\vec{L}_3: y = \frac{-5}{3}x - 4$ which of the following lines are parallel? Why?

$$\vec{L}_1: y = \frac{-5}{3}x + 4 \Rightarrow m_1 = \frac{-5}{3}, \quad k_1 = 4 \quad \text{Its slope and Y-intercept}$$

$$\vec{L}_2: y = \frac{5}{3}x + 4 \Rightarrow m_2 = \frac{5}{3}, \quad k_2 = 4 \quad \text{Its slope and Y-intercept}$$

$$\vec{L}_3: y = \frac{-5}{3}x - 4 \Rightarrow m_3 = \frac{-5}{3}, \quad k_3 = -4 \quad \text{Its slope and Y-intercept}$$

$$\therefore m_1 = m_3 \Rightarrow \vec{L}_1 \parallel \vec{L}_3, k_1 \neq k_3$$

[4-4-2] Perpendicular lines

-You have previously learned that the two perpendicular line meet in one point and form four right angles and they lie in one plane.

In this lesson, we will distinguish the perpendicular lines by their slopes on condition that each one of them will not be parallel to the two coordinate axes.

The two lines will be perpendicular when the slope of them represents the inverse of the slope of the other and the contrary of the sign.

$$\text{The mathematical formula: } \vec{L}_1 \perp \vec{L}_2 \Leftrightarrow m_1 = \frac{-1}{m_2} \text{ or } m_1 \times m_2 = -1$$

Example (5)

Show that the points A(2,4), B(-4,2), C(-2, -4) are vertices of a right - angled triangle. Determine the right angle.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{-The slope of the line which passes through two points.}$$

$$m_{\overline{AB}} = \frac{2 - 4}{-4 - 2} \quad m_{\overline{AC}} = \frac{-4 - 4}{-2 - 2} \quad m_{\overline{BC}} = \frac{-4 - 2}{-2 - (-4)}$$

$$= \frac{-2}{-6}$$

$$= \frac{-8}{-4}$$

$$= \frac{-6}{2}$$

$$= \frac{1}{3}$$

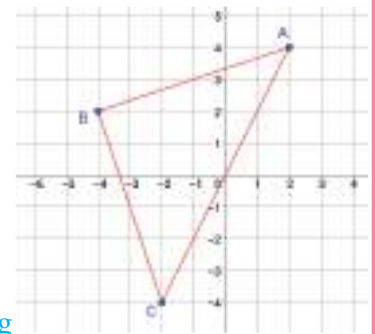
$$= \frac{2}{1}$$

$$= \frac{-3}{1}$$

by substituting

$$\Rightarrow m_{\overline{AB}} \times m_{\overline{BC}} = \frac{1}{3} \times \frac{-3}{1} = -1$$

$$\Rightarrow \overline{AB} \perp \overline{BC} \text{ and } m\angle B = 90^\circ$$



Example (6)

Find the equation of line which passes through the point C (3,-4) and perpendicular to the line which passes through A(0,3),B (2,-2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow m_{\vec{L}_1} = \frac{-2 - 3}{2 - 0} = \frac{-5}{2} \quad \text{Substituting}$$

$$m_{\vec{L}_2} = \frac{2}{5} \quad \text{The two lines are perpendicular. A slope inversion } \vec{L}_1 \text{ on the contrary of the sign}$$

$$y - y_1 = m(x - x_1) \quad \text{An equation of line slope -point}$$

$$y + 4 = \frac{2}{5}(x - 3) \quad \text{Substituting}$$

$$y = \frac{2}{5}x - \frac{26}{5} \quad \text{The equation of the required line}$$

Example (7)

Find the value of (a) which makes the slope of the line which passes through the two points (3, 1), (a, -4) is perpendicular of the line its slope is $-\frac{1}{5}$.

Since the two lines are perpendicular then the slope of the line required is 5.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{The slope of the line which passes through two points}$$

$$\frac{5}{1} = \frac{-4 - 1}{a - 3} \quad \text{By substituting}$$

$$5a - 15 = -5 \quad \text{Commutative multiplication}$$

$$5a = 10 \quad \text{By dividing the two sides of the equation by 5}$$

$$a = 2$$

Make sure of your understanding

Questions 1 -2 are
Similar to example 7

1 The line AB passes through the two points A (-2,4), B(a,6), parallels to the line CD which passes through the two points C(6,-6), D(2,-7). Find the value of (a).

2 Find the value of (a) which makes the slope of the line which passes through the two points (3,2), (6,1), equals $-\frac{1}{4}$.

Question 3 is similar
to example 1

3 Prove that the figure ABCD is a parallelogram, where, A (3,0), B(0,4), C(-3,0), D (0,-4)

4 Prove that the triangle ABC, where, A (-5, -7), B (-8, -2), C (-4, -3) is a right-angled triangle, then determine the right angle.

Question 4 is
similar to example 5

5 Prove that the points A (0, -1), B (4, 2), C (8, 5) are collinear.

Question 5 is
similar to example 2

6 Find the equation of line which passes through the point (-4,0) and perpendicular the line which passes through the two points (3, -2), (6,0).

Question 6 is similar to
examples 5, 6

Solve the Exercises

7 The line AB where A (0, 2), B (3, 0), the line CD where, C (6, -2), D (9, -4) and the line EF? Where, E (0, -5), F (2, -2), what is the relation of (\vec{AB}) with the other two line CD, EF? Explain that.

8 Do the points A (0, -7), B (1, -1), C (2, 3) are collinear? Explain that.

9 Prove that the figure ABCD is a rectangle where A (1, 4), B (2, 6), C (8, 3), D (7, 1): is a right-angled triangle, then determine the right angle.

10 Find the equation of the line which passes through the point (1, -1) and parallel to the line which passes through the two points (3, -2), (6, 0).

Solve the problems

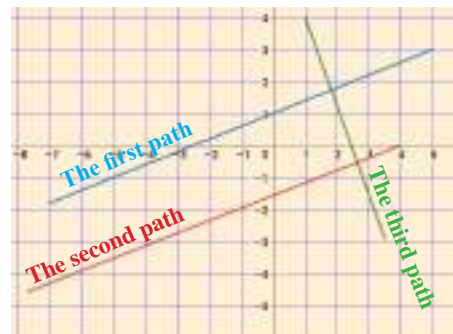
- 11 **Physics :** The nearby table represent the amount of water which flow from one of the dams during certain time .Do the data of the table represent a line ? Explain that.

Flowing water	
Time (seconds)	Water volume
5	75000
10	150000
15	225000

- 12 **Geometry :** Prove that the figure ABCD is a trapezoid , where the coordinates of the upper base are (4,5) ,(6,2) and the lower base coordinates are (2,-1),(-2,5). Does it have a right angle? Explain that.

- 13 **Map:** Use the nearby map to show that:

- The first path parallel to the second one.
- The second path is perpendicular to the third path.
- Is the first path perpendicular to the third path ? Explain that.



Think

- 14 **Challenge:** Do the following points $(-2, -1), (-1, 0), (4, 5), (2, 3)$ are collinear ? Explain that.
- 15 **Correct the mistake :** Ahmed said that the line which passes through the two points $(0, 4), (-3, 0)$ perpendicular to the line which passes through the two points $(1, \frac{3}{4}), (0, 0)$. Discover Ahmeds mistake and correct it.
- 16 **Open problem :-** The two following equations : $3y - 5x = 20, 3y - 5x = 15$ represent two parallel line . What are the similarity and differences between them ? Explain that .
- 17 **Justification :** Why do the following points lie on a line parallel to the X- axis : $(-1, 4), (0, 4), (2, 4)$?
- 18 **Which one is correct:** Sarah said that the slope of line $5y + 2x = 10$ is $(\frac{2}{5})$ and its intercept is 2, Mohammed said that the slope is $(-\frac{2}{5})$ and the intercept is 2 . Show which answer is correct ?
- 19 **Open problem :** ABCD is a rhombus , its vertices $A(0, 3), B(3, 4), C(2, 1), D(-1, 0)$ prove that its two diameters are perpendicular .
- 20 **Open problem :** What are the similarities and differences between any two parallel lines ?

Write

Does the two lines are parallel or perpendicular by using their slope ?

Lesson [4-5]

Distance Between Two Points

Idea of the lesson:

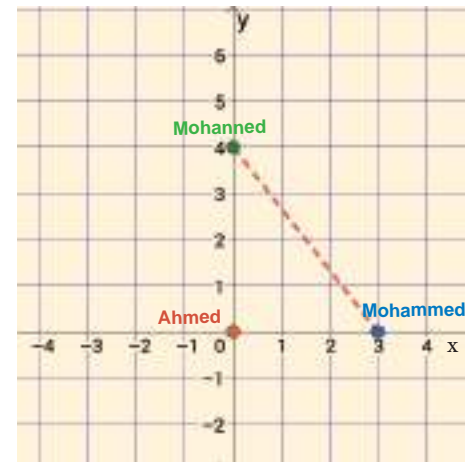
- Identifying the formula of the distance between two points
- Applying the formula of the distance between two points
- Identifying the formula of midpoint.
- Applying the formula of midpoint

Vocabulary:

- The formula of the distance between two points.
- The midpoint
- The formula of the midpoint

Learn

Three friends went out in a journey of discovery ,their sites were determined as in the nearby figure. Mohammed is away from Ahmed in 3 Km and Mohammed is away from Ahmed in 4 Km.
How can you find the distance between Mohammed and Mohammed?



[4- 5- 1] Distance between two points formula

You have previously learned that the distance between two points on the X-axis is $|x_2 - x_1|$

And the distance between two points on the Y-axis is $|y_2 - y_1|$

In this lesson you will identify the formula of the distance in the coordinate plane.

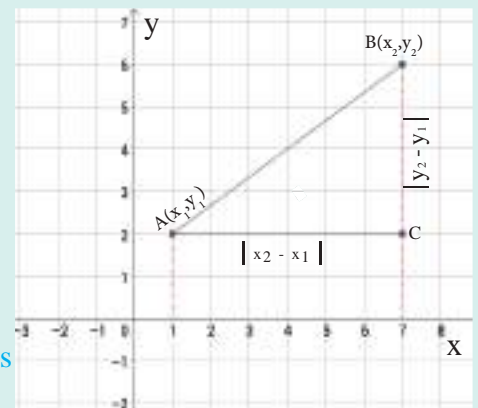
The formula of distance between the two points A,B depends on pythagorean theorem.

The triangle ACB is a right – angled triangle in C.

$$(AB)^2 = (AC)^2 + (BC)^2 \quad \text{pythagorean theorem}$$

$$d^2 = (|x_2 - x_1|)^2 + (|y_2 - y_1|)^2 \quad \text{By substituting and simplifying}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Take the square root of both sides}$$



Example (1)

From the paragraph of learn: We find that the site of Mohammed is the point A(3,0) and the site of Mohammed is the point B(0,4).

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The formula of the distance between two points

$$AB = \sqrt{(0 - 3)^2 + (4 - 0)^2}$$

By substituting

$$AB = \sqrt{9 + 16} = \sqrt{25} = 5$$

By simplifying

∴ The distance between Mohammed and Mohammed is 5 Km.

Example (2)

By using the formula of distance , prove that the points

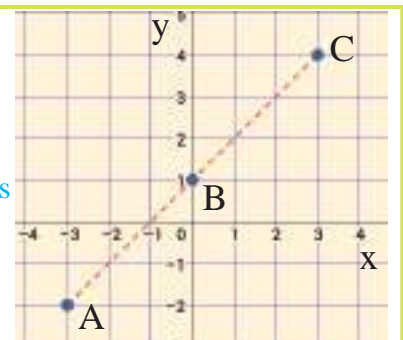
A(-3, -2), B(0,1), C(3,4) are collinear.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{The formula of the distance between two points by substituting the points A,B,C}$$

$$AB = \sqrt{(0 + 3)^2 + (1 + 2)^2}$$

$$BC = \sqrt{(3 - 0)^2 + (4 - 1)^2}$$

$$AC = \sqrt{(3 + 3)^2 + (4 + 2)^2}$$



$$AB = \sqrt{9+9}, BC = \sqrt{9+9}, AC = \sqrt{36+36}$$

$$AB = \sqrt{18}, BC = \sqrt{18}, AC = \sqrt{72}$$

$$= 3\sqrt{2}, \quad = 3\sqrt{2}, \quad = 6\sqrt{2}$$

$$6\sqrt{2} = 3\sqrt{2} + 3\sqrt{2}$$

$$AC = AB + BC$$

Simplification

All equal the sum of parts

so the points A,B,C are collinear.

Example (3)

Show the type of the triangle which its vertices are A(3, -4), B(5, -2), C(5, -6), in term of its sides.

Is the triangle a right -angled one?

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

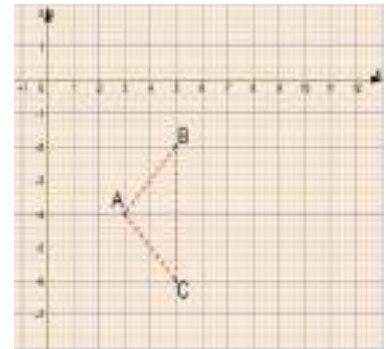
$$AB = \sqrt{(5-3)^2 + (-2+4)^2}$$

$$BC = \sqrt{(5-5)^2 + (-6+2)^2}$$

$$AC = \sqrt{(5-3)^2 + (-6+4)^2}$$

The formula of the distance between two points

By substituting the points A,B,C



$$AB = \sqrt{4+4}, BC = \sqrt{0+16}, AC = \sqrt{4+4}$$

$$AB = \sqrt{8}, BC = 4, AC = \sqrt{8}$$

$$= 2\sqrt{2}, \quad = 4, \quad = 2\sqrt{2}$$

Simplification

$\therefore AB=AC, \therefore$ The triangle is an isosceles triangle

$$(4)^2 = (\sqrt{8})^2 + (\sqrt{8})^2$$

$$(4)^2 = 8 + 8$$

On the converse of pythagorean theorem

The triangle is right -angled triangle

Example(4)

By using the formula of the distance ,show if the following points A(-2,3), B(-1,4), C(2, -1), D(1, -2) represent vertices of a parallelogram .

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The formula of the distance between two points

$$AB = \sqrt{(4-3)^2 + (-1+2)^2}$$

$$= \sqrt{1+1}$$

$$= \sqrt{2}$$

$$DC = \sqrt{(1-2)^2 + (-2+1)^2}$$

$$= \sqrt{1+1}$$

$$= \sqrt{2}$$

$$\therefore AB = DC$$

By the same way

$$AD = \sqrt{(1+2)^2 + (-2-3)^2}$$

$$= \sqrt{9+25}$$

$$= \sqrt{34}$$

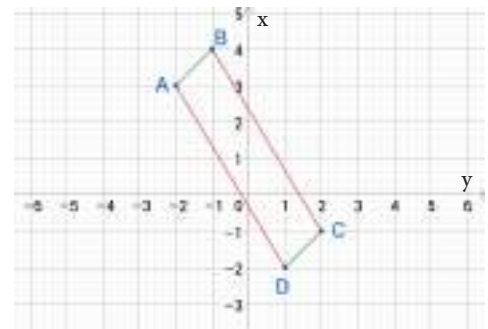
$$BC = \sqrt{(2+1)^2 + (-1-4)^2}$$

$$= \sqrt{9+25}$$

$$= \sqrt{34}$$

$$\therefore AD = BC$$

So the figure ABCD is a parallelogram (the properties of the parallelogram are each two opposite sides is equaled.)

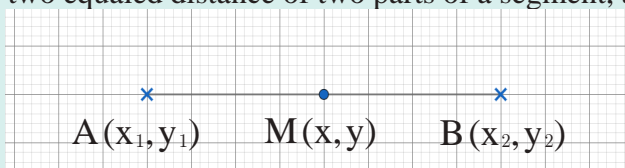


[4 -5 -2] The Midpoint Formula

The midpoint: is the point which lies on two equal distance of two parts of a segment, and it belongs to it.

Coordinates of midpoint

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$



Example (5)

Find the coordinate of midpoint of the segment which connects between A (3, - 8), B (3, 6)

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

The formula of midpoint

$$= \left(\frac{3 + 3}{2}, \frac{6 + (-8)}{2} \right)$$

By substituting the two points

Simplification

$$= \left(\frac{6}{2}, \frac{-2}{2} \right) = (3, -1)$$

(3, -1) is the midpoint of \overline{AB}

Example (6)

If M (1, -3) is the midpoint of \overline{AB} and A (-1, -2), then find the coordinates of the point B.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

The formula of the midpoint

$$(1, -3) = \left(\frac{-1 + x_2}{2}, \frac{-2 + y_2}{2} \right)$$

Assume B(x₂, y₂) by substituting the points

$$1 = \frac{-1 + x_2}{2} \Rightarrow -1 + x_2 = 2 \Rightarrow x_2 = 3$$

Multiplication and simplification

$$-3 = \frac{-2 + y_2}{2} \Rightarrow -2 + y_2 = -6 \Rightarrow y_2 = -4$$

Coordinates of B are: B (3, -4)

Example (7)

By using the formula of the midpoint, show if the points A (-2, 3), B (-2, 3), C (2, -1), D (1, -2) represent vertices of a parallelogram.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

The formula of the midpoint

The midpoint of diameter \overline{AC}

The midpoint of diameter \overline{BD}

$$M_1 = \left(\frac{2 + (-2)}{2}, \frac{-1 + 3}{2} \right)$$

$$M_2 = \left(\frac{1 + (-1)}{2}, \frac{-2 + 4}{2} \right)$$

$$M_1 = \left(\frac{0}{2}, \frac{2}{2} \right)$$

By substituting

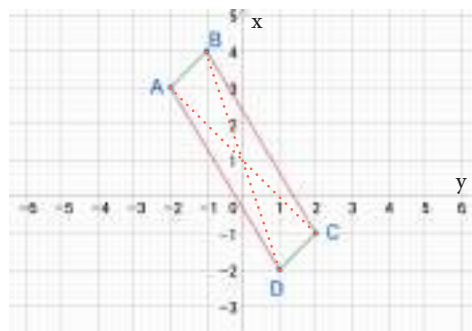
$$M_2 = \left(\frac{0}{2}, \frac{2}{2} \right)$$

$$M_1 = (0, 1)$$

By simplifying

$$M_2 = (0, 1)$$

$\therefore M_1 = M_2$ \therefore The figure ABCD is a parallelogram (the properties of the parallelogram are one of its two diameters is dividing the other into two parts.)



Example (8)

The points A (3, 1), B (5, 3), C (5, -1) are vertices of a triangle, the point M is the midpoint of \overline{BC} . Find the length of \overline{AM} .

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{5 + 5}{2}, \frac{-1 + 3}{2} \right) = (5, 1)$$

The formula of midpoint, simplification

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The formula of the distance between two points

$$= \sqrt{(5 - 3)^2 + (1 - 1)^2}$$

Substitution

$$= \sqrt{4} = 2$$

Simplification

Make sure of your understanding

- | | |
|---|--|
| <p>1 Find the distance between each two points of the following :-
 i) $(0, 0), (3, 8)$ ii) $(-3, -1), (1, -4)$ iii) $(-1, -2), (3, -4)$</p> <p>2 Find the midpoint of the branches (i),(ii),(iii) in question 1.</p> <p>3 By using the formula of the distance between two points, prove that the points $A(-2, -1), B(-1, 0), C(4, 5)$ are collinear.</p> <p>4 Show the type of the triangle which its vertices $A(2, 4), B(-4, 2), C(-1, -2)$ in term of the sides ,it is a right –angled triangle?</p> <p>5 Show if the following points $A(4, 0), B(6, -6), C(-8, 0), D(-10, 6)$ represent vertices of a parallelogram.
 i)By using the formula of the distance between two points
 ii)By using the midpoint formula.</p> <p>6 If $M(-2, 0)$ is the midpoint of \overline{AB} and $A(4, 0)$, find the two coordinates of the point B.</p> | <p>Question 1 is similar to example 1</p> <p>Question 2 is similar to example 4</p> <p>Question 3 is similar to example 2</p> <p>Question 4 is similar to example 3</p> <p>Question 5 is similar to examples 4, 6</p> <p>Question 6 is similar to examples 7</p> |
|---|--|

Solve the exercises

- 7 Find the distance between each two points for each of the following :
 i) $(8, 1), (-4, 3)$ ii) $(6, -9), (0, 2)$ iii) $(-2, 4), (-6, -2)$
- 8 Find the midpoint for the branches (i),(ii),(iii) of the question (7)
- 9 By using the formula of distance between two points , prove that the points $A(1, -3), B(3, -4), C(-1, -2)$ are collinear.
- 10 What is the type of the triangle which its vertices $A(2, -1), B(2, 1), C(-1, -1)$ in term of its sides .Is it a right –angled triangle?
- 11 Show if the following points are vertices of a parallelogram $A(-3, 5), B(2, 7), C(1, 9), D(-4, 7)$
 i)By using the formula of distance between two points ii)By using the midpoint formula.
- 12 If $M(4, -2)$ is the midpoint of \overline{AB} and $B(5, 1)$ find the two coordinates of A.

Solve the problems

- 13 **Geometry:** ABC is a triangle, its vertices are $A(6, 4), B(-2, 6), C(0, -4)$, check if the length of the segment which connects the two midpoints sides of the triangle, is equal to the half of the third side.
- 14 **Determine a site:** Mahmoods house is in the point $(-4, 0)$ and the site of his school is in the point $(0, -3)$. What is the distance which Mahmood cut when he go to school, and you should know that the side length of each square in the coordinate plane represents one Km?

Think

- 15 **Challenge:** A circle, the following points $A(-1, 1), B(5, 1)$ represent the two ends of its diameter. Find i) The coordinates of its center. ii) Its area.
- 16 **Discover the mistake:** Shahad had found the coordinates of the midpoint of the segment which its two ends are $(6, 1), (8, 3)$,she wrote as follow $(\frac{8-6}{2}, \frac{3-1}{2}) = (1, 1)$,discover Shahads mistake, then correct it

Write

Relation of midpoint formula with finding of the arithmetic mean.

Lesson [4-6]

Trigonometric Ratios

Idea of the lesson:

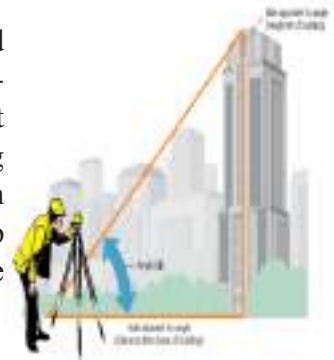
- Identifying the fundamental trigonometric ratios
- The trigonometric ratios for special angles
- Finding the values of statements including special angles

Vocabulary:

- Trigonometric ratios
- sin, cos, tan, sec, csc, cot
- Special angles: $60^\circ, 45^\circ, 30^\circ, 90^\circ, 0^\circ$

Learn

A surveyor stood a distance d meter from a building by using his appliance, he looking at the upper level of the building with a certain angle. How can the trigonometric ratios help him in finding the height of the building?



[4- 6- 1] Trigonometric ratios ($\sin \theta, \cos \theta, \tan \theta$)

You have previously learned the elements of the triangle, where it consists of three angles and three sides. According to its angles, we call the triangle (acute angles, obtuse angle, right angle) or according to its sides, we call it (equilateral, isosceles, scalene)

-Calculating the triangles: Is the study of the relation between the angles and sides of the triangle.

-Trigonometric ratio: Is the ratio which compares between the length of two sides of a right-angled triangle

-The fundamental ratio: Is the sine (sin) and the cosine (cos) and the tangent (tan).

-The sine of $\angle \theta$ (we refer to by $\sin \theta$): Is the ratio between the side which **opposite** to the angle (θ) and the **hypotenuse**

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

-The cosine of $\angle \theta$ (we refer to by $\cos \theta$): Is the ratio between the side which is **adjacent** to the angle (θ) and the **hypotenuse**.

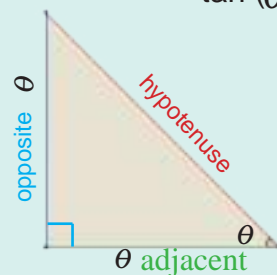
$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

-The tangent of $\angle \theta$ (we refer to by $\tan \theta$): Is the ratio between the side which is **opposite** to the angle (θ) and the **adjacent**.

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

-To find the trigonometric ratios (\sin, \cos, \tan) we follow:-

- 1) Drawing a diagram of a right-angled triangle and we fix the data on it.
- 2) Using Pythagorean theorem to find the missing side.
- 3) Using the trigonometric ratios to find the required.



Example(1)

From the nearby figure, find the three values of the trigonometric ratios of the angle θ .

Use Pythagorean theorem to find the length of the side AB (opposite)

$$(AC)^2 = (AB)^2 + (BC)^2$$

Pythagorean theorem

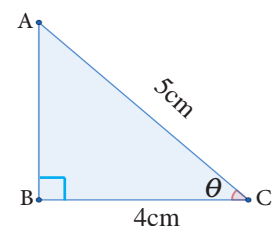
$$(5)^2 = (AB)^2 + (4)^2$$

By substituting and simplifying

$$(AB)^2 = 25 - 16 = 9$$

Take the square root of both sides
(positive sign because it is length)

$$AB = 3$$



-use the trigonometric ratios, then the substitution

$$\sin \theta = \frac{\text{opposite of } \angle \theta}{\text{hypotenuse}} = \frac{3}{5}$$

$$\cos \theta = \frac{\text{adjacent of } \angle \theta}{\text{hypotenuse}} = \frac{4}{5}$$

$$\tan \theta = \frac{\text{opposite of } \angle \theta}{\text{adjacent of } \angle \theta} = \frac{3}{4}$$

Example (2)

The triangle ABC has a right-angle in B, if $\tan A = \frac{15}{8}$, find:

i) $\sin A$ ii) $\cos A$

$$\tan A = \frac{15k}{8k}$$

$$\tan A = \frac{BC}{BA}$$

$$\therefore BC = 15k, AB = 8k$$

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$= (8k)^2 + (15k)^2$$

$$= 64k^2 + 225k^2$$

$$(AC)^2 = 289k^2 \Rightarrow \therefore AC = 17k$$

$$i) \sin A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17} \quad ii) \cos A = \frac{AB}{AC} = \frac{8k}{17k} = \frac{8}{17}$$

By multiplying the numerator and the denominator by the constant K, where K is greater than 0. -Tangent formula

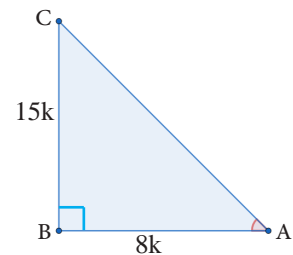
By comparison

Pythagoras theorem

By substituting

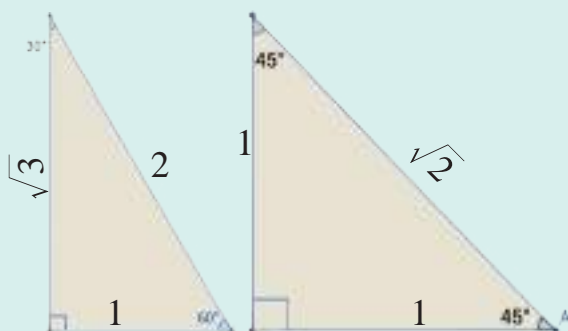
Simplification

Take the square root of both sides



[4 -6 -2]The Trigonometric Ratios for Special Angles.

The nearby table shows the values of the trigonometric for special angles



Trigonometric ratios	30°	60°	45°	90°	0°
Sine (sin)	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	1	0
Cosine (cos)	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	0	1
Tangent (tan)	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1	undifind	0

Example (3) Prove that $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ = \sin 90^\circ$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \cos 30^\circ = \frac{\sqrt{3}}{2}, \cos 60^\circ = \frac{1}{2}, \sin 30^\circ = \frac{1}{2}, \sin 90^\circ = 1 \quad \text{From the table, we find}$$

$$\text{R.H.S: } \sin 90^\circ = 1$$

$$\text{L.H.S: } \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$\frac{3}{4} + \frac{1}{4} = 1$$

$$\therefore \text{L.H.S} = \text{R.H.S}$$

By substituting in the right side and the left side

Example(4) A man stood in front of a building in a distance which is 12 m from its base, he looked at the top of the building in a angle which is (30°). Find the height of the building.

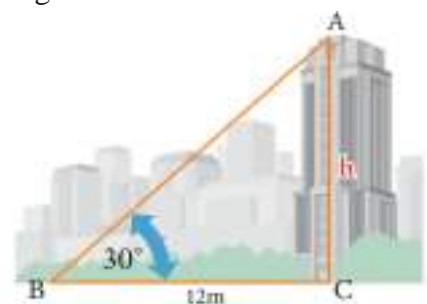
The trigonometric ratio which connects between the height of the building h and the distance of the man from its base, is the tangent ratio.

$$\tan 30^\circ = \frac{h}{12} \quad \text{Tangent formula}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{12} \quad \text{Substituting}$$

$$\sqrt{3}h = 12 \quad \text{Commutative Multiplication}$$

$$h = \frac{12}{\sqrt{3}} = 4\sqrt{3} \text{ m} \quad \text{Simplifying} \quad \text{The height of the building is } 4\sqrt{3} \text{ m}$$



[4-6-3] Relations of Trigonometric Ratios

In this item, we will focus on the inversion of the trigonometric ratios sin, cos, tan, and as it shown in the following table:

Trigonometric ratio	$\sin \theta$	$\cos \theta$	$\tan \theta$
Its inversion	$\csc \theta = \frac{1}{\sin \theta}$	$\sec \theta = \frac{1}{\cos \theta}$	$\cot \theta = \frac{1}{\tan \theta}$

Example(5)

A triangle has a right angle in B, if $\cos A = \frac{\sqrt{3}}{11}$ find i) $\sec A$ ii) $\csc A$ iii) $\cot A$

$$\cos A = \frac{\sqrt{3}k}{\sqrt{11}k} = \frac{AB}{AC} \Rightarrow AB = \sqrt{3}k, AC = \sqrt{11}k$$

$$(AC)^2 = (AB)^2 + (BC)^2$$

Pythagorean theorem

$$(\sqrt{11}k)^2 = (\sqrt{3}k)^2 + (BC)^2$$

By substituting

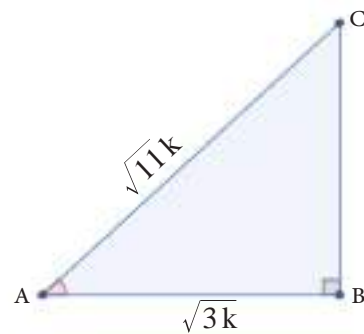
$$11k^2 = 3k^2 + (BC)^2$$

Simplifying

$$(BC)^2 = 8k^2$$

$$\therefore BC = \sqrt{8}k$$

Take the square root of both sides



$$\text{i) } \cos A = \frac{\sqrt{3}}{\sqrt{11}} \Rightarrow \sec A = \frac{1}{\cos A} = \frac{\sqrt{11}}{\sqrt{3}} \quad \text{ii) } \sin A = \frac{\sqrt{8}}{\sqrt{11}} \Rightarrow \csc A = \frac{1}{\sin A} = \frac{\sqrt{11}}{\sqrt{8}}$$

$$\text{iii) } \tan A = \frac{\sqrt{8}}{\sqrt{3}} \Rightarrow \cot A = \frac{1}{\tan A} = \frac{\sqrt{3}}{\sqrt{8}}$$

Inversion of fundamental trigonometric ratios

Example(6)

Find the numerical value for the expression: $(\sin 45^\circ)(\sec 45^\circ) - (\tan 60^\circ)(\cot 30^\circ) + 2 \csc 90^\circ$

$$\left. \begin{aligned} \sin 45^\circ &= \frac{1}{\sqrt{2}}, \sec 45^\circ = \frac{1}{\cos 45^\circ} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2} \\ \tan 60^\circ &= \sqrt{3}, \cot 30^\circ = \frac{1}{\tan 30^\circ} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3} \\ \csc 90^\circ &= \frac{1}{\sin 90^\circ} = \frac{1}{1} = 1 \end{aligned} \right\}$$

From the table, we find the values of the special trigonometric ratios and the inversions of the fundamental trigonometric ratios.

$$\left. \begin{aligned} \tan 60^\circ &= \sqrt{3}, \cot 30^\circ = \frac{1}{\tan 30^\circ} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3} \\ \csc 90^\circ &= \frac{1}{\sin 90^\circ} = \frac{1}{1} = 1 \end{aligned} \right\}$$

$$(\sin 45^\circ)(\sec 45^\circ) - (\tan 60^\circ)(\cot 30^\circ) + 2 \csc 90^\circ$$

$$\left(\frac{1}{\sqrt{2}}\right)(\sqrt{2}) - (\sqrt{3})(\sqrt{3}) + 2(1) \Rightarrow 1 - 3 + 2 = 0$$

The given expression

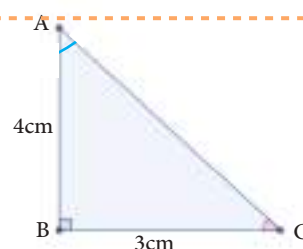
by substituting and simplifying

\therefore The numerical result of the expression equals 0

Make sure of your understanding

1 From the nearby figure, find the following trigonometric ratios

i) $\sin A$ ii) $\cos C$ iii) $\cot C$ iv) $\sec A$



Question 1 is similar to examples 1,2,5

2 The triangle ABC has a right angle in B, if $\cot A = \sqrt{3}$, find :

i) $\tan A$ ii) $\sin A$ iii) $\csc A$ iv) $\sec A$ v) $\cos A$

Question 2 is similar to examples 2,5

3 Prove the following:

Question 3 is similar to examples 6,3

i) $(\cos 30^\circ - \csc 45^\circ)(\sin 60^\circ + \sec 45^\circ) = \frac{1}{2}$, ii) $2\sin 30^\circ \sec 30^\circ = \csc 60^\circ$

iii) $(\cos 45^\circ - \csc 45^\circ)(\tan 45^\circ)(\csc 90^\circ) = -\cos 45^\circ$, iv) $\sqrt{\frac{1 - \cos 60^\circ}{2}} = \sin 30^\circ$

4 A Kite had raised ($3\sqrt{3}$ m) from the surface of the land, if the string which was connected with it formed an angle of 60° with the surface of the land. find the length of the string

Question 4 is similar to example 4

Solve the Exercises

5 From the nearby figure, find the following trigonometric ratios:

i) $\cot A$ ii) $\cot C$ iii) $\sec C$ iv) $\csc A$

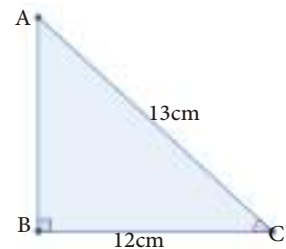
6 The triangle ABC has a right angle in B, if $\sec A = \sqrt{2}$, find :

i) $\sin A$ ii) $\cot C$ iii) $\csc A$ iv) $\cos C$

7 Prove the following :

i) $\cos 60^\circ \csc 60^\circ + \sin 60^\circ \sec 60^\circ = \frac{4}{\sqrt{3}}$

ii) $\sin 45^\circ \sec 45^\circ + \csc 45^\circ \cos 45^\circ = 2$, iii) $\frac{\cot 45^\circ + \sin 90^\circ}{2\sec 30^\circ} = \frac{\sqrt{3}}{2}$



Solve the problems

8 **Sport** : A sporty appliance for practising walking was inclined in angle of 30° , if the height of its edge is 1.5m from the land surface. How long is the appliance belt?

9 **Skiing** : In a position for skiing, the height of the main hill is 500m and the angle of its inclination from the surface of the land is 60° . How long is the skiing surface?

10 **A ladder of fire fighting** : The long of a ladder of fire fighting is 20m and the measurement of the angle which it creates with the land is (45°). Find the height of long of ladder on the building.

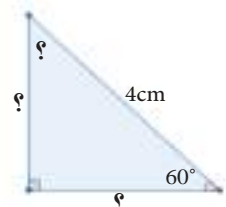
11 **Garden** : Benan stood in a distance of 25m far from base of a tree which its height is 25m. What is the measurement of the angle which she forms with the top of the tree?

Think

12 **Challenge**: In the nearby figure, find the indicated values (?) by using the trigonometric ratios.

13 **Open problem** : The triangle ABC has a right angle is B, $\sin A = \frac{\sqrt{3}}{2}$ how can you find the value of the angle C?

14 **Justification** : If the sine and cosine of a right -angled triangle were equaled, what is the type of the triangle in term of its sides length?



Write

A problem in which you can use the sine ratio to find the length of an unknown side in a right - angled triangle, then solve it.

Chapter Test

1 Represent the following equations in the coordinate plane:

i) $2x-4y=8$

ii) $y=2$

iii) $x=2$

iv) $y=x^2-1$

2 Find the equation of the line which passes through the two points A(-2,-3), B(2,3).

3 Find the X-intercept and Y-intercept for the equation $y-x=4$.

4 Find the equation of the line for each the following:

i) passes through the two points (3,-2), (1,5).

ii) Slope $\frac{3}{2}$ and Y-intercept equals -5.

iii) Slope $-\frac{1}{5}$ and X-intercept equals 3.

5 Use slope equation and point to determine the slope and one of its points:

$$2y-3x=8.$$

6 By using the slope, show the following :

i) The points: A(3,2), B(0,-1), D(1,0) are collinear.

ii) The following points are vertices of parallelogram:

$$A(4,-1), B(2,2), C(-2,4), D(0,1)$$

iii) The line passing through the two points A(3,1), B(4,-1) is perpendicular to the line which passes through the two points C(4,-1), D(0,-3).

7 Find the equation of the line that passes through the point (0,3) and is parallel to the line that has slope $-\frac{2}{3}$.

8 By using the distance between two points formula, prove the branches (i), (ii) in question 6.

9 By using the midpoint formula, prove the branch (ii) in question 6.

10 The triangle ABC has a right-angle in B, if $\sin A = \frac{1}{2}$, find the:

i) $\cos A$ ii) $\tan A$ iii) $\cot C$ iv) $\sec A$

Geometric and Measurement

lesson 5-1 polygons and polyhedrons (Pyramid and Cone)

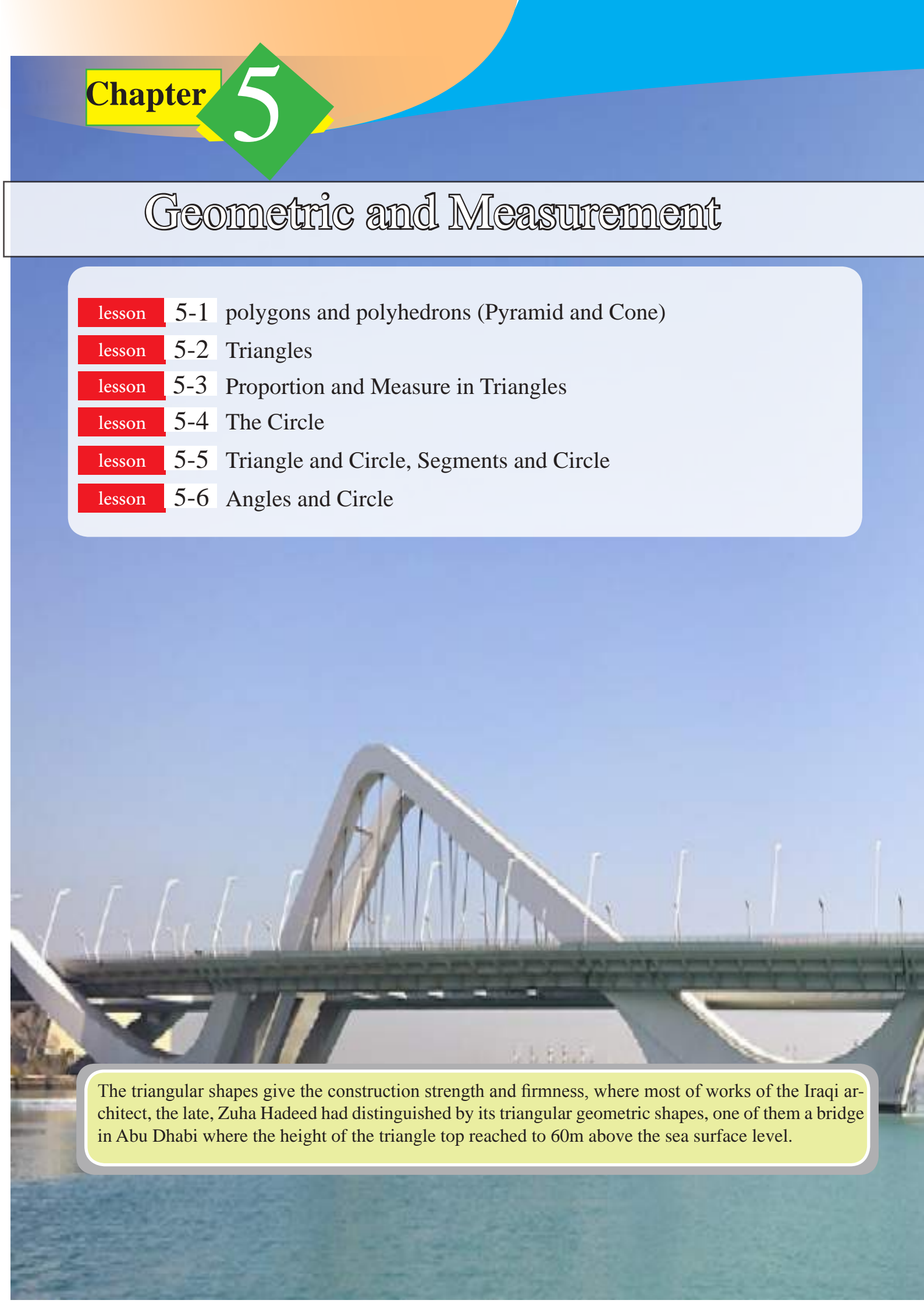
lesson 5-2 Triangles

lesson 5-3 Proportion and Measure in Triangles

lesson 5-4 The Circle

lesson 5-5 Triangle and Circle, Segments and Circle

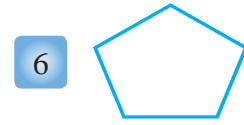
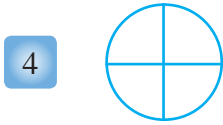
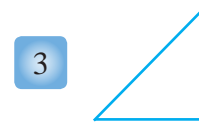
lesson 5-6 Angles and Circle



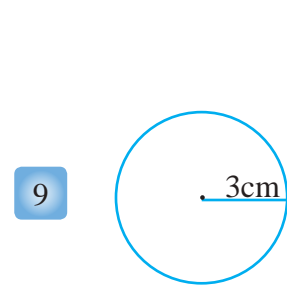
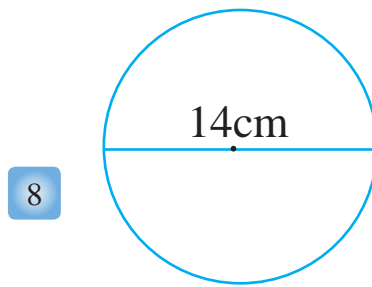
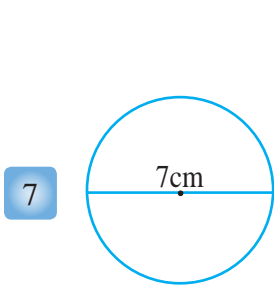
The triangular shapes give the construction strength and firmness, where most of works of the Iraqi architect, the late, Zuhair Hadeed had distinguished by its triangular geometric shapes, one of them a bridge in Abu Dhabi where the height of the triangle top reached to 60m above the sea surface level.

Pretest

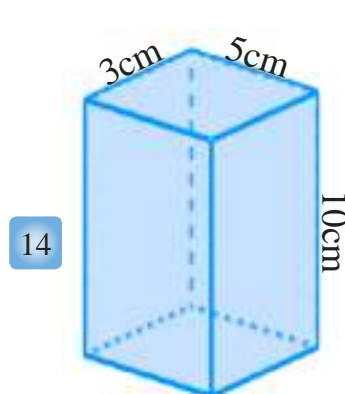
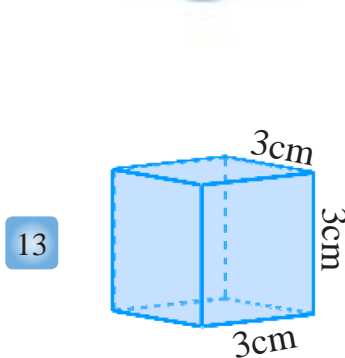
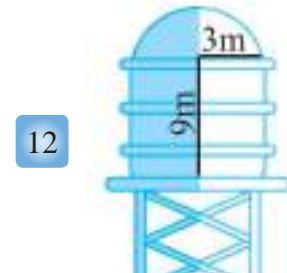
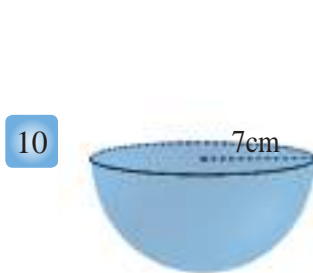
Determine if the figure is polygon or not, and if it is a polygon, then, is it a regular polygon or irregular one?



Find the area and the perimeter of each of the following circles:



Find the surface area and the volume of each of the following:



Find the value of x in each of the following:

16 $\frac{7}{6} = \frac{x-3}{2}$

17 $\frac{7}{x} = \frac{1}{2}$

18 $\frac{3}{16} = \frac{x}{4}$

Find the measure of the central angle and the sum of the interior and exterior angles for each of the following:

19 regular pentagonal

20 regular octagonal

21 regular hexagonal

22 A commercial company includes 20 employees, the ratio of males to females is $\frac{3}{2}$, how many female employees are there? And, how many male employees are there?

23 An equilateral triangle, each side of its three sides equals $(2x - 1)$ cm, and its perimeter is 57 cm, find the value of x and find the length of each side.

Lesson [5-1]

Polygons and Polyhedrons (Pyramid and cone)

Idea of the lesson:

- Finding the perimeter and the area of the regular polygons.
- Finding the volume and the surface area for each of pyramid and cone.

Vocabulary

- Apothem (H)
- Lateral height
- Cone
- Pyramid

Lesson

You have previously learned the regular and irregular polygons and how to find the internal and external of the regular polygon. You have also learned how to find the central angle of the polygon. You have been able to distinguish between the convex and concave polygon. In this lesson you will be able to find the area and the perimeter of the regular polygons.



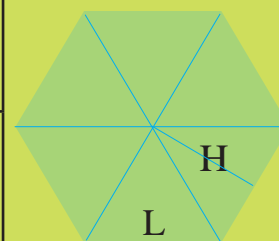
[5- 1- 1] Regular Polygons

The regular polygon perimeter = the number of sides multiplying by the side length

$$P = n \times L$$

The regular polygon area = the triangle area which its vertex is the center of the polygon and its base is the side of the polygon \times the number of its sides.

$$A = \frac{1}{2} L \times H \times n$$



If you know that the side length is L and the apothem is H (it is the descending column from the polygon center on one of the polygon sides).

We can calculate the triangle area as follow: The triangle area = $\frac{1}{2} \times \text{Base} \times \text{height}$ (Apothem),

$$A = \frac{1}{2} L \times H$$

Example (1):

Find the perimeter and the area of the regular hexagonal, its side length is 4m and the length of the apothem is $2\sqrt{3}$ m.

$$P = n \times L$$

$$P = 6 \times 4 = 24\text{m}$$

$$A = \frac{1}{2} L \times H \times n$$

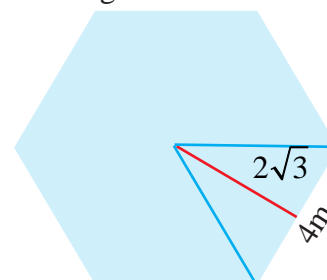
$$= \frac{1}{2} \times 4 \times 2\sqrt{3} \times 6 = 24\sqrt{3}$$

By using the formula of the polygon perimeter

The polygon area

By using the formula of the polygon area

By substituting and simplifying



Example (2):

Find the square area which its apothem length is 4 cm

$$A = \frac{1}{2} L \times H \times n$$

$$L = 4 \times 2 = 8\text{cm}$$

$$A = \frac{1}{2} \times 8 \times 4 \times 4 = 64\text{cm}^2$$

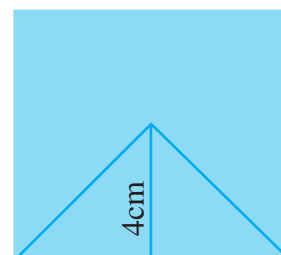
$$A = L \times L$$

$$A = 8 \times 8 = 64\text{cm}^2$$

Method (1) by using the formula of the regular polygon area.

The length of the square side

Method (2) by using the formula of the square area (the side length \times itself)



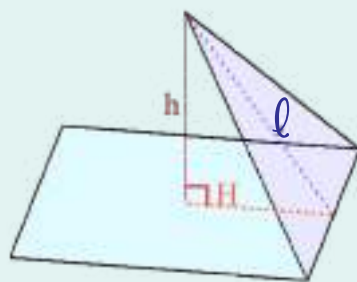
[5-1-2] pyramid and Cone

Pyramid: Is a polyhedron which has, at least, three triangular faces and one base represents a polygon figure (the shape of the base determines the name of the pyramid)

h = height

H = Apothem

ℓ = lateral height (slant height)



$$\ell^2 = h^2 + H^2$$

Cone: Is the polyhedron which has one base represents a circle and has one vertex.

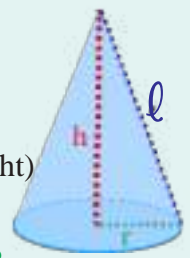
ℓ = the lateral height (slant height)

h = height

r = radius

$$\ell^2 = h^2 + r^2$$

The total area = the lateral area + the base area



The area formula of the regular pyramid and the right circular cone			The volume of pyramid and cone	
	The regular pyramid	The Right cone	The pyramid volume	
Lateral area	$LA = \frac{1}{2}p \times \ell$ The base perimeter p	$LA = \pi r \times l$		$V = \frac{1}{3}b \times h$
Total area	$TA = \frac{1}{2}p \times \ell + b$ The base area b	$TA = \pi r \times l + \pi r^2$	The cone volume	$V = \frac{1}{3}\pi r^2 \times h$

Example (3): Find the lateral area and the total area for a regular pyramid which its lateral height is 8 cm and the side length of its square base is 3cm.

$$LA = \frac{1}{2}p \times \ell$$

Lateral area

$$LA = \frac{1}{2} \times 12 \times 8$$

The base perimeter = the square perimeter = 4X3

$$LA = 48\text{cm}^2$$

The total area

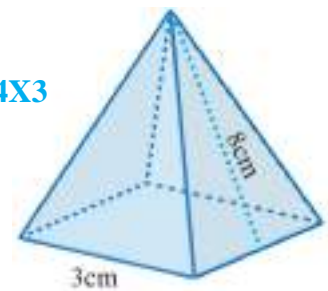
$$TA = \frac{1}{2}p \times \ell + b$$

The base area = the square area = 3X3

$$TA = 48 + 9 = 57\text{cm}^2$$

The total area

$$TA = 57\text{cm}^2$$



Example (4): Use the nearby figure to find i)The lateral area ii)The total area iii)The volume

$$i) LA = \pi r \times \ell$$

$$= \pi \times 3 \times 5 = 15\pi\text{cm}^2$$

The lateral area of the cone

By substituting and simplifying

$$ii) TA = \pi r \times \ell + \pi r^2$$

$$= 15\pi + 9\pi = 24\pi\text{cm}^2$$

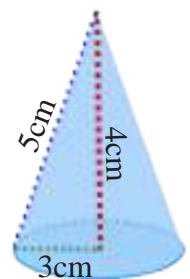
The total area of the cone

By substituting and simplifying the cone volume

$$iii) V = \frac{1}{3}\pi r^2 \times h$$

$$= \frac{1}{3} \times \pi \times 9 \times 4 = 12\pi\text{cm}^3$$

By substituting and simplifying



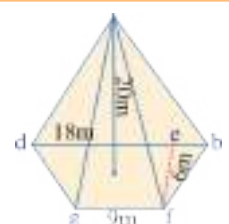
Example (5): Find the volume of the nearby pyramid

$$b = \frac{1}{2}(gf + bd) \times fe = \frac{1}{2}(9 + 18) \times 6 = 81\text{m}^2$$

The area of the trapezoid

$$V = \frac{1}{3}b \times h = \frac{1}{3} \times 81 \times 20 = 540\text{m}^3$$

The pyramid volume



Example (6): Find the volume of the nearby compound polyhedron.
To find the volume of the compound polyhedron, we firstly find the volume of the cylinder and the cone, after that, we add the volumes to find the volume of the compound polyhedron.

$$V_1 = \pi r^2 h \Rightarrow V_1 = 36\pi \times 20$$

$$V_1 = 720\pi \text{ cm}^3$$

$$V_2 = \frac{1}{3} r^2 \pi \times h$$

$$V_2 = \frac{1}{3} \times 36\pi \times 30 = 360\pi \text{ cm}^3$$

$$V = V_1 + V_2$$

$$V = 720\pi + 360\pi = 1080\pi \text{ cm}^3$$

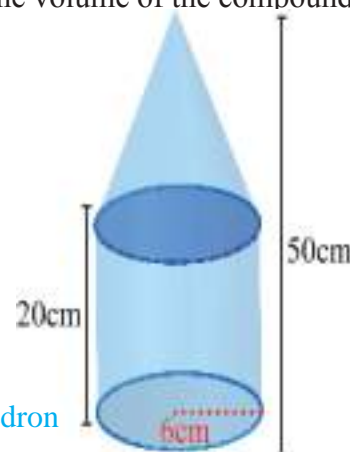
The formula of the cylinder volume

By substituting and simplifying

The formula of the cone volume

By substituting and simplifying

The volume of the compound polyhedron



Make sure of your understanding

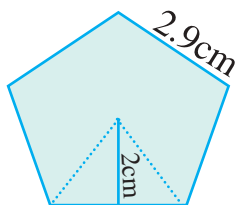
Find the perimeter and the area for each regular polygon:

Questions 1- 2 are similar to example 1 .

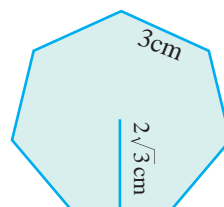
Questions 3 - 4 are similar to examples 6

Questions 6 - 7 are similar to example 5.

1



2



3

Find the volume , the lateral area and the total area for each of the following:

- Right circle cone: its base area is $225\pi \text{ cm}^2$, its base perimeter is $30\pi \text{ cm}$, its height is 20 cm and its lateral height is 25 cm.
- A pyramid; its base area is $54\sqrt{3} \text{ cm}^2$, its perimeter is 36 cm, its height is $3\sqrt{6} \text{ cm}$ and its lateral height is 9 cm.

4

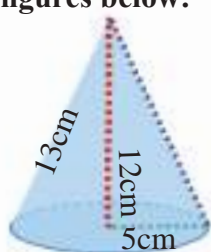
Find the volume and the lateral area and the total area for each of the following:

- Right circle cone; its radius is 7 cm, its height is $\sqrt{5} \text{ cm}$ and its lateral height is 10 cm.
- A pyramid has a square base, the length of its side is 12 cm, its height is 8 cm and its lateral height is 10 cm.

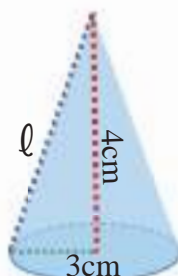
5

Find the volume and the lateral area and the total area for each of the following by using the figures below:

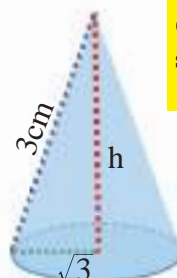
i)



ii)

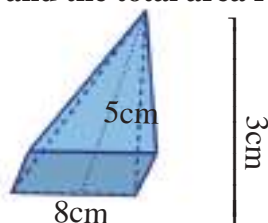


iii)



6

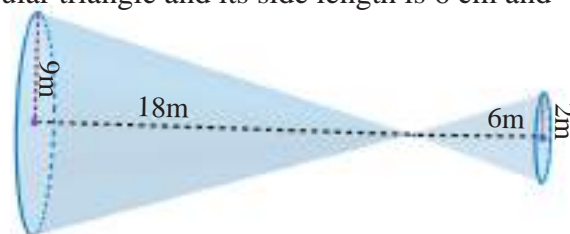
Find the volume and the lateral area and the total area for each of the following by using the figures below:



Its base is a square

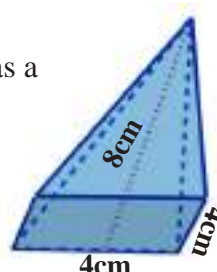
Solve the Exercises

- 7 Find the lateral area of the pyramid which has a square- shaped base, its side length is 8 cm and its lateral height is 7.2 cm.
- 8 Find the lateral area of the pyramid which has a base of a regular octahedral polygon, its side length is 1.16 cm and its lateral height is 2 cm.
- 9 Find the lateral area and the total area of a right circle cone which its diameter is 35m and its lateral height is 20m, and write the answer by π .
- 10 Find the volume of a pyramid which its base is a regular triangle and its side length is 6 cm and its height is 13cm
- 11 Find the volume of the nearby figure Composite form.



Solve the problems

- 12 **Science:** A volcanic model in a shape of a right circle cone, Its radius length is 3 cm, if the model volume is about 203 cm^3 , what is its height?
- 13 **Construction:** The height of the Arab tower is 321m and it represents an arched pyramid, calculate the approximate area of its base if the pyramid volume which represents it is 1904000 m^3 .
- 14 **Geometry:** Find the lateral area of the pyramid which has a square-shaped base, as it is shown in the nearby figure.



Think

- 15 **Challenge:** A cone and a cylinder have the same base and volume, the cylinder diameter is 40 cm and its height is 7 cm, what is the lateral area of the cone?
- 16 **Discover the mistake:** Which of the two solutions is wrong? Clarify your answer.

The first solution

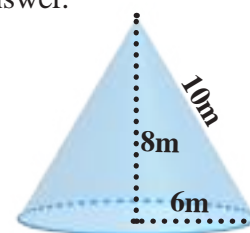
$$V = \frac{1}{3} \times b \times h$$

$$V = \frac{1}{3} \times 36\pi \times 10 = 120\pi \text{ m}^3$$

The second solution

$$V = \frac{1}{3} \times b \times h$$

$$V = \frac{1}{3} \times 6 \times 6 \times \pi \times 8 = 96\pi \text{ m}^3$$



Write

A problem about a regular polygon which its data allows to find the perimeter and the area of the polygon.

Lesson [5-2]

Triangles

Idea of the lesson:

-Identifying the triangles bisections of the angles, and median of a triangle, the similarity of two triangles and using the similarity in solve problems.

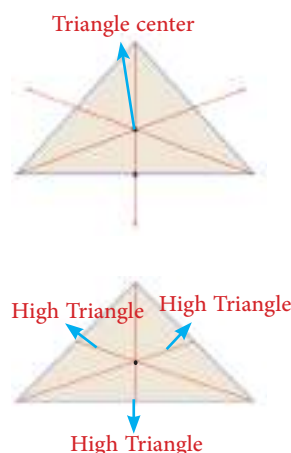
Vocabulary:

- The two similar triangles
- The similarity ratio

Learn

You have previously learned the properties of the triangle. In this lesson, we will learn the median of a triangle: it is a segment, its two ends represent one of the triangle's vertices and the midpoint of the side which is opposite of that vertices. Each triangle has three median intersect in one point which is called the meeting point of the median of the triangle (the triangle center).

The triangle height: is the vertices descending column from one vertex of the triangles on the line which contains the side which is opposite of that vertex, each triangle has three heights which are intersected in one point called (the meeting point of the heights)



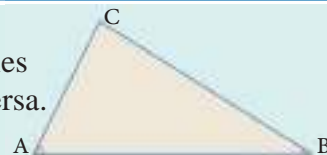
[5-2-1]

Sides and Angles in the triangle

(theorems without reasoning)

A theorem : If two sides of a triangle were variated, then the two opposite angles will be variated too, the great one will be opposite of the great side and vice versa.

$$BC > AC \Leftrightarrow m\angle A > m\angle B$$



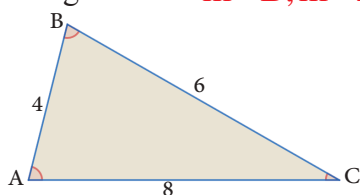
Example(1)

- In the triangle below, arrange the angles from the smallest to the greatest.
- In the triangle below, arrange the sides from the shortest to the longest and calculate the measure of $\angle C$.

The shortest side AB, then the small angle is $\angle C$.

The longest side AC, then the great angle is $\angle B$.

The arrangement is $m\angle B, m\angle A, m\angle C$

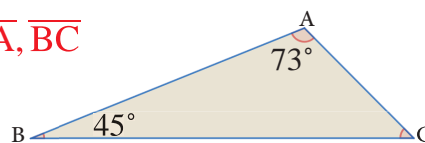


$$m\angle A + m\angle B + m\angle C = 180^\circ$$

$$m\angle C = 180^\circ - (73^\circ + 45^\circ) = 62^\circ$$

$$\therefore m\angle B < m\angle C < m\angle A$$

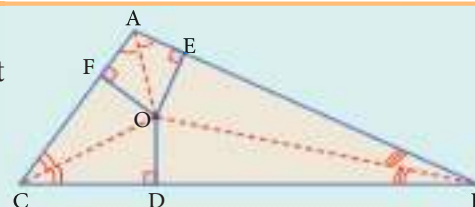
$$\overline{AC}, \overline{BA}, \overline{BC}$$



(theorems without reasoning) in each triangle:

A theorem : The bisectors of the triangles angles meet in one point which is in equidistant from its sides.

If \overline{OA} , \overline{OB} , \overline{OC} , are bisectors of the angles A, B, C respectively, they meet in the point O, then $OD = OE = OF$



Example(2)

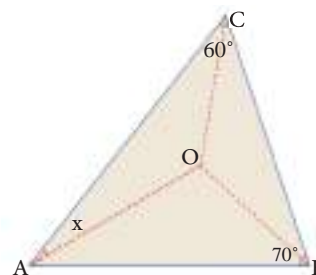
In the nearby triangle, find the value of x.

\overline{BO} bisectors $\angle B$, \overline{CO} bisects $\angle C$, then O is the meeting point of the bisectors of the angles of the triangle ABC

(\overline{AO} bisects $\angle A$) $x = \frac{1}{2} m\angle A$ theorem

The sum of the triangle angles $m\angle A + m\angle B + m\angle C = 180^\circ$

$$m\angle A = 180^\circ - (70^\circ + 60^\circ) = 50^\circ \Rightarrow \therefore x = 25^\circ$$

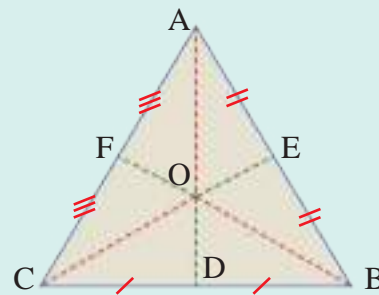


(theorems without reasoning) in each triangle:

A theorem: The medians of the triangle are meet in one point called the center of triangle gravity. It divides each of them in a ration of $\frac{2}{3}$ from the vertices.to tenmid point of the side which is opposite of the vertices.

$$AO = \frac{2}{3}AD, BO = \frac{2}{3}BF, CO = \frac{2}{3}CE$$

$$OD = \frac{1}{3}AD, OF = \frac{1}{3}BF, OE = \frac{1}{3}CE$$



Example(3)

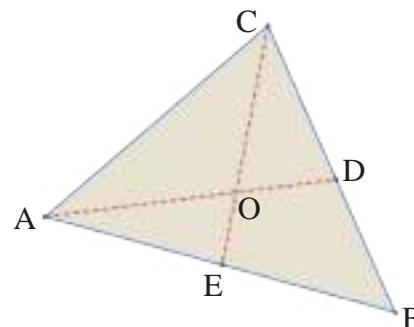
The triangle ABC in which the two medians $\overline{AD}, \overline{CE}$ meet in the point O, $CE=9$ cm, $AD=6$ cm, find the length of $\overline{AO}, \overline{OE}$

$$OE = \frac{1}{3}CE$$

$$\therefore OE = \frac{1}{3} \times 9 = 3\text{cm} \quad \text{A median of } \overline{CE}$$

$$\therefore OA = \frac{2}{3}AD \quad \text{also a median of } \overline{AD}$$

$$\therefore OA = \frac{2}{3} \times 6 = 4\text{cm}$$



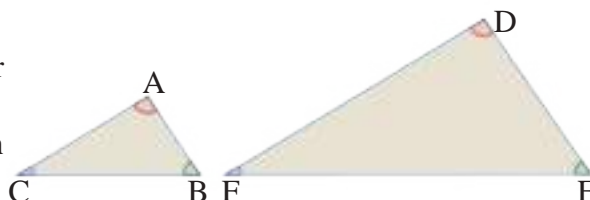
[5-2-2]

Similarity of Triangles

The two similar triangles: are triangles which their sides are proportioned and their angles are congruent, we refer to the similarity by (\sim) theorems without reasoning.

A theorem: If two angles in a triangle are congruent with two angles in another triangle, then the two triangles are similar. $m\angle A = m\angle D, m\angle C = m\angle F, \Rightarrow \triangle ABC \sim \triangle DEF$

A theorem : If three sides of a triangle are proportioned with three sides of another triangle, then the two triangles are similar.



Example(4)

Show if the two triangles in the nearby figure are similar, and write the ratio of the similarity.

i) $\frac{AB}{DE} = \frac{6}{9} = \frac{2}{3}$
 $\frac{AC}{EF} = \frac{4}{6} = \frac{2}{3}$
 $\frac{BC}{FD} = \frac{4}{6} = \frac{2}{3}$

Then the two triangles are similar

ii) $\frac{BC}{EF} = \frac{3}{10}$
 $\frac{AB}{DF} = \frac{4}{6} = \frac{2}{3}$
 $\frac{BC}{EF} \neq \frac{AB}{DF}$

Then the two triangles are not similar

A theorem: If two sides of a triangle the angle were proportioned with the two sides of another triangle and the angle between them is congruent with the angle of the other triangle and the sides, then the two triangles are similar.

Example(5)

In the nearby figure: if $(m\angle C = m\angle FDB, \frac{EC}{FD} = \frac{CD}{DB})$ find the value of x. Since the two triangles BFD, DEC are similar, then their symmetric sides are proportion

$$\frac{x-1}{2} = \frac{9}{3}$$

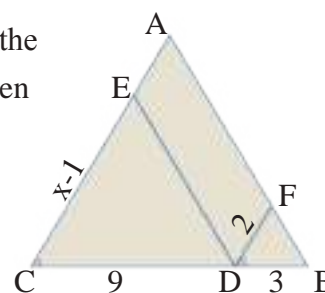
$$3x - 3 = 18$$

$$3x = 21 \Rightarrow x = 7$$

Proportion

Commutative multiplication

Simplifying

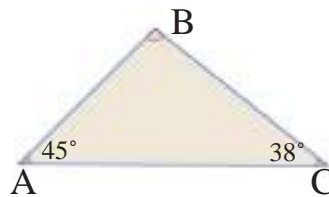


Make sure of your understanding

1 Arrange the sides from the shortest to the longest:

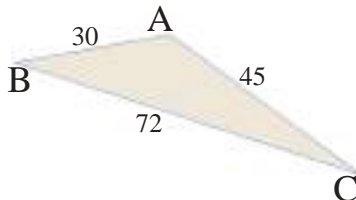


2

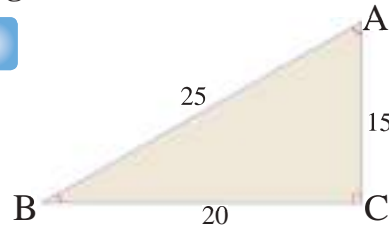


Questions 1- 4 are similar to example 1

3 Arrange the angles from the smallest to the greatest:



4



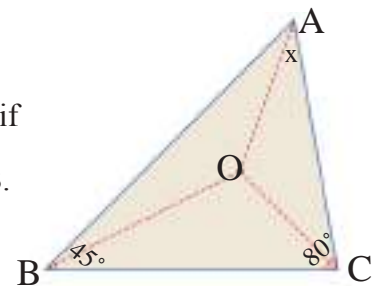
Question 5 is similar to example 2

5 In the nearby triangle, if \overline{AO} , \overline{BO} , \overline{CO} are bisectors of the angles A, B, C find $m\angle X$.

6 ABC is a triangle, O represent a point of intersection of its median, if $BO = 12\text{cm}$, Find the length of the median which one of its to ends is the point B.

7 In the triangle ABC, O is the point of meeting the medians, find the length of \overline{AD} if your knew that

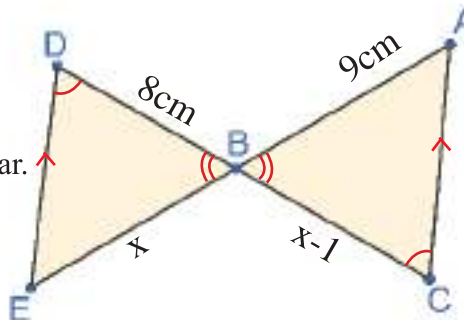
$m\angle COB = 90^\circ$, $\overline{AO} \cap \overline{BC} = \{D\}$, $BC = 6\text{cm}$



Questions 6 - 7 are similar to example 3

8 In the nearby figure:
Show that:

- The two triangles ABC, BDE are similar.
- Find the value of x.
- Find the ratio of the similarity.

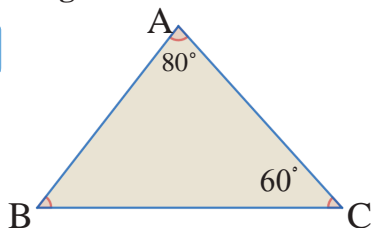


Question 8 is similar to examples 4,5

Solve the Exercises

Arrange the sides from the shortest to the longest:

9

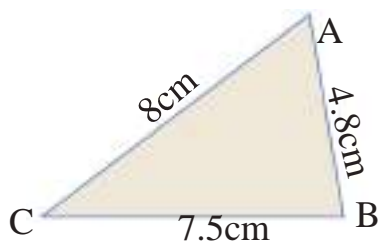


10

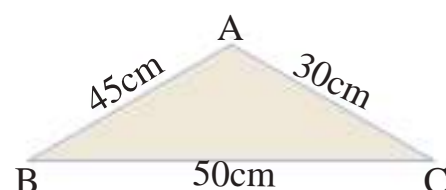


Arrange the angles from the smallest to the greatest:

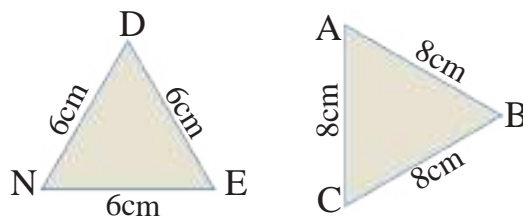
11



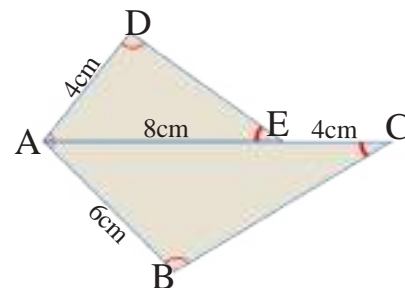
12



- 13 Show if the two triangle ABC, DEN, in the nearby figure, are similar, and write the ratio of similarity, then name the pairs of the congruent angles.

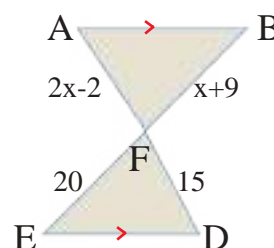


- 14 Show if the two triangles ABC, ADE, in the nearby figure, are similar, and write the ratio of the similarity.

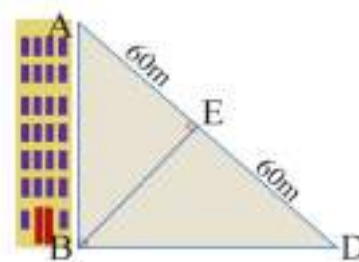


Solve the problems

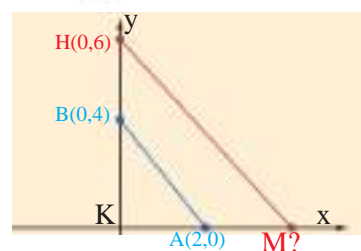
- 15 **Geometry:** If you knew that $\triangle ABF \sim \triangle DEF$, $\overline{AB} \parallel \overline{ED}$ use the information in the nearby figure to find the value of x.



- 16 **Building:** A building which its height represented by a side right_angled triangle as it is shown BE is tringle ABD high in the nearby figure, prove that :
i) $\angle EBA \cong \angle D$
ii) $\triangle ABE \sim \triangle DBE$

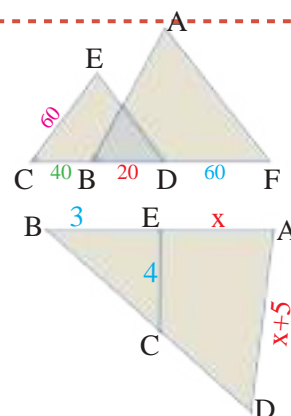


- 17 In the nearby figure, the two triangle KAB, KMH are similar, find the two coordinates of M and the ratio of the similarity.



Think

- 18 **Discover:** What is the length of \overline{AB} in the nearby figure? If you knew that $\triangle ECD \sim \triangle ABF$.
- 19 **Challenge:** (x,15,6) and (10,5,2) are the lengths of the corresponding sides in two similar triangles, what is the value of x?
- 20 **Numerical sense:** Find the value of x in the nearby figure. If the two tringle ABD, EBC are similar $EC \parallel ED$
- 21 **Opened problem:** Illustrate why we need the measurements of the angles to be sure of the similarity of the triangles, give an example.



write

A problem about two isosceles triangles in which the two angles of the vertices are congruent, and find the ratio of the similarity.

Lesson [5-3]

Proportion and Measure in Triangles

Idea of the lesson:

-Using the proportioned parts in the triangles to prove the parallel of two or more of lines.

-Using the proportion to find unknown measures

-Using the geometric proportion in the coordinate plane

Vocabulary:

Geometric proportion

Learn

The planning of cities and streets in the map of the electronic appliance map include parallel and perpendicular lines, the lateral planning represents part of Baghdad city, we see that the streets are parallel and perpendicular.



[5-3-1] Proportion in Triangles

You have previously learned the similar triangles and some of the theorems of the similarity in the triangles, you will learn, in this term, the proportion in the triangle using the previous theorems.

The triangular proportion theorem

The theorem	Given	The result
If a line parallels a side of a triangle and intersects the other two sides in two different point.it divides the two sides into segments which are proportional in (without reasoning)	$\overline{AB} \parallel \overline{EF}$	$\frac{CE}{EA} = \frac{CF}{FB}$

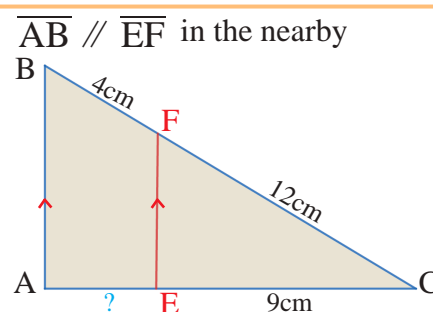
Example(1)

Find the length of the segment AE, if you knew that $\overline{AB} \parallel \overline{EF}$ in the nearby figure.

$$\frac{CE}{EA} = \frac{CF}{FB}$$

$$\frac{9}{EA} = \frac{12}{4} \Rightarrow EA = \frac{4 \times 9}{12} = \frac{36}{12} = 3\text{cm}$$

A theorem of the triangular proportion the substitution and simplifying



Converse the theorem of the triangular proportion

The theorem	Given	The result
If a line divided two sides of a triangle into proportioned segments, then it will be parallel to the third side (without reasoning)	$\frac{CE}{EB} = \frac{CF}{FA}$	$\overline{EF} \parallel \overline{AB}$

Example(2)

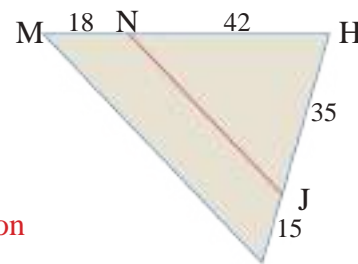
In the nearby figure, prove $\overline{MK} \parallel \overline{NJ}$.
we find the ratio of the proportional parts

$$\frac{HJ}{JK} = \frac{35}{15} = \frac{7}{3}, \frac{HN}{NM} = \frac{42}{18} = \frac{7}{3}$$

$$\therefore \frac{HJ}{JK} = \frac{HN}{NM} = \frac{7}{3}$$

$$\therefore \overline{MK} \parallel \overline{NJ}$$

converse the theorem of the triangular proportion

**Thales theorem**

The theorem	Given	The result
If three or more parallel lines were intersected by two lines, then the segments which are intercepted by the parallel lines will be proportional	$\overrightarrow{AD} \parallel \overrightarrow{BE} \parallel \overrightarrow{CE}$	$\frac{AB}{BC} = \frac{DF}{FE}$

Example (3):

An architect had used the perspective (which is the drawing of the remote bodies where they seem small, and the near bodies where they seem great, and maintaining their structure and the proportion of their measures to seem three-dimensions) to draw initial lines to help him in drawing parallel columns of communication, he checked his drawing in measuring the distance between the columns, how long \overline{FH} ?

$$\overrightarrow{AE} \parallel \overrightarrow{BF} \parallel \overrightarrow{CJ} \parallel \overrightarrow{DH}$$

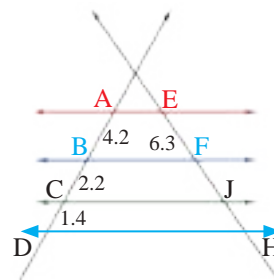
Thale's theorem

$$\frac{AB}{BD} = \frac{EF}{FH}$$

$$BD = BC + CD = 2.2 + 1.4 = 3.6\text{m}$$

By substituting and simplifying

$$\frac{4.2}{3.6} = \frac{6.3}{FH} \Rightarrow FH = \frac{6.3 \times 3.6}{4.2} = 5.4\text{m}$$

**[5-3-2]****Proportion and Measures**

To find the ratio of the two perimeters and the ratio of the two areas of two similar triangles, I can use the following theorem (without reasoning)

A theorem: if two triangles are similar in ratio of $\frac{a}{b}$, then the ratio of the two perimeters of the two triangles equals $\frac{a}{b}$ and the ratio of the two triangle area equals $\frac{a^2}{b^2}$.

If the two triangles are similar, then the ratio between their two perimeters equals The ratio between the length of the corresponding.

Example(4)

If $\triangle WVT \sim \triangle ABC$, find the perimeter of the $\triangle ABC$.
Assume P_1 is the perimeter of the triangle WVT

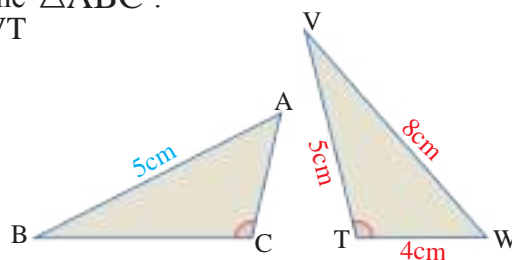
$$P_1 = 8 + 5 + 4 = 17\text{cm}$$

Use the proportion to find the perimeter of the triangle ABC
assume P_2 is the perimeter of

$$\frac{P_2}{P_1} = \frac{AB}{WV} \Rightarrow \frac{P_2}{17} = \frac{5}{8}$$

$$\therefore P_2 = 10.625\text{cm}$$

The triangle ABC



You have previously learned three geometric transformations: the translation with drawaill, the reflection and rotation, these transformations maintain the structure and the measures. In this lesson, you will learn new transformation maintain the structure without the measures, it is the dilation.

[5-3-3]

Dilation in the Coordinate Plane

Dilation: is a transformation which change the measures of the geometric shapes without changing their structure. In the dilation, the preimage and its image will always be similar, the proportion center is the origin.

In this lesson, we will only study the dilation in the coordinate plane, if you deal with a dilation with scale factor M, you will be able to find the image of the point by multiplying its coordinates by M
 $(x, y) \rightarrow (Mx, My)$

Example(5)

the nearby figure shows a position of a picture in the internet, draw the limits of the picture after transform it in a dilation its ratio is $\frac{5}{3}$

The first step:

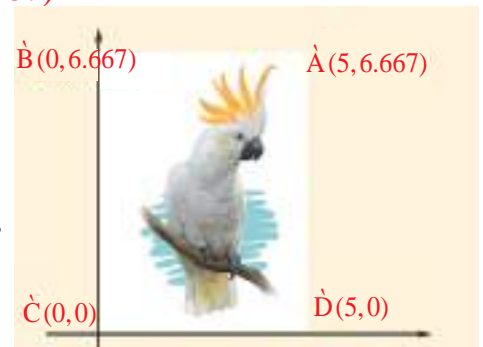
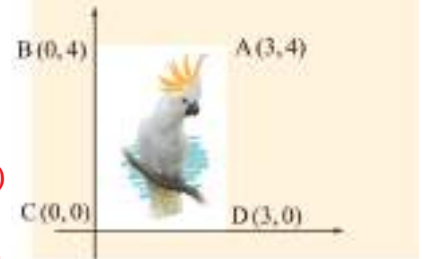
Multiplying the factor of the dilation by the coordinates of the vertices.

$$A(3,4) \rightarrow \left(\frac{5}{3} \times 3, \frac{5}{3} \times 4\right) \rightarrow \hat{A}(5, 6.667)$$

$$B(0,4) \rightarrow \left(\frac{5}{3} \times 0, \frac{5}{3} \times 4\right) \rightarrow \hat{B}(0, 6.667)$$

$$C(0,0) \rightarrow \left(\frac{5}{3} \times 0, \frac{5}{3} \times 0\right) \rightarrow \hat{C}(0,0)$$

$$D(3,0) \rightarrow \left(\frac{5}{3} \times 3, \frac{5}{3} \times 0\right) \rightarrow \hat{D}(5,0)$$

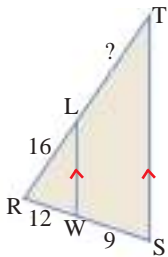


The second step: put the points $\hat{A}, \hat{B}, \hat{C}, \hat{D}$ on the coordinate plane, and then connecting them to get the rectangle $\hat{A} \hat{B} \hat{C} \hat{D}$

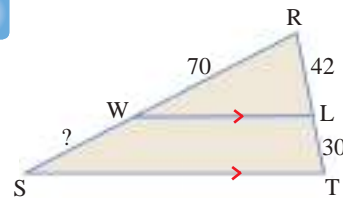
Make sure of your understanding

Find the length of the unknown segment in the following figures:

1



2



Questions 1 - 2 are similar to examples 1 - 3

Questions 3 - 4 are similar to example 2

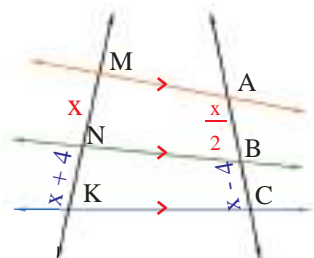
3

In the triangle MQP, $MQ = 12.5$, $\overline{MR} = 4.5$, $MP = 25$, $MN = 9$, does $\overline{RN} \parallel \overline{QP}$ or not?

Explain your answer. $N \in \overline{MP}$, $R \in \overline{MQ}$

4

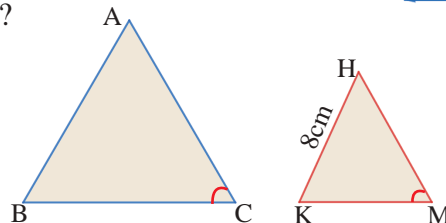
In the nearby figure, find the length of \overline{KN} , \overline{MN} .



Question 4 is similar to example 3

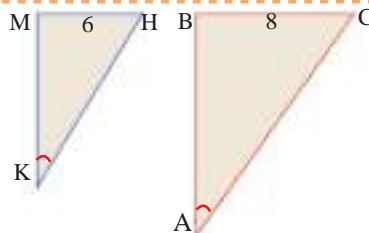
5

The two triangles ABC, HKM are similar, the area of $\triangle ABC$ is the double of the area of $\triangle HKM$, what is the length of \overline{AB} ?



Questions 5, 6 are similar to examples 4,5

- 6 The two triangles ABC, KMH are similar, find the area and the perimeter of the triangle ABC, it should be taken in consideration that the perimeter of the triangle KMH equals 18 cm and its area 15 cm².



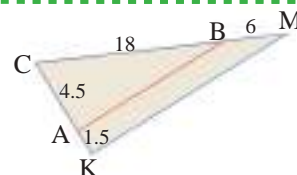
Question 6 is similar to example 4

Question 7 is similar to example 5

- 7 ABC is a triangle, where $A(6, 0)$, $B(-3, \frac{3}{2})$, $C(3, -6)$, find its image after minimizing it in $\frac{1}{3}$, it is worth to be mentioned that the proportion center is the origin.

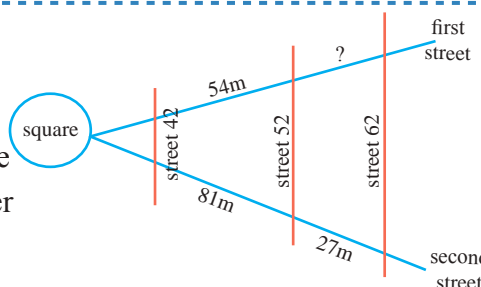
Solve the Exercises

- 8 In the triangle ACD, $\overline{BE} \parallel \overline{CD}$, find the value of x and \overline{ED} if $\overline{ED} = 3x - 3$, $BC = 8$, $AE = 3$, $AB = 2$.
- 9 Determine if $\overline{AB} \parallel \overline{MK}$ in the nearby figure.
- 10 The area ratio of the triangle ABC to the area ratio of the triangle KMH equals $\frac{16}{25}$, what is the ratio of the similarity between the two triangles, and what is the ratio of the similarity between their two perimeters.
- 11 Find the image of the triangle ABC, where $A(-1, -1)$, $B(1, -2)$, $C(1, 2)$ under the effect of the proportion of its factor 2.



Solve the problems

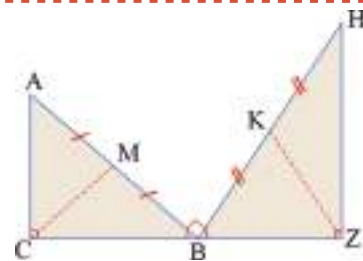
- 12 **Roads:** The nearby map represents some of the parallel streets and two ways crossing them, what is the length of the first way between the street number 62 and the street number 52?



- 13 **Geometry:** Find the image of the quadrangle, where $A(2, 6)$, $B(-4, 0)$, $C(-4, -8)$, $D(-2, -12)$ under the effect of the proportion of its factor $\frac{1}{4}$.

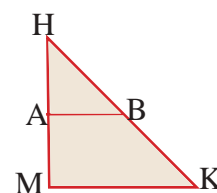
Think

- 14 **Challenge:** In the nearby figure, M is the midpoint of \overline{AB} , and K is the midpoint of \overline{HB} , the angles $\angle Z$, $\angle ABH$, $\angle C$ are right angles, prove that $\left(\frac{KZ}{CM}\right)^2 = \frac{(BZ)^2 + (ZH)^2}{(BC)^2 + (CA)^2}$



Write

As you can of the dilations, if you knew that $\overline{MK} \parallel \overline{AB}$ in the near by figure.



Lesson [5-4]

The Circle

idea of the lesson:

-Finding the measure of the arcs and the central angles of the circles.

Identifying the tangent and the common tangent

Vocabulary:

- Arc chord
- Tangent, common tangent
- Central angles

Learn

Every angle between clock hand is a central angle. The central angle is the angle which intersects the circle in two points and its vertex is the center of the circle, and every central angle in a circle, an arch would be in front of it, it is called the angle's arc. What is the measure of \widehat{AB} which is in front of $\angle AOB$? Are there many types of arcs?



[5-4-1] Arch and chord

You have previously learned the concept of the circle: it is a set of the connected points in the plane which have the same distance from a fixed point called the circle center, and the radius r : is a segment connects between the circle center and a point on the circle, and the circle chord: is a segment which its two ends are on the circle, and the diameter : is a chord passes through the circle center.

In this lesson, you will increase your information by identifying the arc and its measure by using the central angle which is in front of it.



Example(1)

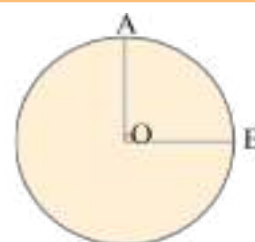
How can I find the measure of the arc \widehat{AB} by using the central angle which is in front of it?

The measure of the central angle is equivalent to the arc measure which is in front of it, and we refer to the arc \widehat{AB}

The angle AOB is a right angle $m\angle AOB = 90^\circ$

Then the measure of the arc which is in front of the angle AOB equals $\widehat{AB} = 90$

There are three types of the arcs in the circle:



The smallest arc (less than 180)	The greatest arc (greater than 180)	Measure of the half of the circle (equals 180)
 $m\widehat{ACB} = m\angle AOB < 180$	 $m\widehat{ACB} = 360 - m\widehat{AB} > 180$	 $m\widehat{AB} = 180$

Example(2)

Find the measure of the angles and the unknown arcs in the nearby figure:

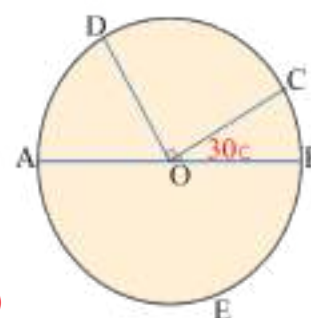
i) \widehat{BC} : $m\angle BOC = 30^\circ \Rightarrow m\widehat{BC} = 30$

ii) \widehat{DC} : $m\angle COD = 90^\circ \Rightarrow m\widehat{DC} = 90$

iii) \widehat{BCD} : $m\angle BOC + m\angle COD = 30^\circ + 90^\circ = 120^\circ$
 $m\widehat{BCD} = 120$

iv) \widehat{BEA} : $m\angle BOA = 180^\circ \Rightarrow m\widehat{BEA} = 180$

v) \widehat{AD} : $m\angle AOD = 180^\circ - 120^\circ = 60^\circ \Rightarrow m\widehat{AD} = 60$



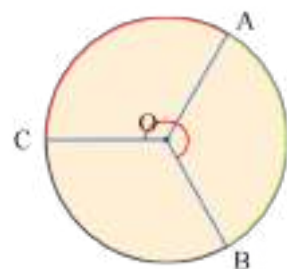
Example (3): The nearby circle is divided into three congruent parts, find the measure of the following arcs:

There are three congruent central angles, its sum is 360° :

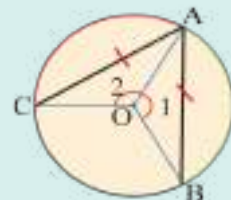
i) \widehat{AB} : $m\angle AOB = \frac{360^\circ}{3} = 120^\circ \Rightarrow \widehat{AB} = 120$

ii) \widehat{ABC} : $m\angle ABC = 120^\circ + 120^\circ = 240^\circ \Rightarrow \widehat{ABC} = 240$

or other method $\widehat{ABC} = 360^\circ - 120^\circ = 240^\circ \Rightarrow \widehat{ABC} = 240$



Notice the two triangles and the two central angles 1,2 and the two arcs \widehat{AB} , \widehat{CA} and the two chords \overline{AB} , \overline{CA} , if the two angles are congruent, then the two arcs will be congruent and the two triangles will be congruent, so the two chords \overline{AB} , \overline{CA} will be congruent. You can use a method like this to get the following theorem. (without reasoning).



The theorem of arcs, chords and the central angles in each circle or in two congruent circles.

If two central angles are congruent, then their two arcs will be congruent and vice versa. $\angle 1 \cong \angle 2 \Leftrightarrow \widehat{AB} \cong \widehat{AC}$

If two central angles are congruent, then their two arcs will be congruent and vice versa. $\angle 1 \cong \angle 2 \Leftrightarrow \widehat{AB} \cong \widehat{AC}$

If two arcs are congruent, then their two chords will be congruent and vice versa. $\widehat{AB} \cong \widehat{AC} \Leftrightarrow \overline{AB} \cong \overline{AC}$

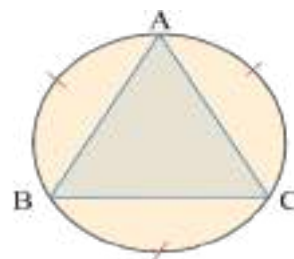
Example (4): Use the theorem of the arcs and the chords to prove that the triangle ABC is an equilateral triangle in the nearby circle, it is worth to be mentioned that:

Given from the question

The theorem of the arcs and the chords

$$\begin{aligned} \widehat{AB} &\cong \widehat{AC} \cong \widehat{CB} \\ \therefore \overline{AB} &\cong \overline{AC} \cong \overline{CB} \\ \therefore \overline{AB} &\cong \overline{AC} \cong \overline{CB} \end{aligned}$$

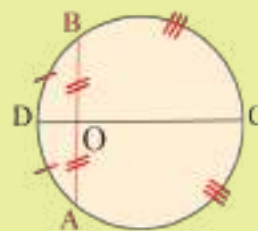
So, the triangle ABC is an equilateral triangle.



A theorem of the perpendicular diameter in each circle

A theorem: The perpendicular diameter to a chord in a circle bisects the chord and the two arcs.

$$\overline{CD} \perp \overline{AB} \Rightarrow AO = BO, \widehat{AD} \cong \widehat{DB}, \widehat{BC} \cong \widehat{AC}$$



Example (5): Use the theorem of the perpendicular diameter to find the chord AB, if you knew that the radius OD equals 5 cm and that DE = 2 cm

OC = OD = 5 cm, DE = 2 cm **The step (1):** draw the radius \overline{OC}

OE = 5 - 2 = 3 cm

$(EB)^2 + (EO)^2 = (OB)^2$

$25 - 9 = (EB)^2$

$(EB)^2 = 16 \Rightarrow EB = 4 \text{ cm}$

$\therefore AB = 2 \times EB = 2 \times 4 = 8 \text{ cm}$

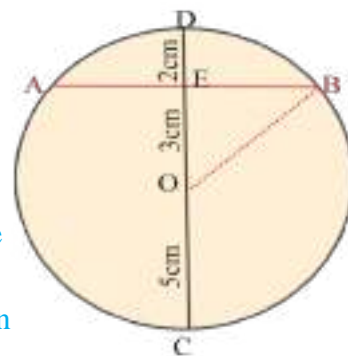
The step (2):

Pythagorean theorem by substitution

By simplifying

E is the midpoint of \overline{AB} , the theorem of the perpendicular diameter

The diameter \overline{DC} is perpendicular on the chord \overline{AB} and bisects it.



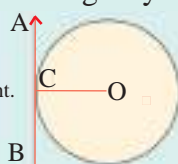
[5-4-2]

Tangent

The circle tangent: is the line which meets the circle in one point and it is vertically to the radius in the tangency point.

The tangency theorem

tangency point.



The common tangent of two circles :is tangent line for the two circles.

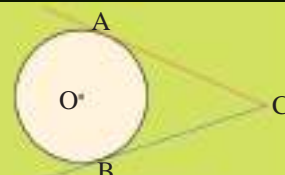


The theorem of the two tangents

A theorem: The two segments of the tangents which are drawn to a circle from outside point are identical.

\overline{CB} , \overline{CA} are two tangents to the circle from the point C.

$$\therefore \overline{CB} \cong \overline{CA}$$



Example (6): A circle, in the nearby figure, its center is O, and \overline{AB} is a tangent of the circle at A and the measure of the angle ABO equals 35° , find the measure of the angle AOB, then find the length of the segment BC.

$$\overline{AB} \perp \overline{AO}, m\angle OAB = 90^\circ$$

\overline{BA} is the circle tangent at the point A

The tangent theorem

Given

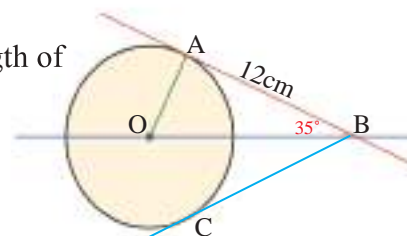
The sum of the triangle angles 180°

The theorem of the two tangents

$$\therefore m\angle OBA = 35^\circ$$

$$\therefore m\angle AOB = 180^\circ - (90^\circ + 35^\circ) = 55^\circ$$

$$BC=12\text{cm}$$



Make sure of your understanding

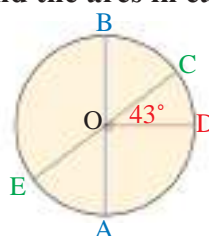
In the circle below, find the measure of the angles and the arcs in each of the following:

1 $\angle AOD$

2 $\angle COB$

3 \widehat{DBE}

4 \widehat{DAB}



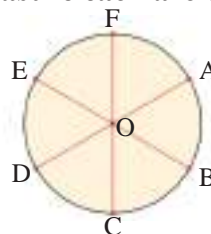
Questions 1- 4 are similar to examples 1- 2

A circle is divided into 6 identical parts, find the measure each arc of the following:

5 \widehat{AB}

6 \widehat{ABC}

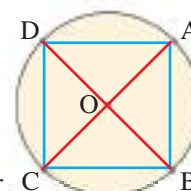
7 \widehat{ABD}



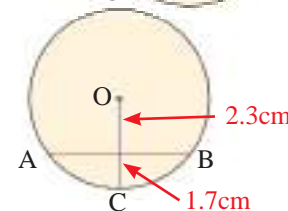
Questions 5- 7 are similar to example 3

Question 8 is similar to example 4

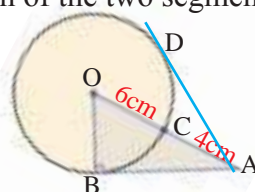
8 The nearby circle is divided into 4 identical parts, prove that the figure ABCD is a square .



9 In the nearby figure, use the theorem of the perpendicular diameter, and find the length of the segment AB in the nearby circle. Approximate the result to the nearest tenth.



10 Use the tangent theorem to find the length of the two segments AB, AD in the nearby figure.



Question 9 is similar to example 5

Question 10 is similar to example 6

Solve the Exercises

Find the measure of the angles and the arcs in the following:

11 $\angle COA$ 12 \widehat{DBE}

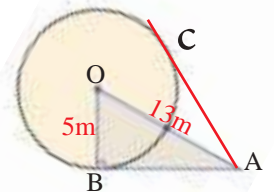
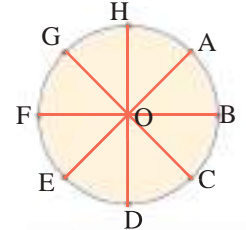
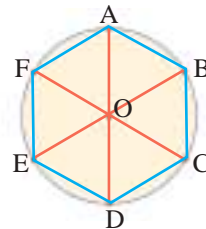
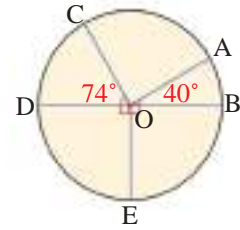
13 \widehat{BAC} 14 \widehat{DCA}

The circle is divided into 8 identical parts, find the measure of each arc in the following:

15 \widehat{AB} 16 \widehat{ABC} 17 \widehat{GDB}

18 The nearby circle is divided into 6 identical parts, prove that the figure ABCDEF is a regular hexagonal.

19 Use the theorem of the tangent to find the length of the two line segments AB, AC in the nearby circle.



Solve the problems

20 **Geography (volcanos):** The vent of the Hulaii volcano rises from the level of the sea surface in 2.52 km, calculate the distance between the top of the volcano and the level of the horizon, if you knew that the radius of the land is 6437 km, approximate the result to the nearest kilometer.



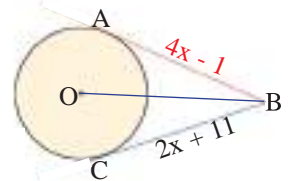
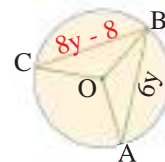
21 **Space station:** Russian station, mir rises from the level of the sea surface in about 390 km, what is the distance between the station and the horizon, approximate the result, if you know that the radius of the land is 6437 km to the nearest kilometer.



Think

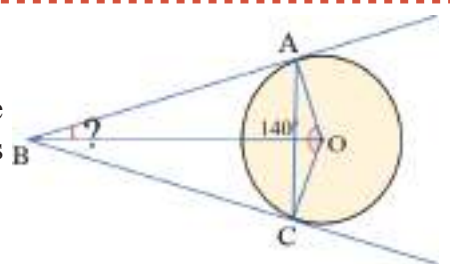
22 **Challenge:** Use the theorem of the two tangents to find the length of \overline{AB} in the nearby circle.

23 **Numerical sense:** If the two angles COB, AOB are congruent, find the length of \overline{CB} in the nearby circle.



Write

The required steps to find the measure of the angle ABC in the nearby figure, if you knew that \overline{BO} bisects the angle AOC which its measure equals 140°



Lesson [5-5]

Triangle and Circle, Line Segments and Circle

Idea of the lesson:

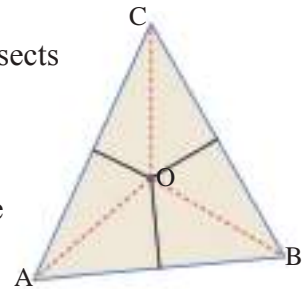
- Using the properties of the axes and the bisectors of the angles to draw the circumscribed circle and the inscribed circle in a triangle
- Finding the lengths of the segments which determine by two lines on a circle.

Vocabulary:

- Circumscribed circle
- Inscribed circle

Learn

- In the nearby $\triangle ABC$, the axis of BC intersects the axis of AB in O
- $OC=OB$ because O in the axis of BC
- $OA = OC$
- So O located on the axis of AC, that is the axis of AC passes through O
- $\therefore OA = OB = OC$
- We can draw a circle, its center is O and passes through the vertices of the triangle ABC



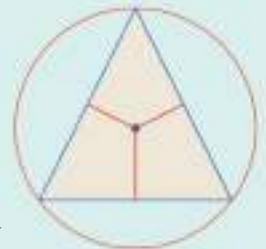
[5-5-1] Triangle and Circle

(the median of the triangle)

(The axes of the three sides of the triangle intersect in one point)

And by them, we can draw the circumscribed circle of the triangle.

The circumscribed circle: of each triangle has one circumscribed circle, its center is a point of intersection the three axes.



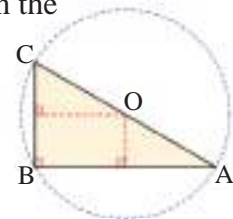
The axes: are the perpendiculars on sides of a triangle from their midpoint, and they meet in one point (O) which has the same distance from its vertices, this point is the center of the circle which passes through the vertices of the triangle.

Example (1): Find the point of intersection of the axes of the triangle ABC as in the nearby figure, and draw the circumscribed circle.

The axis of \overline{AB} passes through the midpoint of \overline{AB} and parallels \overline{BC}

The axis of \overline{BC} passes through the midpoint of \overline{BC} and parallels \overline{AB}

Then the three axes meet in the midpoint of \overline{AC} which represent the center of the circumscribed circle of the triangle.

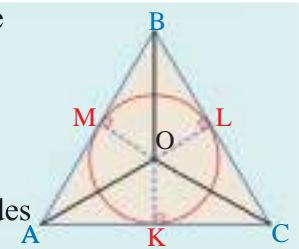


We can benefit from the theorem of the bisectors of the triangles angles to draw the inscribed circle of the triangle

The bisectors of the triangles angles intersect in one point.

The point of intersection of the bisectors of the angles locates on the same distance from the three sides.

In every triangle, there is a circle inside the triangle which is tangent to its three sides and it is called the inscribed circle. $OL = OK = OM$



Example (2): The circle which its center is O surrounded by the triangle ABC, prove that \overline{BO} bisects $\angle LOK$ and the axis of \overline{KL}

The theorem of the two tangents

$$BK=BL$$

The two radius of the circle.

$$OK=OL$$

$\therefore \triangle BOK, \triangle BOL$ congruent (s.s.s)

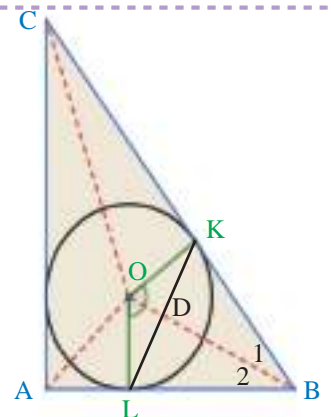
$$m\angle 1 = m\angle 2$$

\overline{BO} bisects $\angle LOK$

The triangles LDB, KDB are congruent (S.A.S)

$$\overline{KL} \perp \overline{BO}$$

$\therefore \overline{BO}$ axis of \overline{KL}



[5-5-2] Line Segments and Circle

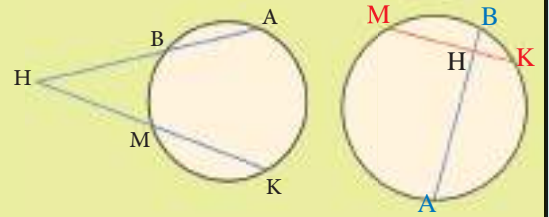
In lesson (5- 4), you have learned how to find the lengths of a chord parts which intersects with a perpendicular diameter on it. But how can we find the lengths of other intersected chords.

Theorem of the two secants of the circle

The theorem

If two intersected lines intersect a circle then the product of the length of the two segments from the point to the circle is constant along any line through the point and circle.

$$HM \times HK = HB \times HA$$



Example (3): Find the value of x and the length of each chord.

$$HM \times HK = HB \times HA$$

A theorem of the two secants in the circle

$$8 \times x = 3 \times 2$$

By substitution and simplifying

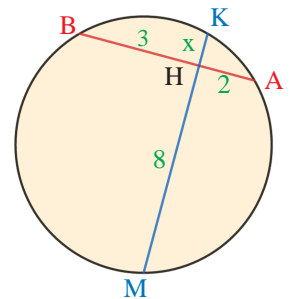
$$x = \frac{6}{8} = \frac{3}{4}$$

$$AB = AH + HB = 2 + 3 = 5$$

The chord length AB

$$MK = MH + HK = 8 + \frac{3}{4} = 8\frac{3}{4}$$

The chord length MK



Example (4): Find the value of x and the length of \overline{AM} , \overline{BM}

$$MD \times MB = MC \times MA$$

A theorem of the two secants in the circle

$$2 \times 9 = 3 \times (3 + x)$$

simplifying

$$18 = 9 + 3x$$

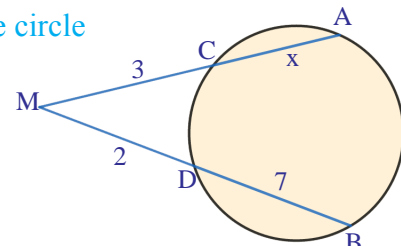
By substitution

$$3x = 18 - 9 = 9$$

Length of $\overline{AM} = 6$

$$x = \frac{9}{3} = 3$$

Length of $\overline{BM} = 9$

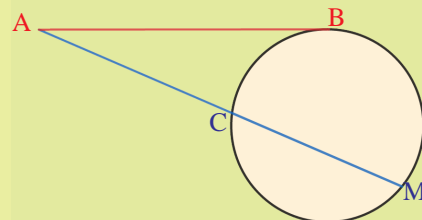


We can use the result of multiplying the two parts of the secant by the theorem of the secant and the tangent, and in this case, the tangent will be the external and total part of the segment itself.

The theorem of the tangent and the secant in circle

The theorem

If a line has a tangency with a circle and another line intersects it, two line segments will be formed on the secant, and the result of multiplying their two lengths will be equaled to the square of the tangent length. $AC \times AM = (AB)^2$



Example (5): Find the length of the tangent AB .

$$AC \times AM = (AB)^2$$

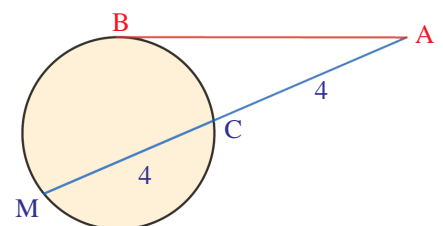
The theorem of the tangent and the secant in the circle

$$4 \times 8 = (AB)^2$$

By substitution

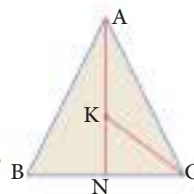
$$\therefore AB = 4\sqrt{2}$$

The length of the tangent AB



Make sure of your understanding

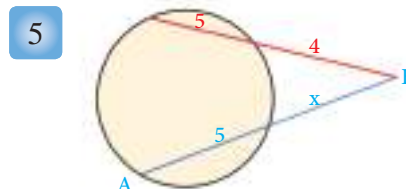
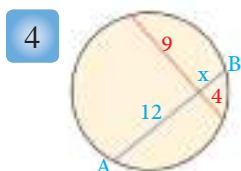
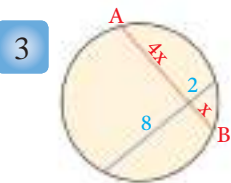
- 1 The triangle ABC is an isosceles triangle $AB = AC$, N is the midpoint of \overline{BC} , $\overline{KA} \cong \overline{KC}$, prove that K is the point of intersection of axes of the triangle ABC, then draw the circumscribed circle.



- 2 ABC is a regular triangle, the length of its side is 12cm, determine the point of intersection of its axes, then draw the circumscribed circle, and find the length of its diameter.

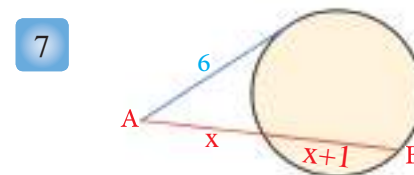
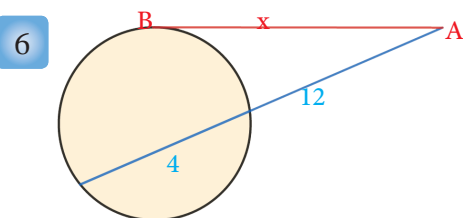
Questions 1- 2 are similar to example 1

Find the value of x and the length of each unknown segments of the following:



Questions 3 - 5 are similar to examples 3,4

Find the value of x and length of \overline{AB} :

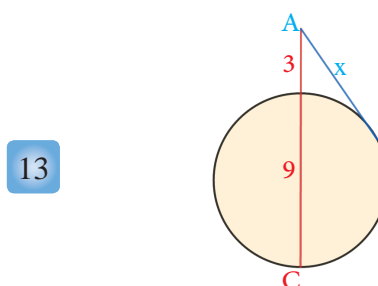
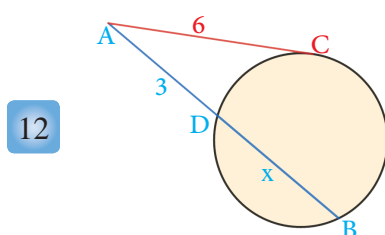
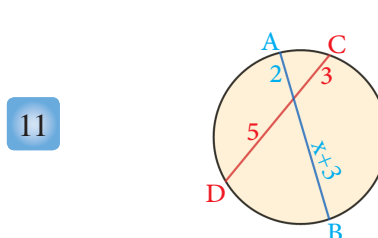
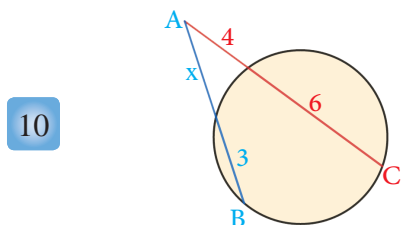


Questions 6-7 are similar to example 5

Solve the Exercises

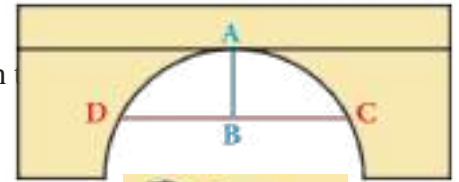
- 8 ABC is a right-angled and isosceles triangle, the length of each of two sides is 6cm, draw the circle that surrounded by the triangle ABC and find the area of the circle.
- 9 ABC is a right-angled and isosceles triangle, its chord is \overline{BC} . Determine the intersection point of the axes of this triangle and draw circumscribed of it.

Find the value of x and the length of the unknown segments of the following:



Solve the problems

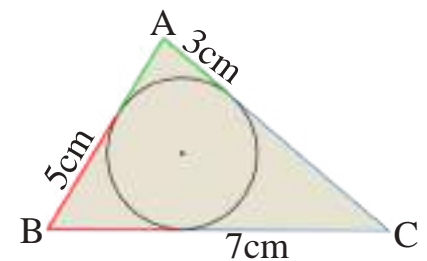
- 14 **Construction:** A bridge bases on an arc a circle, as it is shown in nearby figure, \overline{AB} axis of \overline{DC} , $AB=60\text{m}$, $DC=150\text{m}$, what is the diameter of the circle?



- 15 **Space:** A satellite rotates around the earth in a height of 8200km, if the diameter of the earth is about 12800 km, what is the distance which separates the satellite from the point B in the nearby figure.



- 16 **Geometry:** O is an intersection point of the axes of the triangle ABC, find the perimeter of the triangle ABC by using the nearby figure.



Think

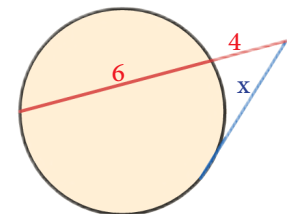
- 17 **Discover the mistake:** Below are two solutions to find the value of X in the nearby figure, which solution is wrong? Explain your answer.

The theorem of the tangent and the secant.

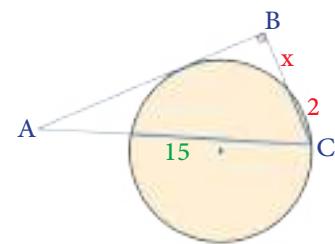
$$\text{i) } 4 \times 6 = x^2$$

$$24 = x^2 \Rightarrow x = 2\sqrt{6}$$

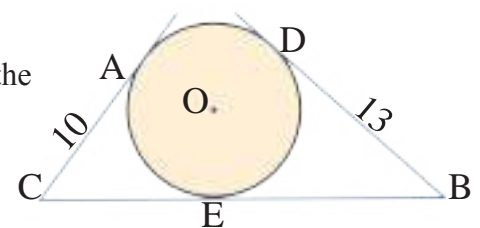
$$\text{ii) } x^2 = 40 \Rightarrow x = 2\sqrt{10}$$



- 18 **Challenge:** In the nearby figure, $AB=10$ and it is the tangent of the circle, find the value of x.



- 19 **Open problem:** In the nearby figure, a circle its center O and \overline{AC} , \overline{BC} , \overline{BD} are tangents of the circle, find the length of the segment BC.



Write

A problem which uses the axes and the bisectors of the angles for a triangle to draw a the circumscribed circle.

Lesson [5-6]

Angles and Circle

Idea of the lesson:

- Finding the measure of inscribed angles and the tangent ones.
- Finding the measure of angles which their sides intersect with a circle.

Vocabulary:

- Inscribed angles
- Tangent angles

Learn

The screwdriver is used as a tool to fix or open the screws, and the gap in this tool takes a hexahedron shape inside a metal cylinder, and every angle in the hexahedron shape represents an inscribed angle inside the circle.



[5-6-1] The Inscribed Angle

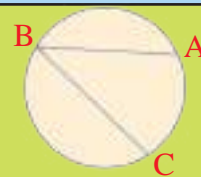
You have previously learned the arc by the indication of the central angle and how to measure the arc. In this lesson, we will learn the inscribed angle: it is an angle which its vertices is a point of the circle points and its two sides are chords in the circle.

We will also learn how to measure it by using the arc which opposite it by the following theorems which are without reasoning.

The theorem of the inscribed angles

The measurement of the inscribed angle equals the half of its intercepted arc.

$$m\angle B = \frac{1}{2} m \widehat{AC}$$



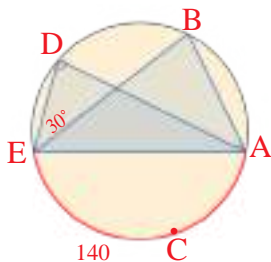
Example (1): Find the measure of the following inscribed angles in the nearby figure.

i) $\angle D$

$$m\angle D = \frac{1}{2} m \widehat{ECA}$$

$$= \frac{140}{2} = 70$$

$$m\angle D = 70^\circ$$



ii) $\angle BAD$

The theorem of the inscribed angles

$$m\angle BAD = \frac{1}{2} m \widehat{BD}$$

By substitution

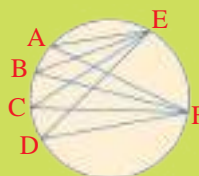
$$m\angle BED = \frac{1}{2} m \widehat{BD}$$

$$\therefore m\angle BED = m\angle BAD = 30^\circ$$

The theorem of the inscribed angles which intercepted the same arc

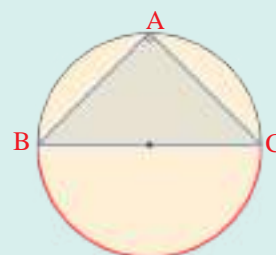
All the inscribed angles which intercept the same arc are congruent.

$$m\angle A \cong m\angle B \cong m\angle C \cong m\angle D = m \widehat{EF}$$



There is a special case for the inscribed angle when it is a right angle each inscribed angle faces a semicircle is a right angle. each right- inscribed angle faces opposite diameter.

$$m\angle A = 90^\circ$$



Example (2): A circle, its diameter is \overline{KH} , intersects \overline{HL} in N and intersects \overline{KL} in M, as in the nearby figure, prove that \overline{HM} and \overline{KN} are altitudes in the triangle HKL.

An inscribed angle HNK opposite the diameter \overline{KH} is a right angle.

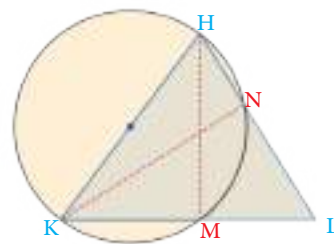
$$\therefore m\angle HNK = 90^\circ$$

\overline{KN} is an altitude in the triangle HKL.

An inscribed angle HMK opposite the diameter \overline{KH} is a right angle

$$\therefore m\angle HMK = 90^\circ$$

\overline{HM} is an altitude in the triangle HKL



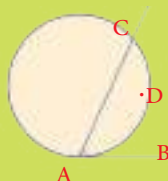
[5-6-2] Tangential Angle

Tangential angle: Is the angle which is formed by the tangent of the circle with another line passes through the point of tangency.

The theorem of the tangent angle

If the circle tangent intersects with a line passes through the point of tangency, the measure of the angle between them will be the half of the intersected arc.

$$m\angle A = \frac{1}{2} m \widehat{ADC}$$



Example (3): By using the theorem of the tangent angles and the nearby figure, find the measurement of the following:

i) $\angle BAC$

ii) \widehat{NC}

The theorem of the tangent angles

$$m\angle BAC = \frac{1}{2} m \widehat{CA}$$

$$m\angle CNM = \frac{1}{2} m \widehat{CN}$$

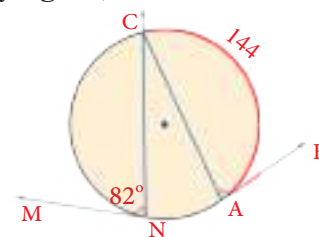
By substitution

$$= \frac{144}{2} = 72$$

$$82^\circ = \frac{1}{2} m \widehat{CN}$$

$$\therefore m \angle BAC = 72^\circ$$

$$\therefore m \widehat{CN} = 164$$

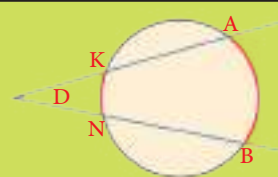


[5 -6- 3] Internal and Exterior Angles in the Circle

The theorem of the exterior angle in the circle

If two lines intersected outside a circle, the measure of the angle between them equals the half of the difference between the two intersected arcs.

$$m\angle D = \frac{1}{2} (m \widehat{AB} - m \widehat{KN})$$



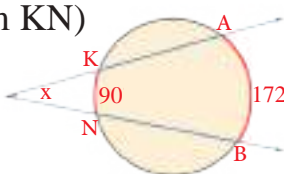
Example (4): Find the measure of the exterior angle x in each of the following:

i) By using the theorem of the exterior angle in the circle and by the substitution of the arcs value, in the figure, we find the measure of the angle x.

$$m\angle x = \frac{1}{2} (m \widehat{AB} - m \widehat{KN})$$

$$= \frac{1}{2} (172 - 90)$$

$$\therefore m\angle x = \frac{82^\circ}{2} = 41^\circ$$



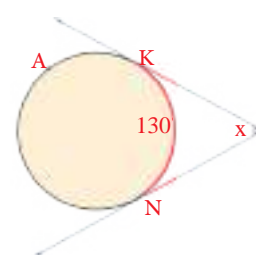
ii) By using the theorem of the exterior angle in the circle and by the substitution of the value \widehat{KAN} by 360, we find the measure of the angle x.

$$m\widehat{KAN} = 360^\circ - 130^\circ = 230^\circ$$

$$m\angle x = \frac{1}{2} (m \widehat{KAN} - m \widehat{KN})$$

$$= \frac{1}{2} (230 - 130)$$

$$\therefore m\angle x = \frac{100^\circ}{2} = 50^\circ$$

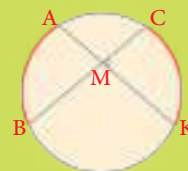


The theorem of the interior angle in a circle

If two lines intersected inside a circle, the measure of the angle between them equals a half of the sum of the measure for the cutting arches.

$$m\angle CMK = \frac{1}{2}(m\widehat{CK} + m\widehat{AB})$$

$$m\angle CMA = \frac{1}{2}(m\widehat{AC} + m\widehat{BK})$$



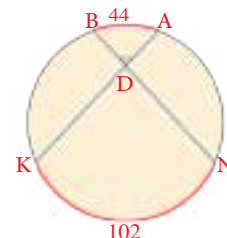
Example (5): Find the measure $\angle ADB$, using theorem of the interior angle in the circle.

The theorem of the interior angle in a circle

By substituting and simplifying

$$\begin{aligned} m\angle ADB &= \frac{1}{2}(m\widehat{KN} + m\widehat{AB}) \\ &= \frac{1}{2}(102 + 44) \end{aligned}$$

$$\therefore m\angle ADB = \frac{146^\circ}{2} = 73^\circ$$



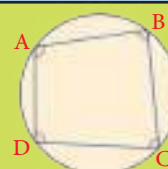
We can draw a circle passes through the four vertices a quadrinomial this quadrinomial is called the circular quadrinomial.

The theorem of the circular quadrinomial

In each circular quadrinomial, the sum of measuring each two opposite angles equals 180° .

$$m\angle A + m\angle C = 180^\circ$$

$$m\angle B + m\angle D = 180^\circ$$



Example (6): Find the value of x , and (a) in the nearby figure.

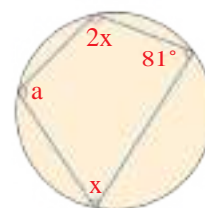
The theorem of the circular quadrinomial

$$\therefore a + 81^\circ = 180^\circ$$

$$\therefore a = 180^\circ - 81^\circ = 99^\circ$$

$$\therefore x + 2x = 180^\circ \Rightarrow 3x = 180^\circ$$

$$\therefore x = 60^\circ$$



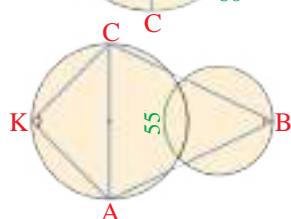
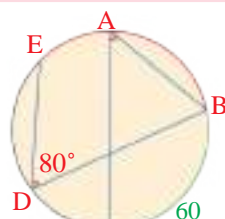
Make sure of your understanding

1 $m\widehat{BE}$

3 $m\angle CAB$

6 $m\angle CKA$

8 $m\angle CBA$



Find the measure of each of the following:

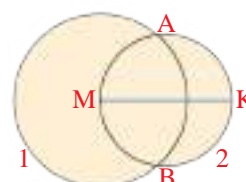
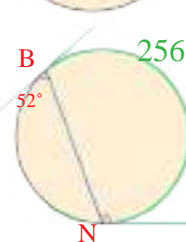
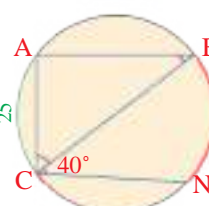
2 $m\angle ABC$

4 $m\angle ACB$

5 $m\widehat{BN}$

7 $m\angle MNB$

9 $m\widehat{BN}$



Questions 1,5 are similar to example 1

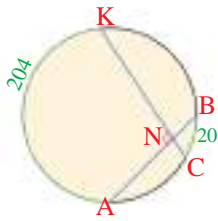
Questions 6,7,10 are similar to example 2

Questions 8, 9 are similar to example 3

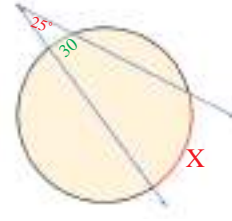
- 10 If you knew that M is a center of the circle 1 and \overline{MK} is the diameter of the circle 2, prove that \overline{KA} and \overline{KB} are two tangents to the circle 1.

Find the measure for each of the following:

11 $m\angle KNA$



12 $m\widehat{X}$

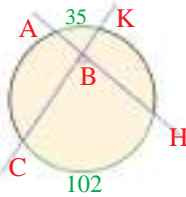


Questions 11 - 12 are similar to examples 4,5,6 respectively

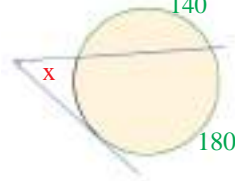
Solve the Exercises

Find the measure for each of the following:

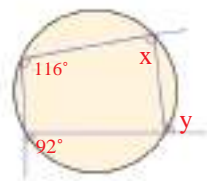
13 $m\angle HBC$



14 $m\angle x$

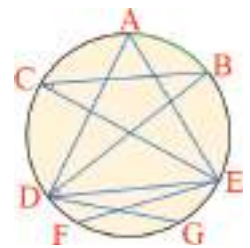


15 $m\angle x, m\angle y$

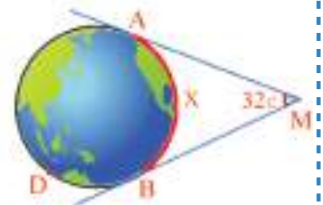


Solve the problems

- 16 **Glass:** one of the artists had drawn the nearby diagram on glass, find the measure of $\angle ADE$ if you knew that $m\angle BCE = 30^\circ$ and the measure $\widehat{AB} = 42$



- 17 **Space:** A satellite rotates around the earth, and when it reaches to the point M, its height will be 14000 km above the earth. What is the measure of the arch that we can see by the camera of the satellite on the earth?

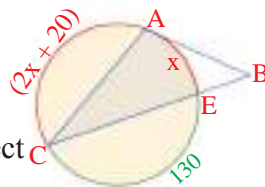


Think

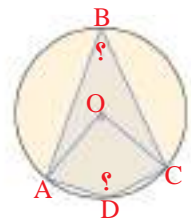
- 18 **Discover the mistake:** Saeed wrote

$$m\angle CAB = \frac{160^\circ}{2} = 80^\circ$$

show the mistake and find the correct answer.

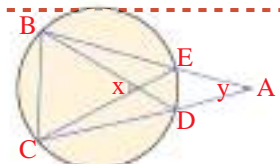


- 19 **Numerical sense:** Find the value of the unknown angles



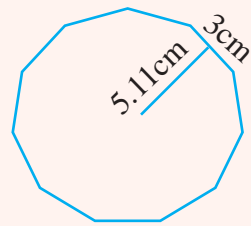
Write

The theorem of the interior and exterior angles to compare the two angles x,y ?



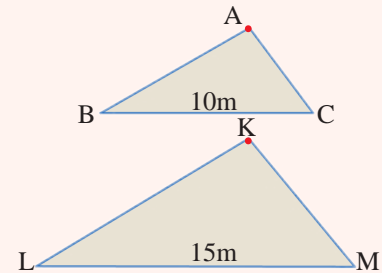
Chapter Test

- 1 Find the area and the perimeter of a regular polygon, if you were given the data in the nearby figure.

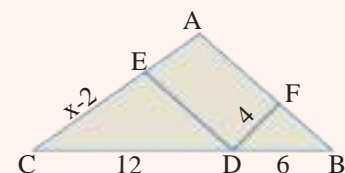


- 2 Find the surface area and the volume of the cone if you knew that the area of its base is $9\pi \text{ cm}^2$ and its lateral height is 5 cm.

- 3 The two triangles ABC, KLM are similar, the area of the triangle $ABC = 24 \text{ m}^2$,
What is the area of the triangle KLM?

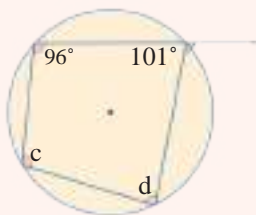


- 4 Show if the two triangles ABC, FBD in the nearby figure are similar, and write the ratio of the similarity if $\overline{AC} \parallel \overline{FD}$, and find the value of x.

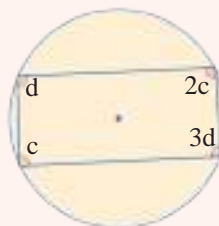


- 5 Find the measure of the unknown angles in the following figures:

i)

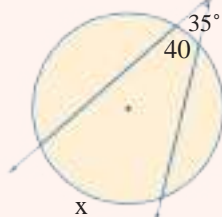


ii)

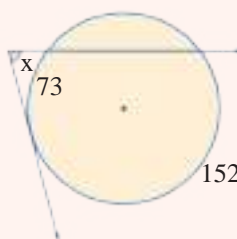


- 6 Find the value of x in each of the following:

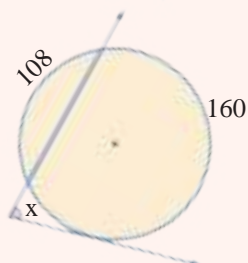
i)



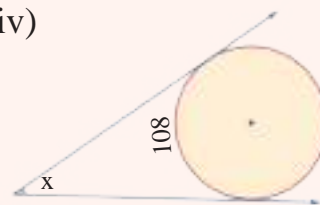
ii)



iii)



iv)



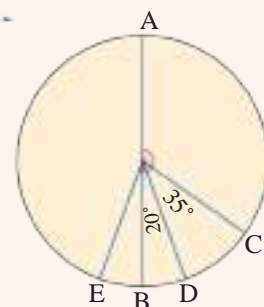
- 7 Find the measure of the unknown angles and arcs in the nearby figure.

i) $m\angle AOC$

ii) $m\widehat{DC}$

iii) $m\widehat{DB}$

iv) $m\angle DOA$



Statistics And Probabilities

- lesson 6-1 Design a Survey Study and Analysis its results
- lesson 6-2 Graphs and Misleading Statistic
- lesson 6-3 Permutations and Combinations
- lesson 6-4 Experimental Probability and Theoretical Probability
- lesson 6-5 Compound Events

Before marketing its production, the cars factories usually investigate some issues to make sure of the quality, such as the durability of cars engines, the quality of cars electricities, colors and affairs related to design, like cars lights and others.

Pretest

Find the arithmetic mean, median, the mode and the range for each of the following:

- 1 9,6,8,5,5,8,7,6,9,7 2 20,17,42,26,27,12,13 3 8,7,5,8,2,8,9,1,4,3,3,5

- 4 Represent the following graphs by points, then find the arithmetic mean, median, the mode and the range.

0,2,5,3,1,4,5,3,4,3

Write each fraction as percentage :

- 5 $\frac{1}{4}$ 6 $\frac{13}{20}$ 7 $\frac{27}{100}$ 8 $\frac{3}{25}$

- 9 A box contains five red balls, and three white balls. Find a probability of withdrawal.

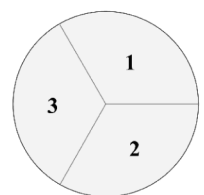
- i) One red ball.
ii) One white ball after returning the red ball to the box.
iii) One white ball in case the red ball without returning to the box.

- 10 a,b two perfect events, find:

- i) $p(a)$ if $P(b) = \frac{2}{7}$
ii) $p(b)$, $p(a)$ if $p(a)$ three times $p(b)$.

Determine if the two events are independent or dependent:

- 11 After flapping a coin, writting (tail) was appeared, while a picture (head) was appeared after the second flapping
- 12 Drop a yellow ball, then red ball from a sack which contains 3 yellow balls and 5 red balls.
- 13 The number 5 was a appeared when the dice was thrown for the first time, but when it was thrown for the second time the number 6 appeared.
- 14 Drop a card, which a name of Jamana was written on, from a sack without returning it, then another card, which contains the name of Sally, was dropped from the same sack.
- 15 A disc spinner was stopped on the number 3, and appearance of number 3 when a dice was thrown for once.
- 16 Three cards contain the letters A ,B, C, in how many ways we can arrange the cards on a straight line.



Lesson [6-1]

Design a Survey Study and Analysis its Results

Idea of the lesson:

- *Designing a survey study
- *Analyzing the results

Vocabulary:

- *A survey study
- *Society
- *Sample

Learn

Al- Najaf factory for men's suits, one of most important factories. people who are concerned in the factory affairs interested in quality of production by checking the type of clothes, colors, modern designs and others. The process of checking all cloths will be illogical , so samples of suits would be checked instead to see If the product needed to be developed or not.



[6- 1- 1]] Design a Survey Study

Sample:- It is a subest from the society. By analyzing the results of the sample, we can get conclusions about the society as a whole. The conclusions would be more representative to the society in any of the following two cases:

- Size of sample is biggest.
- Using more samples.

To the type of the sample has an effect on its conclusions which be got, and they are two types:

The unbiased sample: If each one has the same probability of choice.

The unbiased sample: If each one has different probabilities of choice.

Example (1):

A school headmaster distributed 100 questionnaires to the students to know their opinions about the quality of food in school shop stall.

i)Determine the sample and the society which the sample was taken from.

ii)Describe the method of gathering data which the headmaster used.

iii)Determine if the sample was biased or unbiased.

i)The sample: The students who received the questionnaires, their number was 100 students.

Society: All students of school

ii)Method of gathering data is a survey study, where they would be taken from the answers of students who represent the sample.

iii)The sample is unbiased: because this sample consists of the students who were chosen randomly.

Example (2):

A shop seller wants to give a gift for every customer shopping from his shop, so he stood in front of the shop and asked 20 customers about the type of gift that everyone wish to be presented to him.

- i) Determine the sample and the society that the shop seller chose.
- ii) Determine the method of gathering data which the shop seller used.
- iii) Determine if the sample was biased or unbiased.
- i) The sample: The customers who were asked, their number was 20.

Society: The customers who visited the shop.

- ii) The method of gathering data is a survey study, where the answers were taken from the selected sample (20 people)
- iii) The sample is unbiased because people who visited the shop were chosen randomly.

Example (3):

Ten persons, who were in a restaurant for kabab, were asked about their favorite meals

- i) Determine the sample and the society who were chosen by the owner of the restaurant.
 - ii) Describe the method of gathering data which the owner of restaurant used
 - iii) Determine if the sample was biased or unbiased.
 - i) The sample: The ten persons who entered the restaurant.
- Society: All people who entered the restaurant.
- ii) The method of gathering data is a survey study, where the answers were taken from the selected sample.
 - iii) The sample is biased: Because the favorite meal for the persons who were sitting in the restaurant is kabab.

[6-1-2] Analysing the results

After gathering the data by the survey study, the data would be summarized to be meaningful. All that will be done by using the measurements of the central tendency (arithmetic mean, median, the mode) which was studied previously in different ways and choosing the suitable measurement to represent the data.

The type	When it better to be used
Arithmetic mean	When there are no extreme values in the data set
Median	When there are extreme values in the data set, but there are no big gaps in the middle of data
The Mode	When there are repeated numbers in data set

Example (4):

Which of the measurements of central tendency is (if there is) more suitable to represent the data in each of the following:

i) The near by data show the weights of 10 boxes in kg: 3, 2, 3, 6, 5, 5, 21, 4, 3, 5

Arithmetic mean: is not suitable to represent the data because there is a big extreme value which is: 21

The mode: is unsuitable to represent the data because there are more than one

Mode: 3, 5

Median: is the suitable measurement to represent the data because there is no big gap in the middle of set 2, 3, 3, 3, 4, 5, 5, 5, 6, 21

ii) Mohammed had got the following marks in five tests of Maths: 90, 93, 85, 86, 9

$$\text{Arithmetic mean} = \frac{90+93+85+86+91}{5} = \frac{445}{5} = 89$$

Arithmetic mean: 89 is a suitable measurement to represent the data because there is no extreme value

Median: 90 is a suitable measurement for representing the data because it comes in the middle of data and no big gap is there.

Both of them suitable measurement for representing data.

The Mode: there is no mode because there is no repetition in the data.

Make sure of your understanding

Determine the sample and the society, then describe the style of gathering data and differentiate the biased sample from the unbiased one in each of the following, illustrate your answer :

- 1 30 persons entered a library, the sixth person from each six persons was asked about his or her favorite hobby.
- 2 100 questionnaires were distributed to a group of workers in a factory. They were asked about the conditions of work inside the factory.
- 3 In a zoo, animals were distributed; the animal from each group was chosen randomly to make tests on it.

Which measurement of central tendency (if there is) is suitable represent the following data?

Illustrate your answer :

- 4 8, 10, 14, 8, 13, 6 5 8, 10, 8, 9, 11, 4, 6, 54 6 8, 9, 8, 6, 10, 9, 11, 13, 14, 8, 6, 7, 19

Solve the Exercises

Determine the sample and the society, then describe the style of gathering the data and differentiate the biased sample and the unbiased one in each of the following. Illustrate your answer :

- 7 A factory owner wants to check if the workers work well or not. He watches a worker for two hours.
- 8 A number of students stands at the main gate of school; they were asked the tenth placed student who entered school about her favourite hobby.

Which measurement of the central tendency (if there is) is more suitable to represent the following data? Illustrate your answer.

- 9 34, 47, 41, 49, 39, 26, 40 10 6, 2, 4, 4, 3, 2, 6, 2, 4, 4, 20 11 5, 3, 5, 8, 5, 3, 6, 7, 4, 5

Solve the problems

Hospital: Medical city hospital is a complete medical center that offers its services to all citizens in Baghdad and other governorates. In an introductory seminar, a doctor was selected randomly to present an overview about the services that his department offers within the hospital.



12 Describe the sample and the society.

13 Is the sample biased or not? Illustrate that.

14 **Shopping:** The table below shows the number of customers who visit a shop for selling electric appliances in each hour of a day. Which measurement of the central tendency is more suitable to represent the data.

Number of customers			
79	71	86	86
88	32	79	86
71	69	82	70
85	81	86	86



15 **Feeding:** the table below shows the calories for some vegetables in the plate for each sample. Which measurement of the central tendency is more suitable to represent the data.

vegetables	calories	vegetables	Calories
onion	16	cucumber	13
pepper	20	corn	66
cabbage	17	spinach	9
carrot	28	pumpkin	17



Think

16 **Challenge:** Find a set of numbers in which its median is smaller than its arithmetic mean.

17 **Correct the mistake:** Snaria says that the arithmetic mean is the more suitable measurement of the central tendency to represent data 3,5,4,8,20. Determine Snaria,s mistake, then correct it.

18 **Numerical sense:** In a survey study about the daily going to a secondary school, a questionnaire was distributed to 50 students, the result was that 74% prefer to go to school in the morning . It this study reliable? Explain that.

Write

A question about a meaning whcih you want to get an answer about it by a survey study.

Lesson [6-2]

Data and Misleading statistic

Idea of the lesson:

- * Discrimination the misleading data
- * Discrimination the misleading statistics

Vocabulary:

- * Misleading data
- * Misleading statistics

Learn

- We often see advertisements for sale on the shopwindow of commercial shops at the end of each season.
- These advertisements attract the customers to buy goods



[6- 2- 1] Discrimination the Misleading Graphs

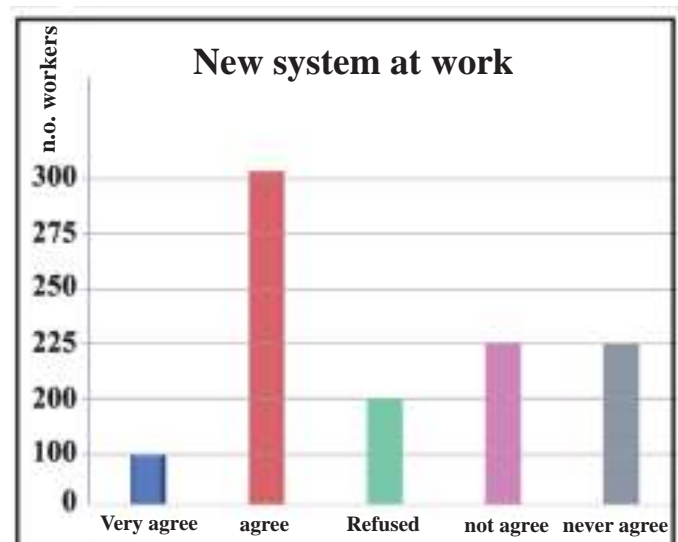
The misleading data: Represent the data which focus on a certain item of the , goods in an extreme ways by showing facts in a way that would misleading the customers and encourage them to buy goods.

Example (1):

An owner of a factory thinks to apply a new system at work. He distributed a questionnaire in which he asked the workers their opinions in the new system.

Does the representation, in the nearby figure, give the correct form about the outcomes of the questionnaire?

At the first time, it seems that most workers are agree with the new system, although it is knowing that the longest period of grading is unconstant.



Note: there are 450 workers either not agree with this new system or they are never agree with this system, while the number of workers who said yes is more than 400 workers. So, the nearby graphic diagram considers misleading one, and the conclusion is untruthful.

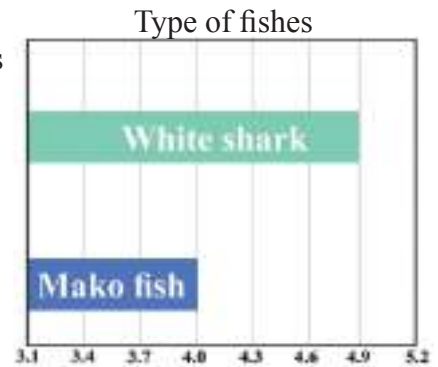
Note: (the graphic diagram may be misleading, by extending or shortening the periods among the values of data to give a certain impression)

Example (2):

The nearby graphic diagram shows the relation between the two lengths of the big white shark and the Mako fish. Show if the graphic diagram is misleading, explain that.

- From the nearby figure, we see that the length of the upper column is twice the length of the lower column double.

- But the corresponding value to the length of the upper column is 4.9 and value to the length of the lower column is 4, and certainly the value 4.9 is not double the value 4. So the nearby graphic diagram is misleading.



Note: (when the graphic diagram started from zero, it will not be misleading)

[6-2-2]

Discrimination Misleading Statistics

The misleading statistics: in addition to the misleading diagrams, the misleading statistics use to promote of a company or certain goods. By looking carefully at the given informations of the advertisement we can discriminate the misleading statistics.

Example (3):

An owner of a men's suits shop put the following advertise ment (New mens suits, the average of price is 45000 dinars) There are five types of the suits inside the shop which their prices reach up to (in thousands dinars) 53, 48,20,50,54

$$\frac{54 + 50 + 20 + 48 + 53}{5} = 45$$

-The average price of suits is 45000 dinars, but there is only one suit which its price is less than this average, and that will make the customer pay more than this price for the suit.



Example (4):

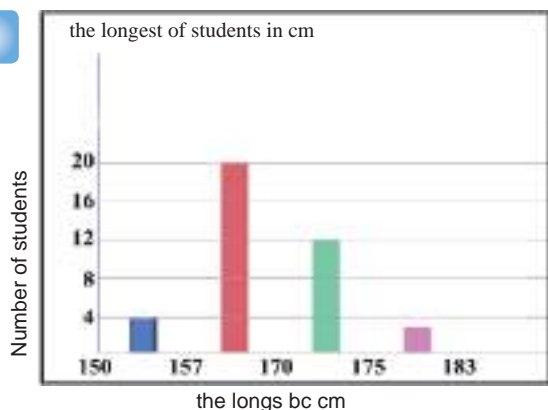
A survey had been done on 800 students from the preparatory stage, 70 students wished to join the engineering college, while 50 student wished to enter the college of medicine. The results reveal that students prefer engineering college to the perecentage of studens who participated in the survery = $\frac{120}{800} \times 100$ college of medicine.The real number $(50 + 70) = 120$ of students who participated in the survey is 120 students from 800 students, which means that the random sample was so small .



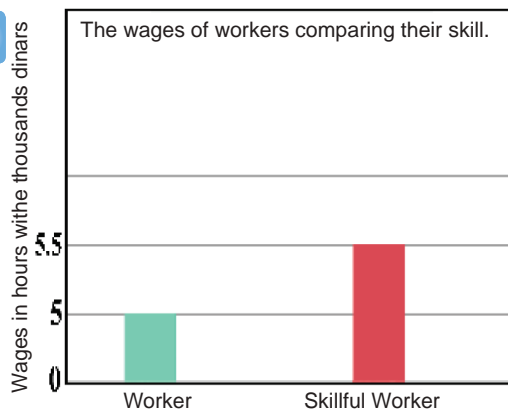
Make sure of your understanding

Clarify how can the following two graphic diagrams give a misleading impression?

1



2



Illustrate why the following statistics are misleading:

3

An article was shown to 20 persons to evaluate it, 13 persons were admired in the article. According to that, the person who wrote the article announced that the article is durable for publishing because the ratio of people who support the article is 13 to 7.

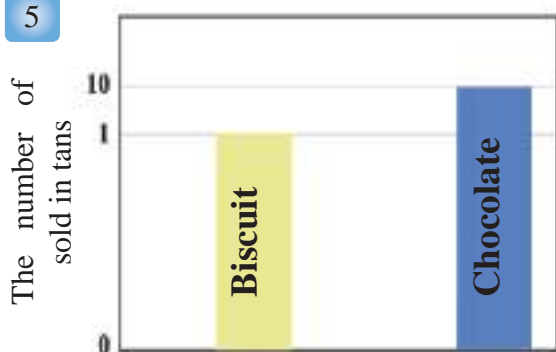
4

A sport store sold for a certain period 390 sport suits, while a store for entertainment sold for the same period 90 sport suits

Solve the Exercises

Clarify how can the following two graphic diagrams give a misleading impression:

5



6



7

In a survey for 6 persons about reading a daily newspaper, 4 of them said that they prefer reading the newspaper (x), at the end of the survey the following sentence was mentioned:

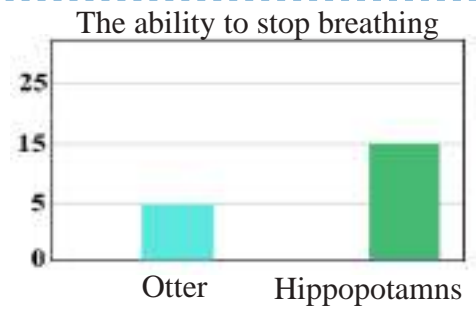
2 from 3 persons prefer reading the newspaper (x) , why does this advertisement consider misleading?

8

100 students were asked about the way they preferred to come to school. 60 students of them their answer were as the followings. 32 of them prefer coming by taxis, 18 students prefer walking and 10 students prefer their own cars. conclude that half of the students prefer taxis. Illustrate why the statistics are misleading.

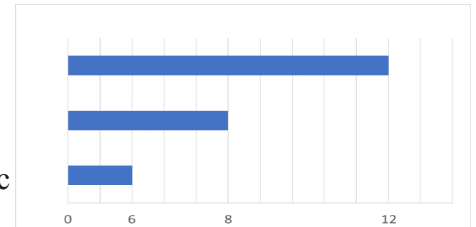
Solve the problems

- 9 **Biology:** the nearby graphic diagram represents the ability to stop breathing for Hippo and other. Why the data in the diagram are misleading? Clarify that.

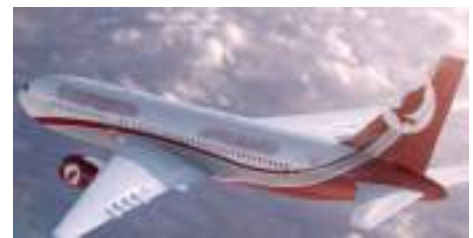


- 10 **Reading:** The nearby graphic diagram represents people who prefer reading literature, scientific and art books. Illustrate why the data are misleading ?

Art
literature
Scientific



- 11 **Transportation:** the profits of the aviation company A reached to 5500 million dinars in the two months July and August. While the profits of the aviation company B were 7500 million dinars in the two months April and May. Illustrate why the statistics are misleading ?



- 12 **Feeding:** the pack of broccoli contains 477mg of potassium, a big carrot contains 230mg, while the cauliflower contains 803mg of potassium. Illustrate why the statistics are misleading ?



Think

- 13 **Discover the mistake:** Mohammed says that the diagram will not be misleading if it started from zero for columns regardless of proving the long of periods. Discover Mohammed's mistake.
- 14 **Numerical sense:** A seller had got the following amounts of thousands dinars: February 965, March 170, April 120, July 125, May 100. He told his friends that the average of his monthly commission is 265000 dinars. Illustrate why this statistic is misleading.
- 15 What is the thing that you should be sure of to decide that the diagram is misleading or not?

Write

A question from everyday life which we need to make misleading diagrams.

Lesson [6-3]

Permutations and Combinations

Idea of the lesson:

- *Identify the factorial of the nonnegative integer.
- *Identify the concept of the Permutations
- *Knowing the concept of the combinations

Vocabulary:

- *Factorial of the number
- *Permutations
- *Combinations
- *Sample space

Learn

- Four persons entered a room which has four chairs in one row. They were asked to sit down on the four chairs.
- How many methods can they use to sit down on the chairs?



[6- 3- 1] Factorial

If n is any positive integer, then $n! = n(n-1)(n-2)... (3)(2)(1)$, $n \in \mathbb{Z}^+$ and $1! = 1, 0! = 1$

Example (1):

Four persons entered a room contains a four-chairs row. They were asked to sit down on those chairs. How many ways can they follow to sit down on the chairs?

- The first person who entered the room has the choice to sit down on any chair he likes, that means he has 4 choices
- The second person has the right to sit down on any of the three chairs, that means he has 3 choices
- The third person has the right to sit down on any of the two chair, that means he has two choices
- While the fourth person, for sure he will sit down on the last chair, that means he has only one choice.

Then, the number of the possible ways for sitting down equals $4 \times 3 \times 2 \times 1 = 24$

Note that you got the previous result by multiplying consecutive numbers starting from (4) and then decreasing till number (1)

A form like this is called the factorial of number (4) and we can use the symbol (4!) to refer to it.



Example (2):

Find the value of each of the following:

i) $5!$ ii) $4! - 2!$ iii) $\frac{7!}{5!}$ iv) $3! \times 2!$ v) $\frac{(6-2)!}{0!}$ vi) $\frac{6!}{3 \times 6}$

i) $5! = (5)(4)(3)(2)(1) = 120$ ii) $4! - 2! = (4)(3)(2)(1) - (2)(1) = 24 - 2 = 22$

iii) $\frac{7!}{5!} = \frac{(7)(6)(5)(4)(3)(2)(1)}{(5)(4)(3)(2)(1)} = (7)(6) = 42$ iv) $3! \times 2! = ((3)(2)(1))((2)(1)) = (6)(2) = 12$

v) $\frac{(6-2)!}{0!} = \frac{(4)!}{0!} = \frac{(4)(3)(2)(1)}{1} = 24$ vi) $\frac{6!}{3 \times 6} = \frac{(6)(5)(4)(3)(2)(1)}{3 \times 6} = 40$

[6-3-2] Permutations

How many an ordered pair it can be configure from the letters: a, b, c ? by using tree rules.



There are six ordered pairs and this gives a simple idea for the permutations which we will study it later. Number of Permutations for elements which its number n is taken r in each time and it is the result of dividing $n!$ on $(n-r)!$ the symbols of Permutations P_r^n or $p(n,r)$ where

$$P_r^n = \frac{n!}{(n-r)!} \quad 0 \leq r \leq n \quad \text{Note that} \quad P_0^n = 1, P_1^n = n, P_n^n = n!$$

Example (3): Find the value of each of the following:

i) P_2^7 ii) P_3^3 iii) P_1^9 iv) P_0^{10}

$$\text{i) } P_2^7 = \frac{7!}{(7-2)!} = \frac{7!}{5!} = \frac{(7)(6)(5)(4)(3)(2)(1)}{(5)(4)(3)(2)(1)} = 42$$

$$7! = (7)(6)(5!)$$

Or by simplest method by making

$$P_2^7 = \frac{7!}{(7-2)!} = \frac{7!}{5!} = \frac{(7)(6)(\cancel{5!})}{\cancel{5!}} = (7)(6) = 42$$

ii) $P_3^3 = 3! = (3)(2)(1) = 6$ iii) $P_1^9 = 9$ iv) $P_1^{10} = 1$

The branch ii, iii, iv is from applying the notice directly

Example (4):

The digits plate of cars: to make digits plates for cars which consist of five digits from the series 1 to 9. What is the number of the possible different orderings?

Since the ordering of digits is important, then this case represents permutations.

$$P_r^n = \frac{n!}{(n-r)!}$$

Writing the Permutations formula

$$P_5^9 = \frac{9!}{(9-5)!} \quad r = 5, n = 10$$

By substituting

$$P_5^9 = \frac{9!}{4!}$$

Simplify

$$= \frac{(9)(8)(7)(6)(5)(4!)}{4!}$$

Dividing the common factors

$$= 15120$$

Then there are 15120 orderings

[6-3-3] Combinations

How many set of two elements can we make from the letters a,b,c? Since in the sets, the ordering is not important, there are three set: {a,b}, {b,c}, {a,c} that give a simple idea for the combinations which will study it later.

The number of combinations for the elements which its number n is taken r in each time equals the result of dividing $n!$ on $(n-r)!r!$, the symbol of combinations is C_r^n or $\binom{n}{r}$

$$\binom{n}{r} = C_r^n = \frac{n!}{(n-r)!r!}, \quad 0 \leq r \leq n$$

Note that: $C_0^n = 1$, $C_1^n = n$, $C_n^n = 1$

In combinations, the ordering is not important

Example (5):**Find the value of each of the following:**

i) C_2^8 ii) C_{12}^{12} iii) C_1^9 iv) C_0^{50}

i) $C_r^n = \frac{n!}{(n-r)!r!}$ Writing the combinations formula

$$C_2^8 = \frac{8!}{(8-2)!2!} = \frac{8!}{6!2!}$$
 Substituting $n = 8$, $r = 2$

$$= \frac{(8)(7)(6!)}{6!(2)(1)} = 28$$
 Simplify
Solve the factorial and then simply

ii) $C_{17}^{17} = 1$ iii) $C_1^9 = 9$ iv) $C_0^{50} = 1$ As a note

Example (6):**Jobs:** A company announced 4 jobs, 10 persons were applied for, In how many methods, the vacancies can be? filled the four jobs?

Since the ordering of jobs is not important, that this case represents a combinations.

$$C_r^n = \frac{n!}{(n-r)!r!}$$
 Writing the combinations law

$$C_4^{10} = \frac{10!}{(10-4)!4!}$$
 Substituting $n = 10$, $r = 4$

$$C_4^{10} = \frac{10!}{6!4!}$$
$$= \frac{(10)(9)(8)(7)(6!)}{(6!)(4)(3)(2)(1)}$$
 Simplify
$$= 210$$

Then, there are 210 ways to fill the four jobs.

Make sure of your understanding**Find the value of each of the following:**

1 $4! \times 2!$ 2 $(3+2)!$ 3 $\frac{9!}{6!}$

4 $(7-5)!$ 5 $3! + 2!$ 6 P_8^8

7 P_4^{10} 8 C_3^8 9 $\binom{9}{0}$

Solve the Exercises**Find the value of each of the following:**

10 $2! \times 6!$ 11 $4! \times 3!$ 12 $0! \times 1!$

13 P_0^{10} 14 $\binom{10}{1}$ 15 C_5^9

16 P_3^7 17 P_1^{15} 18 C_{100}^{100}

Solve the Problems

- 19 **Committees:** - In how many ways can we choose a triple committee among a group consists of 5 persons?
- 20 **Committees:** - In how many ways can we choose a triple committee which consists of, president, vice- president and treasurer among a group consists of 5 persons?
- 21 **Paintings:** - An artist drew 7 paintings. In how many ways can we select 5 paintings to be shown in an exhibition?
- 22 **Test:** - A questions paper contains 12 questions, it is required to answer 10 questions. In how many methods can the questions be selected?



Think

- 23 **Challenge: Find the value:**
- i) $\frac{15! \cdot 9!}{14! \cdot 10!}$ ii) $\frac{5!}{3! \times 1!} \times \frac{6!}{5! \times 4!}$
- 24 **Which one is correct?** Four students from seven students were chosen to form a committee, the number of choices either C_4^7 or P_4^7 . Illustrate your answer.
- 25 **Justification:-** When the statements $C_r^n = P_r^m$ can be happened?
- 26 **Critical thinking:-** What is the relation among 3 orderings out of 5?, and 3 combinations out of 5? Write the relation by calculating each of them.
- 27 **Numerical Problem:-** Find the value n which makes $\frac{n!}{(n-1)!} = 9$

Write

A question to choose 3 things out of 5, but you should pay attention to the ordering.

Lesson [6-4]

Experimental Probability and Theoretical Probability

Idea of the lesson:

- * Counting the experimental probability
- * Counting the theoretical probability

Vocabulary:

- * The experimental Probability
- * The theoretical Probability
- Sample space

Learn

Mohanned tossed two coins 13 times and he wrote down the results which shown in the nearby table.

results	frequency
H,H	7
H,T	3
T,H	1
T,T	2

1- find the ratio $\frac{\text{number of appearance (H,T)}}{\text{number of element of the sample space}}$

2-Find the ratio $\frac{\text{number of appearance (H,T)}}{\text{number of experiments}}$

Does the ratio in question one equal the ratio in the second question? Explain that.

[6- 4- 1] Experimental probability and Theoretical probability.

You have learned counting the experimental and theoretical probabilities, where the determining of the probability in the paragraph (learning) by implementing the experimental and the results in this way is called the experimental probability. While the probabilities which based on facts and well-known properties are called the theoretical probabilities.

Example (1):

The sample space for the experiment of tossed two coins is:

Then, the elements number of the sample space equals 4

$$\Omega = \{(H,H), (H,T), (T,H), (T,T)\}$$

From the table, the number of times, in which event HT appeared, equals 3

Then, the theoretical probability $P(H, T) = \frac{3}{4}$

The ratio in the second question. $\frac{3}{13}$ In the table, the number of times, in which the event HT appeared, equals 3

The number of times of the experiment equals 13

Then the experimental probability $P(H, T) = \frac{3}{13}$

The theoretical probabilities: provide us with the results of experiment without implementing it
(depending on the sample space by experiment)

experimental probability: provide us with the results of experiment by repeating it many times
(depending on the repeating of experiment)

Example (2):

A researcher found in a factory of cars batteries that probability in which the battery is not good is $\frac{3}{20}$, is the probability theoretical or experimental?

and if the factory want to get 240 batteries which are not good. How many batteries

the factory has to produce? this probability is experimental, because it depends on what had really happened, we use proportionality for the all second part of the example.

And each 3 batteries out of 20 are not good.

Then, 240 batteries are not good out of x batteries produced by if factory.

$$\frac{3}{20} = \frac{240}{x}$$

$$3x = 4800$$

$$x = \frac{4800}{3}$$

$$x = 1600$$

write the proportion

cross product

dividing the equation on 3

Simplifying X= 1600

the factory must produce 800 batteries

Example (3):

When rolled the two dices at once, Find the probability of :

- i) The event: we get the sum 5 on the two faces of the dices.
- ii) The event: the digit on the face of the first dice is the double of the digit on the face of the second dice.

This is a theoretical probability: because the two dices were at once.

The number of digits of the first dice=6, the number of digits of the second dice=6

Then, according to the fundamental counting principle: the elements number of the sample space equals $6 \times 6 = 36$.



$$\Omega = \left\{ \begin{array}{l} (1,1) \dots\dots (1,6) \\ (2,1) \dots\dots (2,6) \\ \vdots \\ (6,1) \dots\dots (6,6) \end{array} \right\}, \quad n = 36$$

- i) $E_1 = \{(1,4), (4,1), (2,3), (3,2)\}, m = 4, n = 36$ Event :The sum on the two faces of the dices is 5

$$P(E_1) = \frac{m}{n}$$

Probability formula

$$P(E_1) = \frac{4}{36} = \frac{1}{9}$$

Substitution and simplify

- ii) $E_2 = \{(2,1), (4,2), (6,3)\}, m = 3, n = 36$

Event :digit on first dice is double digit of second dice

$$\therefore P(E_2) = \frac{3}{36} = \frac{1}{12}$$

[6-4-2] Disjoint Events

The two disjoint events: are two events which can not be happened together in one experiment.

Example:- when we rolled the dice for once, the opportunity of getting an odd number and an even number at the same time impossible.

Then they are two disjoint events

Calculate the probability of the two disjoint events

If E_1, E_2 are two disjoint events, then the probability of happening E_1 or E_2 equals the sum of the two events probability

That means $P(E_1 \text{ or } E_2) = P(E_1) + P(E_2)$

Example (4):

When rolled the dice for one time, find the probability of getting the number 3 or an even number.

Because it is impossible that the number 3 appears on the face of dice, and at the same time an even number appears too! then these two events are disjoint.

$$\Omega = \{1, 2, 3, 4, 5, 6\} \quad \text{the sample space}$$

$$P(E_1) = \frac{m}{n} \Rightarrow m = 1, n = 6 \Rightarrow P(E_1) = \frac{1}{6}$$

$$P(E_2) = \frac{m}{n} \Rightarrow m = 3, n = 6 \Rightarrow P(E_2) = \frac{3}{6}$$

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2)$$

$$P(E_1 \text{ or } E_2) = \frac{1}{6} + \frac{3}{6} = \frac{4}{6} = \frac{2}{3}$$

Probability of getting the number 3
Probability of getting the even number
Probability of the disjoint events
By substitution and simplifying

Then, the Probability of appearing the number 3 or an even number when the dice was rolled equals $\frac{2}{3}$

Example (5):

If we rolled the two dice for once, find the Probability of getting two equaled numbers or the sum of two equaled numbers is 3

The number of the sample space elements when the two dice are rolled is 36

$$E_1 = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$$

$$P(E_1) = \frac{\text{number of elements of } E_1}{\text{sample space}} = \frac{6}{36}$$

$$E_2 = \{(1,2), (2,1)\}$$

$$P(E_2) = \frac{\text{number of elements of } E_2}{\text{sample space}} = \frac{2}{36}$$

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2)$$

$$= \frac{6}{36} + \frac{2}{36} = \frac{8}{36} = \frac{2}{9}$$

E_1, E_2 are two disjoint events
Probability of the disjoint events
By substitution and simplifying

Make sure of your understanding

In the experiment of rolled the two dices for once, find the probability of happening the following events:

- 1 The two numbers on the face of the two dice are equaled.
- 2 The number on the face of the first dice represents the half of the second dice number
- 3 The sum of the two numbers on the faces of the two dices is 10.
- 4 The sum of the two numbers on the faces of the two dices is less than 5.
- 5 Are the previous probabilities experimental or theoretical?
- 6 A sack contains 4 red balls and green ball, how many blue balls should be added to the sack to make the probability of pulling a red ball is $\frac{2}{3}$? is the probability experimental or theoretical?
- 7 A man was standing at one of Baghdads squares, he counted 25 cars: 13 cars are yellow, 7 cars are white and 5 cars are grey. Guess that the next car which crosses the squar is yellow one, and what is the probability experimental or theoretical? Write the ratio in form of decimal fraction and percentage.
- 8 When two dices are rolled, find the probability of getting two numbers which their sum is 5 or 11, are the two events disjoint? Explain that.

Solve the Exercises

In the experiment of rolled the two dices at the once, find the probability of happening the following events:

- 9 The sum of the two numbers on the two faces of the two dices is more than 8.
- 10 The sum of the two numbers on the two faces of the two dices equal 12.
- 11 A study includes 100 persons had been done, 15 persons side that they use their left hands. If this study involves 400 persons, then according to your expectation how many persons use their left hands?

Solve the exercises

- 12 Find the probability of pulling a card with an odd number or with a number which represents the multiples of 2 from a group of cards which were numbered from 1 to 9.

Solve the problems



- 13 **Entertainment:** In which colour we can paint the space to say that the probability of coming the spinner to this colour is $\frac{1}{4}$?
- 14 **Stamps:** Mohannad likes to collect stamps; among 60 stamps he collected 25 stamps from the Arab countries, 15 stamps from African countries and 20 stamps from European countries. Guess the probability that the next stamp will be European one
- 15 **Sport:** In atraining of basketball, the player scored 15 balls from 25 throws, what is the experimental probability in which the basketball player will score the next throw? Write the answer as a fraction, a decimal number and percentage.
- 16 **Study:** A man said that there are three persons in his family have blue eyes from each 22 persons. If he expected a new born, what is the probability that the new baby's eyes will be blue?

Think



- 17 **Challenge:** A disc with a spinner. It was divided into three parts as in the nearby figure: Its half is green, third of it is red and sixth of it is blue . what is the probability in which the spinner will refer to the green or red colour?
- 18 **Discover the mistake:** Sarah and Mohannad want to determine the probability of choosing red or blue ball randomly from a sack contains 5 blue balls. 4red balls and 6 yellow balls. Which answer is correct? Illustrate your answer.



Mohannad	Sarah
$P(R \text{ or } B) = P(R) \times P(B) = \frac{4}{15} \times \frac{5}{15} = \frac{4}{45}$	$P(R \text{ or } B) = P(R) + P(B) = \frac{4}{15} + \frac{5}{15} = \frac{9}{15} = \frac{3}{5}$

Write

Explain what each number in $\frac{2}{9}$ which represents in both, the experimental and theoretical probability.

Lesson [6-5]

Compound Events

Idea of the lesson:

- Calculate the probability of independent events.
- Calculate the probability of depended events.

Vocabulary:

- Independent events.
- Dependent events.

Learn

The reports of the Iraqi airlines company refer to the accurateness of the arrival of its airplanes which $\frac{19}{20}$ as a ratio, and the not 2% refer to lose baggaes. What is the probability of arrival an airplane in its accurate time and without losing baggaes?



[6 - 5- 1] Independent Events

You have previously learned the concept of the independent events (the result of one event doesn't affect on the result of the other). In this lesson, we will learn how to calculate the probability of independent events, If E_1, E_2 were two independent events, then the probability of that they can be happened together equals the result of multiplying the probability E_1 by the event probability E_2 . That means $P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2)$

Example (1):

In the paragraph of learn:-

The probability of arrival the airplane in its accurate time is $P(E_1) = \frac{19}{20}$

The probability of the baggages will be lost is $P(E_2) = \frac{1}{50}$

The arrival of the airplane in its accurate time

will not affect on the losing of baggages, which means that the two events are independent.

$$P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2)$$

The probability of the independent events

$$= \frac{19}{20} \times \frac{1}{50}$$

By substitution and simplifying

$$= \frac{19}{1000} = 0.019 = 1.9\%$$

Example (2):

A sack contains 3 red balls, 4 green balls and 5 blue balls. A ball was taken randomly, then it was returned and another ball was taken. Find the probability of taking a red ball at first, and then a green ball.

$$P(R) = \frac{\text{number of red ball}}{\text{total number of balls}} = \frac{3}{12} = \frac{1}{4}$$

Taking red ball

$$P(G) = \frac{\text{number of green ball}}{\text{total number of balls}} = \frac{4}{12} = \frac{1}{3}$$

Taking green ball

$$P(R \text{ and } G) = P(R) \times P(G)$$

The two events are independent

$$= \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

The probability of the independent events

By substitution and simplifying

then the probability of taking a red ball and then a green ball with returing the red ball equal $\frac{1}{12}$

Example (3):

If one of the numbered cards was chosen, and then the spinner of disc was rotated, as shown in the nearby figure.

What is the probability that the result will be an even number and the colour will be blue?

-We suppose that $P(E_1)$ is a probability of the even number

$$P(E_1) = \frac{2}{4} = \frac{1}{2}$$

-We suppose that $P(E_2)$ is a probability of stopping the spinner at

the blue colour. $P(E_2) = \frac{1}{4}$

The probability of the independent events

$$P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2)$$

$$= \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

By substitution and simplifying

So, the probability (an even number and blue colour) is $\frac{1}{8} = 12.5 \%$

[6-5-2] Dependent Events

The dependent events (they affected by the results of each other)

If E_1 and E_2 are two dependent events, the probability of happening them together is the result of multiplying the probability of the first event E_1 by (the probability of the event E_2 after happening the event E_1)

That means:- $P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2 \text{ after } E_1)$

Example (4):

In example (2) if we didn't return the red ball into the sack.

What is the probability of taking a red ball and then a green ball?

$$P(R) = \frac{\text{number of red ball}}{\text{total number of balls}} = \frac{3}{12} = \frac{1}{4}$$

Taking the red ball

If we don't return the red ball to the sack. That means the number of red balls becomes 2 and the total number of the balls, in this case, is 11 instead of 12.

$$P(G \text{ after } R) = \frac{\text{number of green ball}}{\text{total number of balls}} = \frac{4}{11}$$

Taking a green ball

The two event are dependent

$$P(R \text{ and } G) = P(R) \times P(G \text{ after } R)$$

The probability of the dependent event

$$= \frac{1}{4} \times \frac{4}{11} = \frac{1}{11}$$

By substitution and simplifying

Then, the probability of taking a red ball and then a green ball without returning the red ball equal . $\frac{1}{11}$

Example (5):

A box contains 5 red balls, 3 blue balls and 8 yellow balls. A ball was taken from the box, and then another one was taken without returning the first one. Find $P(\text{yellow, and then red})$.

Suppose $P(Y)$ is taking yellow ball $\Rightarrow P(Y) = \frac{8}{16} = \frac{1}{2}$

Without returning the yellow ball, the total number of balls in the box becomes

15 balls, as follow:- 5 red balls, 3 blue balls and 7 yellow balls.

A red ball was taken from the box

$$P(R \text{ after } Y) = \frac{5}{15} = \frac{1}{3} \quad \text{The two events are dependent}$$

$$P(Y \text{ and } R) = P(Y) \times P(R \text{ after } Y) \quad \text{The probability of the dependent events}$$

$$= \frac{1}{2} \times \frac{1}{3} = \frac{1}{6} \quad \text{By substitution and simplifying}$$

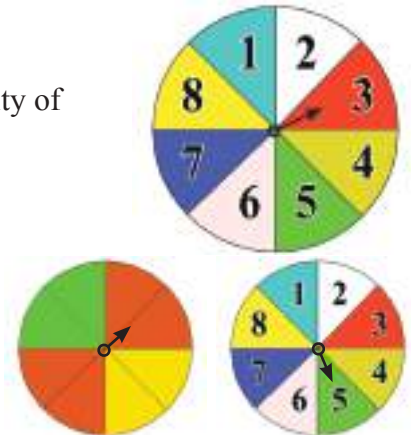
Then, the probability of taking a yellow ball and then a red ball without returning the yellow ball is $\frac{1}{6}$

Conclusion

1. We find $P(E_2), P(E_1)$
2. If E_1, E_2 are independent, then $P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2)$
3. If E_1, E_2 are dependent, then $P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2 \text{ after } E_1)$

Make sure of your understanding

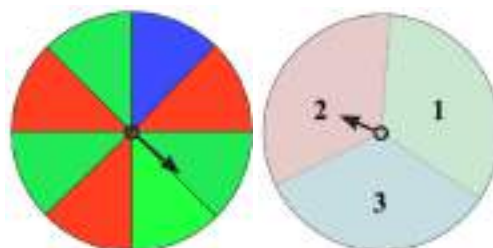
- 1 A box contains 3 red balls and 3 green balls, what is the probability of taking two green balls without returning the first ball?
- 2 Spinner in the two opposite discs were released at the same time, what is the probability in which the first spinner comes to the red colour and the second spinner comes to number 5?
- 3 If we flap two coins at once, what is the probability of appearing a picture (head) in the first coin and writing (tail) in the second coin?

**Solve the Exercises**

- 4 A box contains 5 red cards, 4 black cards and 6 green cards. A card was taken and then a second card was taken without returning the first one. What is the probability that the first card was red and the second was black?

5 A spinner was released in the two nearby discs at once, what is the probability in which the spinner of the first disc comes to the green colour and the second spinner comes to the number of 3?

6 The two dices were rolled at once, what is the probability of appraring a number which can be divided on 3 in the first dice and a number which can be divided on 5 in the second dice?



Solve the problems

- 7 **Sweets:** A box contains 10 pieces of sweets with strawberry flavor, 15 pieces with chocolate flavor and 5 pieces with lemon flavor. What is the probability of choosing two pieces randomly, the first with chocolate flavor without returning and the second with lemon flavor?
- 8 **Books:** Suha chose a book from a shelf in her room and returned it to take another one, what is the probability in which the chosen book would be from Maths? It is worth to be mentioned that the shelf contains 5 books of Maths, 2 books of English language and 3 books of science.

Think

- 9 **Discover the mistake:** Jumana and her sister Sally want to determine the probability of choosing a red ball and a yellow ball randomly by taking them from a sack contains 4 red balls and 5 yellow balls without returning the ball after taking it.

Jumana

P(red and yellow)

P(yellow) \times P(red)

$$\frac{4}{9} \times \frac{5}{9}$$

Sally

p(red and yellow)

p(yellow) \times p(red)

$$\frac{4}{9} \times \frac{5}{8}$$

Which one of them is correct?

- 10 **Challenge:** A dice and a coin were thrown, what is the probability of appearing a digit which is greater than 2 and less than 6 on the dice and the writing (tail) on the coin?

- 11 **Open problem:** 10 cards with three different shapes. Write a problem related to pull two cards randomly without returning them, the probability have to be $\frac{1}{15}$.

Write

An example on two independent events and another example on two dependent events.

Chapter Test

1 Distributed questionnaires to 30 students from 100 students answer the following:

- i) Determine the sample and society which the sample was taken from.
- ii) Describe the method of the questionnaires distributed.
- iii) Determine if the sample was biased or unbiased.

2 How to distinguish between the misleading data graphics and unmisleading data graphics.

3 Find the value of the following:

- i) C_0^5
- ii) P_0^5
- iii) C_{10}^{10}
- iv) P_{10}^{10}
- v) C_5^7
- vi) P_5^7

4 In how many ways can choose committee of 3 students among a group consists of 8 students?

5 When throwing the dice for 25 times, the results shows in the following table:

Result	1	2	3	4	5	6
Count	2	6	3	5	2	7

- i) What is the type of the probability?
- ii) Find the probability to getting the number 8.

6 In the experiment of rolled the dice for once, find:

- i) Type of the probability experiment or theoretical.
- ii) Probability to getting a number divided by 4.

7 Mohannad was standing at one of Baghdad's squares, he counted 20 cars, 10 cars are sedan, 7 cars are small transportation, 3 cars are truck, guess that the next car which crosses the squares.

Exercises of chapters



Unit 1

**Relations and Inequalities
in Real Numbers**



Unit 2

Algebraic Expressions



Unit 3

Equations



Unit 4

Coordinate Geometry



Unit 5

Geometric and Measurement



Unit 6

Statistics and Probabilities

Multiple Choice

[1-1] Ordering Operations in Real Numbers

Choose the correct answer for each of the following:

Simplify the following numerical sentences by using the ordering of operations in the real numbers:

1 $(\sqrt{2} + \sqrt{7})(\sqrt{2} + \sqrt{7}) = \dots$ a) $2+9\sqrt{7}$ b) $2+9\sqrt{2}$ c) $9+2\sqrt{14}$ d) $2+9\sqrt{14}$

2 $\frac{6\sqrt{50}}{3\sqrt[3]{-8}} \div \frac{2\sqrt{14}}{\sqrt{7}} = \dots$ a) $\frac{-5}{2}$ b) $\frac{-2}{2}$ c) $\frac{\sqrt{2}}{5}$ d) $\frac{-\sqrt{2}}{5}$

3 $(-27)^{\frac{1}{3}} (\frac{1}{6}\sqrt{2} - \frac{1}{4}\sqrt{32}) = \dots$ a) $\frac{-5}{\sqrt{2}}$ b) $\frac{5}{\sqrt{2}}$ c) $\frac{\sqrt{2}}{5}$ d) $\frac{-\sqrt{2}}{5}$

Simplify the following numerical sentences by using rationalizing the denominator and ordering the operations in the real numbers:

4 $\frac{(\sqrt{2} - \sqrt{3})}{(\sqrt{2} + \sqrt{3})} = \dots$ a) $5 + 6\sqrt{2}$ b) $5 - 6\sqrt{2}$ c) $2\sqrt{6} - 5$ d) $2\sqrt{6} + 5$

Use the ordering of operations and write the result neared to two decimal places using the calculator for each of the following:

5 $(\frac{1}{3})^2 - 3^{-2} - (5)^{\frac{3}{2}} \approx \dots$ a) -18.11 b) 18.11 c) 11.18 d) -11.18

Multiple Choice

[1-2] Mappings

Choose The Correct Answer for each of the following:

1 If , $f: Z \rightarrow R$ where $f(x) = 3x - 2$ then the number 10 is the image of the number:

- a) 5 b) 4 c) 3 d) 2

2 If , $f: A \rightarrow B$, where $A = \{2, 3, 4, 5\}$, $B = \{4, 6, 8\}$ and $f = \{(2, 4), (3, 6), (4, 8), (5, 8)\}$

then f is surjection mapping because:

- a) Range \neq co-domain b) f does not injection
c) The range is the domain of A d) Range = co-domain

3 If , $f: Z \rightarrow Z$ where $f(x) = 2x - 3$, $g: Z \rightarrow Z$ where $g(x) = x + 1$ then the mapping $(g \circ f)(x)$ is:

- a) $2x - 2$ b) $2x - 4$ c) $2x + 2$ d) $2x + 4$

4 Let $f: \{2, 3, 5\} \rightarrow N$ where $f(x) = 3x - 1$

and $g: N \rightarrow N$ where $g(x) = x + 1$ then the range of $(g \circ f)$ is:

- a) $= \{5, 8, 14\}$ b) $= \{5, 6, 9\}$
c) $= \{6, 12, 15\}$ d) $= \{6, 9, 12\}$

5 If the mapping $f: Q \rightarrow Q$ where $f(x) = 4x + 1$ and the mapping $g: Q \rightarrow Q$ where

$g(x) = \frac{1}{3}x^2 - 1$ if $(f \circ g)(x) = 45$ then the value of x is :

- a) ± 5 b) ± 6 c) ± 7 d) ± 8

Multiple Choice

[1-3] The sequences

Choose The Correct Answer for each of the following:

Write the first five terms for each of the following sentences:

1 $\{5n - 2\} = \dots$

a) $\{2, 6, 12, 16, 20\dots\}$

b) $\{3, 8, 13, 18, 23\dots\}$

c) $\{4, 8, 12, 18, 22\dots\}$

d) $\{5, 10, 16, 20, 24\dots\}$

2 $\{\frac{n}{2} + 1\} = \dots\dots$

a) $\{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}\}$

b) $\{\frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \dots\}$

c) $\{\frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2} \dots\}$

d) $\{2, \frac{5}{2}, 3\frac{7}{2}, 4 \dots\}$

3 An arithmetic sequence, its second term is 3 and its common difference is 3 .

a) $\{0, 3, 6, 9, 12, \dots\}$

b) $\{2, 5, 8, 11, 14, \dots\}$

c) $\{3, 6, 9, 12, 15, \dots\}$

d) $\{1, 4, 7, 10, 13, \dots\}$

4 An arithmetic sequence the ninth term and fifteenth term which its second term is 2 and common difference 2 is :

a) $u_9 = 12, u_{15} = 20$

b) $u_9 = 14, u_{15} = 24$

c) $u_9 = 16, u_{15} = 28$

d) $u_9 = 18, u_{15} = 32$

5 The terms between U_2 and U_6 for an arithmetic sequence which its second term is $\frac{9}{5}$ and its common difference 2 .

a) $\{\frac{9}{2}, \frac{19}{2}, \frac{29}{2}, \dots\}$

b) $\{\frac{19}{2}, \frac{29}{2}, \frac{39}{2}, \dots\}$

c) $\{\frac{9}{5}, \frac{19}{5}, \frac{29}{5}, \dots\}$

d) $\{\frac{19}{5}, \frac{29}{5}, \frac{39}{5}, \dots\}$

Multiple Choice

[1-4] Compound Inequalities

Choose The Correct Answer for each of the following:

Solve the compound inequalities which include (and) algebraically:

1 $-10 < x$ and $x \leq -2$ a) $\{x: -10 \leq x\} \cap \{x: x \leq -2\}$ b) $\{x: -10 < x\} \cap \{x: x \leq -2\}$

c) $\{x: -10 \leq x\} \cup \{x: x \leq -2\}$ d) $\{x: -10 < x\} \cup \{x: x \leq -2\}$

2 $16 < 3z + 9$ and $3z + 9 < 30$ a) $\{z: \frac{3}{7} \leq z < 7\}$ b) $\{z: \frac{7}{3} < z \leq 7\}$

c) $\{z: \frac{3}{7} < z < 7\}$ d) $\{z: \frac{7}{3} < z < 7\}$

Solve the compound inequalities which include (or) algebraically:

3 $\frac{y+5}{3} < \frac{1}{3}$ or $\frac{y+5}{3} > \frac{7}{3}$ a) $\{y: y < 4\} \cap \{y: y > 2\}$ b) $\{y: y > -4\} \cup \{y: y < 2\}$

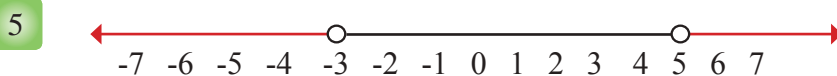
c) $\{y: y < -4\} \cap \{y: y > -2\}$ d) $\{y: y < -4\} \cup \{y: y > 2\}$

Write the compound inequality which shows the range of the third side length in a triangle

which the lengths of its other two sides are known:

4 8cm, 2cm a) $6 \leq x < 10$ b) $6 \leq x \leq 10$ c) $6 < x < 10$ d) $6 < x \leq 10$

Write the inequalities represent the solution set in the line of numbers:



a) $y \leq -3$ or $y > 5$ b) $y \leq -3$ or $y \geq 5$ c) $y < -4$ or $y \geq 5$ d) $y < -3$ or $y > 5$

Multiple Choice

[1-5] Absolute Value Inequalities

Choose The Correct Answer for each of the following:

Solve the following absolute value inequalities:

1 $|y - 8| < 13$ a) $5 < y < -21$ b) $-5 \leq y \leq 21$

c) $-5 < y < 21$ d) $-5 < y \leq 21$

2 $|3x| - 7 < 1$ a) $-\frac{8}{3} \leq x < \frac{8}{3}$ b) $-\frac{8}{3} < x \leq \frac{8}{3}$

c) $-\frac{8}{3} \leq x \leq \frac{8}{3}$ d) $-\frac{8}{3} < x < \frac{8}{3}$

3 $|6 - 3y| \geq 9$ a) $y \leq 1$ or $y \geq -5$ b) $y < -1$ or $y > 5$

c) $y > -1$ or $y < 5$ d) $y < -1$ or $y \geq 5$

4 $|\frac{7 - 2y}{3}| \geq 3$ a) $y \leq -1$ or $y \geq 8$ b) $y < -1$ or $y \geq 8$

c) $y < -1$ or $y > 8$ d) $y < -1$ or $y > 8$

Multiple Choice

[2-1] Multiplying Algebraic Expressions

Choose The Correct Answer for each of the following:

Find the result of multiplying and algebraic expression by another algebraic expression:

1 $(x + 5)^2$ a) $x^2 - 10x + 25$ b) $x^2 + 10x + 25$ c) $x^2 + 5x + 25$ d) $x^2 - 5x + 25$

2 $(z - \sqrt{7})^2$ a) $z^2 - 7z + 49$ b) $z^2 + 7z + 49$ c) $z^2 - \sqrt{7}z + 7$ d) $z^2 - 2\sqrt{7}z + 7$

3 $(x + 8)(x - 8)$ a) $x^2 - 64$ b) $x^2 + 64$ c) $x^2 + 16$ d) $x^2 - 16$

4 $(y + \sqrt{6})(y - \sqrt{6})$ a) $y^2 - \sqrt{12}$ b) $y^2 - 6$ c) $y^2 + \sqrt{12}$ d) $y^2 + 6$

5 $(y - 2)(y^2 + 2y + 4)$ a) $y^3 + 8$ b) $y^3 - 8$ c) $y^3 - 4$ d) $y^3 - 16$

6 $(y + \frac{1}{5})^3$ a) $y^3 - \frac{3}{3}y^2 + \frac{3}{25}y - \frac{1}{125}$ b) $y^3 + \frac{3}{5}y^2 - \frac{3}{25}y + \frac{1}{125}$

c) $y^3 + \frac{3}{5}y^2 + \frac{3}{25}y + \frac{1}{125}$ d) $y^3 - \frac{3}{5}y^2 - \frac{3}{25}y - \frac{1}{125}$

Multiple Choice

[2-2] Factoring the Algebraic Expression by using a greater Common Factor .

Choose The Correct Answer for each of the following:

Factoring each expression by using the greatest common factor:

1 $6y^2(3y - 4) + 36y$

a) $6y(3y^2 + 4y + 6)$

b) $6y(3y^2 + 4y - 6)$

c) $6y(3y^2 - 4y - 6)$

d) $6y(3y^2 - 4y + 6)$

Factoring each expression by using the binomial term as a greatest common factor:

2 $\frac{1}{4}(x + 9) - \frac{1}{2}x^2(x + 9)$

a) $(x + 9)(\frac{1}{4} - \frac{1}{2}x^2)$

b) $(x - 9)(\frac{1}{4} - \frac{1}{2}x^2)$

c) $(x + 9)(\frac{1}{4} + \frac{1}{2}x^2)$

d) $(x + 9)(\frac{1}{2} - \frac{1}{4}x^2)$

3 $\sqrt{2}v(x - 1) - \sqrt{3}t(x - 1)$

a) $(x + 1)(\sqrt{2}v - \sqrt{3}t)$

b) $(x - 1)(\sqrt{2}v - \sqrt{3}t)$

c) $(x - 1)(\sqrt{2}v + \sqrt{3}t)$

d) $(x + 1)(\sqrt{2}v + \sqrt{3}t)$

Factoring each expression by using the property of grouping, then check the correct of solution:

4 $3y^3 - 9y^2 + 5y - 15$

a) $(y + 3)(3y^2 + 5)$

b) $(y - 3)(3y^2 - 5)$

c) $(y - 3)(3y^2 + 5)$

d) $(y - 3)(3y^2 - 5)$

Factoring the expression by using the property of grouping with the inverse:

5 $20y^3 - 4y^2 + 3 - 15y$

a) $(5y + 1)(4y^2 - 3)$

b) $(5y - 1)(4y^2 + 3)$

c) $(5y - 1)(4y^2 - 3)$

d) $(5y + 1)(4y^2 + 3)$

Multiple Choice

[2-3] Factoring the Algebraic Expression by using Special Identities.

Choose The Correct Answer for each of the following:

Factor each of the following algebraic expressions:

- | | | | |
|---|----------------------------------|--|--|
| 1 | $12y^3z - 3yz^3$ | a) $3y(2y - z)(y + 2z)$ | b) $3z(2y - z)(2y + z)$ |
| | | c) $3yz(2y - z)(2y + z)$ | d) $3yz(y - 2z)(y + 2z)$ |
| 2 | $\frac{1}{6}x^3 - x\frac{1}{24}$ | a) $\frac{x}{6}(x + \frac{1}{2})(x - \frac{1}{2})$ | b) $\frac{x}{6}(x + \frac{1}{4})(x - \frac{1}{4})$ |
| | | c) $\frac{x}{3}(\frac{1}{2}x + \frac{1}{2})(\frac{1}{2}x - \frac{1}{2})$ | d) $\frac{x}{6}(\frac{1}{4}x + \frac{1}{4})(\frac{1}{4}x - \frac{1}{4})$ |
| 3 | $4x^2 + 24x + 36$ | a) $(x + 6)^2$ | b) $(x - 6)^2$ |
| | | c) $4(x - 3)^2$ | d) $4(x + 3)^2$ |

Determine which of the following algebraic expressions represents a perfect square:

- | | | | |
|---|-------------------|------------------------------------|--------------------------|
| 4 | $64 - 48y + 9y^2$ | it is not a perfect square because | a) $2(4)(3y) \neq -48y$ |
| | | a perfect square because | b) $2(8)(4y) = 48y$ |
| | | it is not a perfect because | c) $-2(8)(3y) \neq -48y$ |
| | | a perfect square because | d) $-4(4)(3y) = 48y$ |

Write the missing term in the algebraic expression $ax^2 + bx + c$ to become a perfect square:

- | | | | | | |
|---|--------------------|-----------|------------|-----------|------------|
| 5 | $z^2 + \dots + 49$ | a) $14z$ | b) $-10z$ | c) $7z$ | d) $-7z$ |
| 6 | $36 - 24x + \dots$ | a) $2x^2$ | b) $-2x^2$ | c) $4x^2$ | d) $-4x^2$ |

Multiple Choice

[2-4] Factoring the Algebraic Expression of three terms by Trial and Error (Experiment).

Choose The Correct Answer for each of the following:

Factor each of the following algebraic expressions:

1 $x^2 + 7x + 12$ a) $(x - 3)(x + 4)$ b) $(x + 3)(x + 4)$

c) $(x - 1)(x + 7)$ d) $(x - 3)(x - 4)$

2 $x^2 - 5x - 36$ a) $(x - 6)(x + 6)$ b) $(x + 12)(x - 3)$

c) $(x - 9)(x + 4)$ d) $(x + 9)(x - 4)$

3 $y^2 + 4y - 21$ a) $(y - 7)(y + 3)$ b) $(y + 7)(y - 3)$

c) $(y - 7)(y - 3)$ d) $(y + 7)(y + 3)$

Put signs between the terms inside brackets to make the factoring of the algebraic expression

correct:

4 $4y^2 - 2y - 12 = (2y \dots 3)(2y \dots 4)$ a) $(2y - 3)(2y + 4)$ b) $(2y + 3)(2y + 4)$

c) $(2y - 3)(2y - 4)$ d) $(2y + 3)(2y - 4)$

5 $48 - 30z + 3z^2 = (6 \dots 3z)(8 \dots z)$ a) $(6 - 3z)(8 - z)$ b) $(6 + 3z)(8 + z)$

a) $(6 - 3z)(8 + z)$ b) $(6 + 3z)(8 - z)$

Multiple Choice

[2-5] Factoring the Algebraic Expressions Sum of Two

Cubes or difference Between Two Cubes.

Choose The Correct Answer for each of the following:

Factor each of the following algebraic expressions in simplest form:

- 1 $8 + x^3$ a) $(2 - x)(4 + 2x + x^2)$ b) $(2 + x)(4 - 2x + x^2)$
c) $(2 - x)(4 - 2x + x^2)$ d) $(2 + x)(4 + 2x + x^2)$
- 2 $\frac{1}{z^3} + \frac{1}{64}$ a) $(\frac{1}{z} + \frac{1}{4})(\frac{1}{z^2} + \frac{1}{4z} + \frac{1}{16})$ b) $(\frac{1}{z} - \frac{1}{4})(\frac{1}{z^2} - \frac{1}{4z} + \frac{1}{16})$
c) $(\frac{1}{z} - \frac{1}{4})(\frac{1}{z^2} + \frac{1}{4z} + \frac{1}{16})$ d) $(\frac{1}{z} + \frac{1}{4})(\frac{1}{z^2} - \frac{1}{4z} + \frac{1}{16})$
- 3 $\frac{27}{125} + \frac{8}{x^3}$ a) $(\frac{3}{5} - \frac{2}{x})(\frac{9}{25} + \frac{6}{5x} + \frac{4}{x^2})$ b) $(\frac{3}{5} - \frac{2}{x})(\frac{9}{25} - \frac{6}{5x} + \frac{4}{x^2})$
c) $(\frac{3}{5} + \frac{2}{x})(\frac{9}{25} - \frac{6}{5x} + \frac{4}{x^2})$ d) $(\frac{3}{5} + \frac{2}{x})(\frac{9}{25} - \frac{6}{5x} - \frac{4}{x^2})$
- 4 $9 - \frac{1}{3} z^3$ a) $\frac{1}{3} (3 - z)(9 + 3z - z^2)$ b) $\frac{1}{3} (3 - z)(9 + 3z + z^2)$
c) $\frac{1}{3} (3 + z)(9 + 3z + z^2)$ d) $\frac{1}{3} (3 - z)(9 - 3z + z^2)$
- 5 $0.008x^3 - 1$ a) $(0.02x - 1)(0.04x^2 + 0.002x + 1)$ b) $(0.02x - 1)(0.04x^2 + 0.02x + 1)$
c) $(0.2x + 1)(0.4x^2 - 0.2x + 1)$ d) $(0.2x - 1)(0.04x^2 + 0.2x + 1)$

Multiple Choice

[2-6] Simplifying Rational Algebraic Expressions

Choose The Correct Answer for each of the following:

Write each of the following expressions in simplest form:

1 $\frac{x+3}{4x} \times \frac{4x-12}{x^2-9}$ a) $\frac{3}{x}$ b) $\frac{x}{4}$ c) $\frac{1}{4}$ d) $\frac{1}{x}$

2 $\frac{z^2-2z-15}{9+3z} \times \frac{5}{z^2-25}$ a) $\frac{5}{z+5}$ b) $\frac{3}{5(z+5)}$ c) $\frac{5}{3(z+5)}$ d) $\frac{3}{z+5}$

3 $\frac{1-z^3}{1+z+z^2} \div \frac{(1-z)^2}{1-z^2}$ a) $1-z$ b) $1+z$ c) $1+z+z^2$ d) $1-z+z^2$

Write each of the following expressions in simplest form:

4 $\frac{2y^2+1}{y^3-1} - \frac{y}{y^2+y+1}$ a) $\frac{y}{y+1}$ b) $\frac{1}{y+1}$ c) $\frac{1}{y-1}$ d) $\frac{y}{y-1}$

5 $\frac{3y+1}{y+4} - \frac{y-4}{3y-1} - \frac{10+8y^2}{3y^2+11y-4}$ a) $\frac{5}{(y+4)(3y-1)}$ b) $\frac{3}{(y+4)(3y-1)}$
c) $\frac{-3}{(y+4)(3y-1)}$ d) $\frac{-5}{(y+4)(3y-1)}$

Multiple Choice

[3-1] Solving system of two linear equations with two variables

Choose The Correct Answer for each of the following:

Find the set of solution of the system graphically in R:

1 $\left. \begin{array}{l} y = 4x - 6 \\ y = x \end{array} \right\}$ a) $\{(-2, -2)\}$ b) $\{(-2, 2)\}$ c) $\{(2, -2)\}$ d) $\{(2, 2)\}$

Find the set of solution of the system in R by using the substitution for each of the following:

2 $\left. \begin{array}{l} 3x + 4y = 26 \\ 5x - 2y = 0 \end{array} \right\}$ a) $\{(2, 5)\}$ b) $\{(-2, -5)\}$ c) $\{(2, -5)\}$ d) $\{(-2, 5)\}$

3 $\left. \begin{array}{l} \frac{3x}{4} - \frac{y}{2} = 4 \\ \frac{y}{2} - \frac{x}{4} = 2 \end{array} \right\}$ a) $\{(12, -10)\}$ b) $\{(-12, -10)\}$ c) $\{(12, 10)\}$ d) $\{(-12, 10)\}$

Find the set of solution of the system in R by using the elimination for each of the following:

4 $\left. \begin{array}{l} 7x - 4y = 12 \\ 3x - y = 5 \end{array} \right\}$ a) $\{(-\frac{8}{5}, \frac{1}{5})\}$ b) $\{(-\frac{8}{5}, -\frac{1}{5})\}$ c) $\{(\frac{8}{5}, \frac{1}{5})\}$ d) $\{(\frac{8}{5}, -\frac{1}{5})\}$

5 $\left. \begin{array}{l} 6y - 2x - 8 = 0 \\ y + x - 12 = 0 \end{array} \right\}$ a) $\{(8, -4)\}$ b) $\{(8, 4)\}$ c) $\{(-8, 4)\}$ d) $\{(-8, -4)\}$

Multiple Choice

[3-2] Solving Quadratic Equations with one variable

Choose The Correct Answer for each of the following:

Solve the following equations in \mathbb{R} by using the greatest common factor and the difference between two squares:

1 $7z^2 - 21 = 0$ a) $s = \{7, -7\}$ b) $s = \{3, -3\}$ c) $s = \{\frac{1}{3}, -\frac{1}{3}\}$ d) $s = \{\sqrt{3}, -\sqrt{3}\}$

2 $4(x^2 - 1) - 5 = 0$ a) $s = \{\frac{3}{2}, -\frac{3}{2}\}$ b) $s = \{\frac{1}{2}, -\frac{1}{2}\}$ c) $s = \{\frac{3}{2}, \frac{3}{2}\}$ d) $s = \{\frac{1}{2}, \frac{1}{2}\}$

3 $(y + 7)^2 - 81 = 0$ a) $s = \{2, -2\}$ b) $s = \{16, -16\}$ c) $s = \{2, -16\}$ d) $s = \{-2, 16\}$

Solve the following equations in \mathbb{R} by using the rule of square root:

4 $4(y^2 - 1) = 45$ a) $s = \{\frac{7}{2}, -\frac{7}{2}\}$ b) $s = \{\frac{7}{2}, \frac{7}{2}\}$ c) $s = \{\frac{2}{7}, -\frac{2}{7}\}$ d) $s = \{\frac{7}{4}, -\frac{7}{4}\}$

5 $x^2 - \frac{13}{16} = \frac{3}{16}$ a) $s = \{\frac{3}{4}, -\frac{3}{4}\}$ b) $s = \{\frac{\sqrt{3}}{4}, -\frac{\sqrt{3}}{4}\}$ c) $s = \{2, -2\}$ d) $s = \{1, -1\}$

Multiple Choice

[3-3] Solving the quadratic equations by the experiment

Choose The Correct Answer for each of the following:

Solve the following equations in R by factoring in the experiment:

1 $y^2 + 10y + 21 = 0$ a) $s = \{3, -7\}$ b) $s = \{-3, 7\}$ c) $s = \{-3, -7\}$ d) $s = \{3, 7\}$

2 $x^2 - 5x - 36 = 0$ a) $s = \{7, -8\}$ b) $s = \{-4, 9\}$ c) $s = \{4, -9\}$ d) $s = \{-4, -9\}$

3 $32 + 12x - 9x^2 = 0$ a) $s = \{\frac{4}{3}, \frac{8}{3}\}$ b) $s = \{\frac{-4}{3}, \frac{-8}{4}\}$ c) $s = \{\frac{4}{3}, \frac{-8}{3}\}$ d) $s = \{\frac{-4}{3}, \frac{8}{3}\}$

4 What is the number which its square increases in 42?

a) $s = \{7, 6\}$ b) $s = \{7, -6\}$ c) $s = \{-7, 6\}$ d) $s = \{-7, -6\}$

5 Two numbers its product is 54 , one of them increases in 3 to the other number , what are the two

numbers?

a) $s = \{6, 9\}$ b) $s = \{6, -9\}$ c) $s = \{-6, 9\}$ d) $s = \{-6, -9\}$

Multiple Choice

[3-4] Solving the quadratic equations by the perfect square

Choose The Correct Answer for each of the following:

Solve the following equations in R by the perfect Square:

- 1 $x^2 + 6x + 9 = 0$ a) $x = 6$ b) $x = -3$ c) $x = 4$ d) $x = 3$
- 2 $4z^2 - 20z + 25 = 0$ a) $z = \frac{-5}{2}$ b) $z = \frac{-2}{5}$ c) $z = \frac{5}{2}$ d) $z = \frac{2}{5}$
- 3 $\frac{1}{16} - \frac{1}{2}x + x^2 = 0$ a) $x = \frac{1}{4}$ b) $x = \frac{-1}{4}$ c) $x = \frac{1}{2}$ d) $x = \frac{-1}{2}$

Solve the following equations in R by completing the square:

- 4 $x^2 - 12x = 13$ a) $s = \{13, 1\}$ b) $s = \{13, -1\}$ c) $s = \{-13, 1\}$ d) $s = \{-13, -1\}$
- 5 $y^2 - \frac{1}{3}y = \frac{2}{9}$ a) $\{\frac{3}{2}, \frac{1}{3}\}$ b) $\{\frac{-3}{2}, \frac{1}{3}\}$
c) $\{\frac{2}{3}, \frac{-1}{3}\}$ d) $\{\frac{-2}{3}, \frac{1}{3}\}$
- 6 $z^2 + 2\sqrt{5}z = 4$ a) $s = \{3 + \sqrt{5}, 3 - \sqrt{5}\}$ b) $s = \{\sqrt{5} - 3, 3 - \sqrt{5}\}$
c) $s = \{3 - \sqrt{5}, -3 - \sqrt{5}\}$ d) $s = \{\sqrt{5} + 3, \sqrt{5} - 3\}$

Multiple Choice

[3-5] Using General Law to Solve the Equations.

Choose The Correct Answer for each of the following:

The solution set for the following equations by using the general law in R:

- 1 $y^2 - 5y - 5 = 0$ a) $s = \left\{ \frac{3+5\sqrt{5}}{2}, \frac{3-5\sqrt{5}}{2} \right\}$ b) $s = \left\{ \frac{5+3\sqrt{5}}{4}, \frac{3-5\sqrt{5}}{4} \right\}$
c) $s = \left\{ \frac{5+3\sqrt{5}}{2}, \frac{5-3\sqrt{5}}{2} \right\}$ d) $s = \left\{ \frac{5+3\sqrt{3}}{2}, \frac{3-3\sqrt{3}}{2} \right\}$
- 2 $2x^2 - 8x = -3$ a) $s = \left\{ \frac{4+\sqrt{10}}{2}, \frac{4-\sqrt{10}}{2} \right\}$ b) $s = \left\{ \frac{2+\sqrt{10}}{2}, \frac{4+\sqrt{10}}{2} \right\}$
c) $s = \left\{ \frac{4+\sqrt{5}}{4}, \frac{4-\sqrt{5}}{4} \right\}$ d) $s = \left\{ \frac{2+\sqrt{5}}{2}, \frac{2-\sqrt{5}}{2} \right\}$
- 3 $3x^2 - 6(2x+1) = 0$ a) $s = \{2 + \sqrt{3}, 2 - \sqrt{3}\}$ b) $s = \{2 + \sqrt{2}, 2 - \sqrt{2}\}$
c) $s = \{2 + \sqrt{6}, 2 - \sqrt{6}\}$ d) $s = \{6 + \sqrt{6}, 6 - \sqrt{6}\}$

Determine the roots of equation by using the distinctive:

- 4 $x^2 - 6x - 7 = 0$
- a) Two rational real roots b) Two irrational real roots
c) One real root $\left(\frac{-b}{2a}\right)$ d) Two unreal roots (the solution set in R = \emptyset)

- 5 What is the value of the constant K which makes the two roots of the equation

$$y^2 - (k + 10)y + 16 = 0 \text{ are equaled?}$$

- a) $k = 2, -18$ b) $k = -2, -18$ c) $k = 6, 18$ d) $k = -6, -18$

Multiple Choice

[3-6] Solving the Fractional Equations

Choose The Correct Answer for each of the following:

Find the solution for each of following equations in R:

1 $\frac{2}{12x^2} - \frac{1}{6} = \frac{1}{4x}$ a) $s = \{2, \frac{1}{2}\}$ b) $s = \{-2, \frac{1}{2}\}$ c) $s = \{2, \frac{-1}{2}\}$ d) $s = \{-2, \frac{-1}{2}\}$

2 $\frac{8x}{5} = \frac{5}{8x}$ a) $s = \{\frac{5}{8}, \frac{-8}{5}\}$ b) $s = \{\frac{5}{8}, \frac{8}{5}\}$ c) $s = \{\frac{5}{8}, \frac{-5}{8}\}$ d) $s = \{\frac{8}{5}, \frac{-8}{5}\}$

3 $\frac{16x - 64}{x^2} = 1$ a) $x = -8$ b) $x = 8$ c) $x = -6$ d) $x = 6$

Find the solution for each of following equations in R:

4 $\frac{2}{x-2} - \frac{3}{x-1} = 1$ a) $s = \{2 + \sqrt{7}, 2 - \sqrt{7}\}$ b) $s = \{1 + \sqrt{3}, 1 - \sqrt{3}\}$

c) $s = \{1 + \sqrt{7}, 1 - \sqrt{7}\}$ d) $s = \{2 + \sqrt{3}, 2 - \sqrt{3}\}$

5 $\frac{3y}{y-4} + \frac{y}{y-2} = \frac{-4y+8}{y^2-5y^2+8}$ a) $s = \{4, -2\}$ b) $s = \{-4, -2\}$ c) $s = \{-4, 2\}$ d) $s = \{4, 2\}$

Multiple Choice

[4 - 1]

Graphical Representation of the Equation in the Coordinate Plane

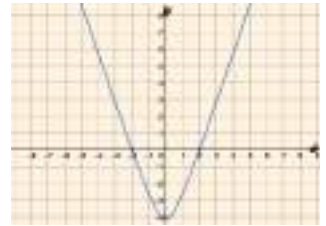
Choose the correct answer for each of the following:

1 The line which its equation is $y = \frac{3}{2}$

- a) Intersects two axes b) Parallels Y-axis c) Parallels X-axis
d) Does not intersect any of the two axes

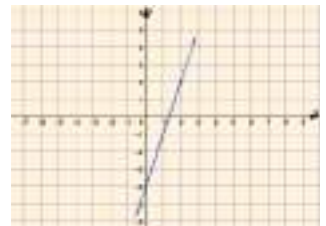
2 Which of the following equations express the equation which is graphically represented in the nearby figure?

- a) $y = -3x^2$ b) $y = 2x^2 + 4$
c) $y = x^2 - 4$ d) $y = 3x^2 - 4$



3 Which of the following equations express the equation which is graphically represented in the nearby figure?

- a) $y = 3x + 4$ b) $y = 4x + 3$
c) $y = -3x + 4$ d) $y = 3x - 4$

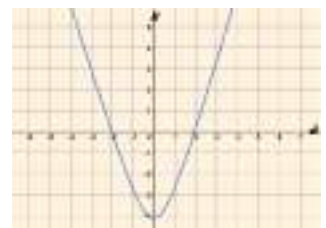
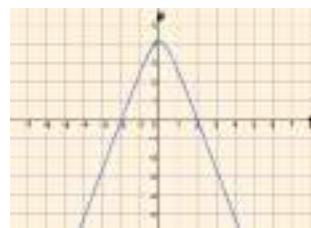
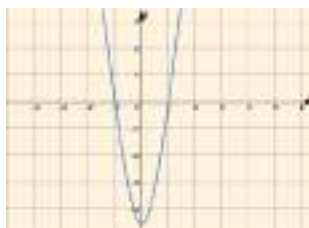
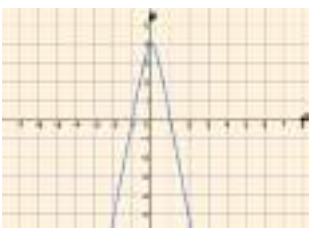


4 Which of the following equations express a linear equation?

- a) $y = x^2 + 1$ b) $y^2 = x + 1$ c) $y^2 = x^2 + 1$ d) $y = x + 1$

5 Which of the following graphic equations express the equation $y = -x^2 + 4$

- a) b) c) d)

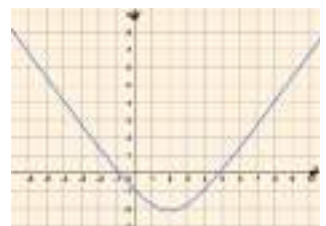


6 We need one of the following to represent the nonlinear equation:

- a) One point at least
b) Maximum ,two points
c) Only two points
d) At least, three points

7 What are the coordinates of the curved vertex which is represented in the nearby figur

- a) (2,-1) b) (1,2) c) (2,-2) d) (0,2)



Multiple Choice

[4 - 2]

Slop of the a Line

Choose the correct answer for each of the following:

- 1 Which slop expresses the slope of the line which passes through the two points $(-1, 3), (5, -2)$

a) $\frac{5}{6}$ b) $-\frac{6}{5}$ c) $-\frac{5}{6}$ d) $\frac{6}{5}$

- 2 The slop of the line which parallels the y_ axis will be:

a) Zero b) Unfind c) Negative d) Positive

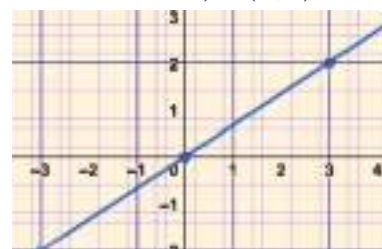
- 3 The Y- Intercept of the line which its equation $3x-5y=15$ is:

a) -5 b) 3 c) 5 d) -3

- 4 The intersection point of the line which its equation $x + y = 6$ with the X- axis is.

a) (0,6) b) (-6,0) c) (6,0) d) (0,0)

- 5 Which of the following lines expresses the line which is represented aside?



a) $2x - 3y = 0$ b) $3y + 2x = 0$ c) $3y - 2x = 0$ d) $2x + 3y = 0$

- 6 The slop of the line which parallels the X- axis is :

a) Zero b) Unfind c) Negative d) Positive

- 7 What is the slope of the line $3x-2y= -6$?

a) $-\frac{3}{2}$ b) $-\frac{2}{3}$ c) 3 d) $\frac{3}{2}$

- 8 The slope of the line which passes through the two points $(8,-3),(5,-3)$ is:

a) Positive b) Negative c) Zero d) Unfind

Multiple Choice

[4 - 3]

The Equation of the Line

Choose the correct answer for each of the following:

- 1 The line equation which passes through the two points $(-2,-3)$, $(-1,-7)$ is:
a) $y - 4x = -11$ b) $y - 4x = 11$ c) $4y + x = -11$ d) $y + 4x = -11$
- 2 The line which its equation is $y + x = 0$, its slope and one of its points are:
a) $m = -1$, $(4,4)$ b) $m = 1$, $(4,4)$ c) $m = -1$, $(4,-4)$ d) $m = 1$, $(-4,-4)$
- 3 a) $5y + 3x = -6$ b) $5y - 3x = 6$ c) $3y - 5x = -6$ d) $3y + 5x = -6$
- 4 Use the line equation $y = mx + k$ and find the value of K, m of the line $7y - 3x = 21$
a) $m = \frac{3}{7}$, $k = -3$ b) $m = \frac{7}{3}$, $k = 3$ c) $m = \frac{3}{7}$, $k = -3$ d) $m = \frac{3}{7}$, $k = 3$
- 5 Which of the following points locates on the line which its equation is: $y + 4x = 0$
a) $(1,4)$ b) $(4,-1)$ c) $(4,1)$ d) $(1,-4)$
- 6 The line equation which its slope is (-1) and its Y-Intercept equals (-2) is :
a) $y + x - 2 = 0$ b) $y + x + 2 = 0$ c) $y + x - 2 = 0$ d) $y - x - 2 = 0$
- 7 Which equation in form slope - intercept to line through two points $(-1,-2)$, $(1,6)$
a) $y = -3x + 6$ b) $y = 4x - 2$ c) $y = 4x + 2$ d) $y = 2x + 4$
- 8 The cost of a meal a restaurant is 25,000 dinars, in addition to 3,000 dinars for any extra type of appetizers, which of the equation represents the cost of a meal with (x) of the appetizers?
a) $y = 25x + 3$ b) $y = 25x - 3$ c) $y = 3x + 25$ d) $y = 3x - 25$

Multiple Choice

[4 - 4]

Parallel and Perpendicular Lines

Choose the correct answer for each of the following:

- 1 The line which passes through the two points (1,9) ,(7,1) parallels the line which its slope is:
- a) $-\frac{3}{4}$ b) $-\frac{4}{3}$ c) $\frac{3}{4}$ d) $\frac{4}{3}$
- 2 If m_2 , m_1 represent two slopes of two perpendicular lines, then:
- a) $m_1 + m_2 = -1$ b) $\frac{m_1}{m_2} = -1$ c) $m_1 \times m_2 = -1$ d) $m_1 - m_2 = -1$
- 3 Is the value of (a) which makes the line slope which passe through the two points (-1,4) , (a,-1) equals $-\frac{5}{3}$
- a) 4 b) -2 c) -4 d) 2
- 4 Is the line equation which passes through the point (0,3) and perpendicular on the line which its slope $\frac{4}{3}$
- a) $3y + 4x = 12$ b) $3y + 4x = -12$ c) $4y - 3x = 12$ d) $4y + 3x = 12$
- 5 If $m_1=m_2$ represent two slopes of the two lines L_1 , L_2 , then:
- a) $\vec{L_1} \perp \vec{L_2}$ b) $\vec{L_1} \parallel \vec{L_2}$ c) Intersected d) $\vec{L_2}, \vec{L_1}$ No relation between them
- 6 Which of the following lines parallels the line which its equation is $6y - 5x = 30$
- a) $6y + 5x = 30$ b) $5y - 6x = 30$ c) $6y - 5x = 25$ d) $6y + 5x = 25$
- 7 Which of the following lines perpendicular the line which its equation is:
- a) $3y + 2x = -6$ b) $3y - 2x = -6$ c) $2y + 3x = 6$ d) $2y - 3x = 6$

Multiple Choice

[4 - 5]

Distance between two Points

Choose the correct answer for each of the following:

- 1 The distance between the two points (0,3),(2,-5) equals:
- a) $-2\sqrt{17}$ b) $\sqrt{10}$ c) $17\sqrt{2}$ d) $2\sqrt{17}$
- 2 The midpoint of the segment which connects between the two points (3,-1),(7,-3) is:
- a) (5,2) b) (-2,5) c) (5, - 2) d) (-5, - 2)
- 3 If the point (2,1) is the midpoint of the segment \overline{AB} , where A(a,b), B (3,2), then the value of b,a are :
- a) a = 1, b = 1 b) a = 1, b = -1 c) a = -1, b = 0 d) a = 1, b = 0
- 4 The distance formula between the two points $(x_2, y_2), (x_1, y_1)$ equals:
- a) $\sqrt{(x_2 - x_1)^2 + (y_2 + y_1)^2}$ b) $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$
c) $\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$ d) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- 5 The midpoint formula between the two points $(x_2, y_2), (x_1, y_1)$ equals:
- a) $(\frac{x_2 - x_1}{2}, \frac{y_2 - y_1}{2})$ b) $(\frac{x_1 + x_2}{3}, \frac{y_1 + y_2}{3})$
c) $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$ d) $(\frac{y_1 + y_2}{2}, \frac{x_1 + x_2}{2})$
- 6 The point (2, - 2) is the midpoint of the segment between :
- a) (-8, - 1), (4, - 3) b) (8, 1), (1, - 3)
c) (8, 1), (4, - 3) d) (8, - 1), (-4, - 3)
- 7 By using the distance formula : A(3, - 1), B (-3,3), C (-3, - 1) are vertices of the:
- a) Isocseles Triangle b) Equilateral Triangle
c) Scalene Triangle d) Scalene Triangle, right angle

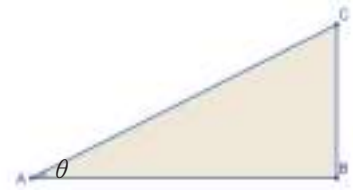
Multiple Choice

[4 - 6]

Trigonometric Ratios

Choose the correct answer for each of the following:

1 In the nearby figure, the trigonometric ratio $\sin \theta$ writes as:



- a) $\frac{AB}{AC}$ b) $\frac{BC}{AB}$ c) $\frac{BC}{AC}$ d) $\frac{AB}{AC}$

2 ABC is a right _angled triangle in B , if $\cos A = \frac{3}{5}$ Then $\tan C$ equals:

- a) $\frac{4}{5}$ b) $\frac{5}{4}$ c) $\frac{4}{3}$ d) $\frac{3}{4}$

3 If $\tan \theta = \frac{1}{\sqrt{3}}$ Then the value of the angle θ equals:

- a) 45° b) 60° c) 90° d) 30°

4 The numerical value of the expression $\sin 30^\circ \cos 30^\circ$ equals:

- a) $\frac{1}{\sqrt{3}}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{\sqrt{3}}{4}$ d) $\frac{2}{\sqrt{3}}$

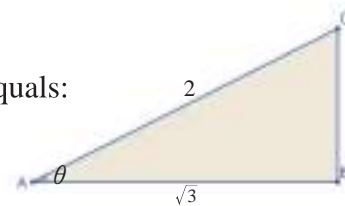
5 The inverse of the ratio $\cos \theta$ is:

- a) $\sin \theta$ b) $\sec \theta$ c) $\csc \theta$ d) $\cot \theta$

6 The numerical value of the expression $(\sec 60^\circ)^2 - (\tan 60^\circ)^2$ equals:

- a) -1 b) 0 c) 2 d) 1

7 ABC is a right _angled triangle in B, as in the nearby figure:
The numerical value of the expression $(\sin \theta)^2 + (\cos \theta)^2$ equals:



- a) -1 b) 0 c) 2 d) 1

8 If $\csc \theta = 2$, then the value of the angle θ is :

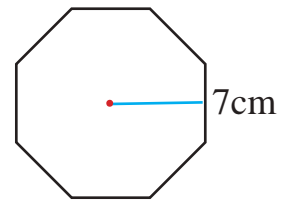
- a) 45° b) 60° c) 90° d) 30°

Multiple Choice

[5 - 1]

Polygons and Polyhedrons (Pyramid and Cone)

Choose the correct answer for each of the following:



1 What is the perimeter of the nearby regular octagonal?

- a) 45.5 cm b) 48 cm c) 38.3 cm d) 56 cm

2 The perimeter of a square which its area is 225m^2 is :

- a) 25m b) 20 m c) 15 m d) 60 m

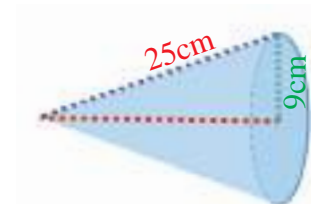
3 The perimeter of a regular pentagonal which its apothem is 3m and the radius of its circle is 5m, is:

- a) 16.2 m b) 40 m c) 16 m d) 10.49 m

4 The area of a regular 7-gon which its apothem is 6cm and the length of its side is 7.5 cm, is:

- a) 157.5 cm^2 b) 28.5 cm^2 c) 28 m^2 d) 9975 m^2

5 The lateral area of the cone in the nearby figure is:



- a) $360\pi\text{ cm}^2$ b) $450\pi\text{ cm}^2$ c) $369\pi\text{ cm}^2$ d) $1640\pi\text{ cm}^2$

6 The volume of a pyramid which the length of each side of its square base is 18cm and its height is 20cm, is:

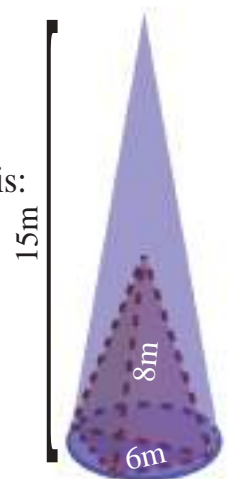
- a) 2160cm^3 b) 120 cm^3 c) 260 cm^3
d) 134 cm^3

7 The total area of the cone its base area is $25\pi\text{ cm}^2$ and its height is 12cm, is:

- a) $108\pi\text{ cm}^2$ b) $27\pi\text{ cm}^2$ c) $208\pi\text{ cm}^2$
d) $90\pi\text{ cm}^2$

8 The difference in volume of the two cones is:

- a) $27\pi\text{ cm}^3$ b) $75\pi\text{ cm}^3$ c) $48\pi\text{ cm}^3$ d) $180\pi\text{ cm}^3$



Multiple Choice

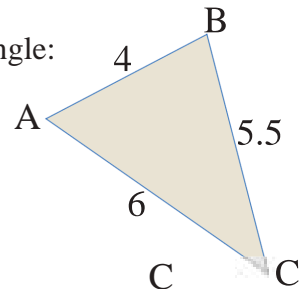
[5 - 2]

Triangles

Choose the correct answer for each of the following:

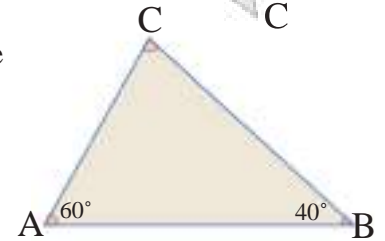
- 1 Arrange the angles from the smallest to the greatest in the nearby triangle:

a. $m\angle C, m\angle A, m\angle B$
 b. $m\angle A, m\angle B, m\angle C$
 c. $m\angle B, m\angle C, m\angle A$
 d. $m\angle C, m\angle B, m\angle A$



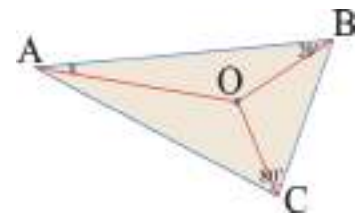
- 2 Arrange the sides from the longest to the shortest in the nearby triangle

a. $\overline{BC}, \overline{AC}, \overline{AB}$
 b. $\overline{AB}, \overline{BC}, \overline{AC}$
 c. $\overline{AC}, \overline{BC}, \overline{AB}$
 d. $\overline{AB}, \overline{AC}, \overline{BC}$



- 3 If O is the meeting point of the angles bisectors of the triangle ABC in the nearby figure, then the value of x is:

a) 20° b) 40° c) 30° d) 50°



- 4 The triangle ABC has two median segments $\overline{AD}, \overline{CE}$ Which meet in point O ($AD=36\text{cm}, CE=24\text{cm}$), B is vertex of ABC, then the value of \overline{OE} is:

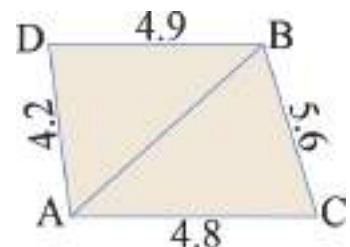
a) 8 cm b) 24 cm c) 16 cm d) 12 cm

- 5 In the question (4), the value of \overline{AO} is:

a) 6 cm b) 12 cm c) 24 cm d) 14 cm

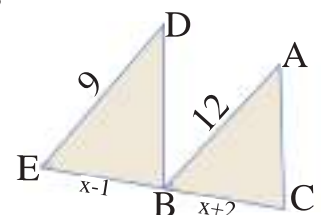
- 6 The similarity ratio between the two triangles ADB, ABC is:

a. $\frac{8}{7}$ b. $\frac{7}{8}$
 c. 7 d. 8



- 7 If the two triangles DBE, ABC are similar and there were the two angles, $m\angle ABC = m\angle DEB$ then the value of x is:

a) 8 b) 12 c) 10 d) 6



Multiple Choice

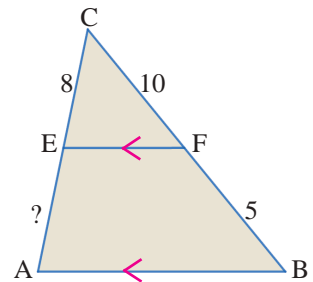
[5 - 3]

Proportion and measurement in Triangles

Choose the correct answer for each of the following:

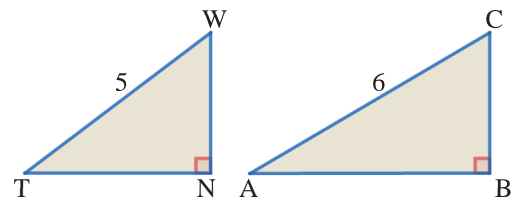
1 If $\overline{AB} \parallel \overline{EF}$, then the length of the line segment AE is:

- a) 4 b) 5 c) 2 d) 10



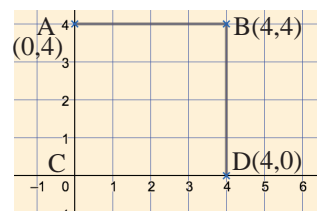
2 If $\triangle ACB \sim \triangle TWN$, If you knew that the height of the triangle TWN is (3), then are of the triangle the perimeter of ABC

- a) 42.3 cm^2 b) 43.2 cm^2 c) 40.2 cm^2 d) 40.3 cm^2



After dilation it in a geometric proportion its ratio is $\frac{4}{3}$

A picture was drawn to be as it is shown in the nearby figure:



Choose the correct answer for the questions (4_7)

3 The coordinates of the point A before the dilation are:

- a) (0,3) b) (3,0) c) (3,3) d) (0,0)

4 The coordinates of the point B before the dilation are:

- a) (0,3) b) (3,0) c) (3,3) d) (0,0)

5 The coordinates of the point C before the dilation are:

- a) (0,3) b) (3,0) c) (3,3) d) (0,0)

6 The coordinates of the point D before the dilation are:

- a) (0,3) b) (3,0) c) (3,3) d) (0,0)

Multiple Choice

[5 - 4]

The Circle

Look at the nearby figure and choose the correct answer for the questions (1_4) :

1 The measure of $\angle AOB$ is:

- a) 180° b) 135° c) 90° d) 45°

2 The measure of \widehat{AB} is:

- a) 180 b) 90 c) 135 d) 45

3 The measure of \widehat{ABC} is:

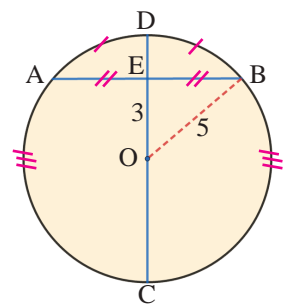
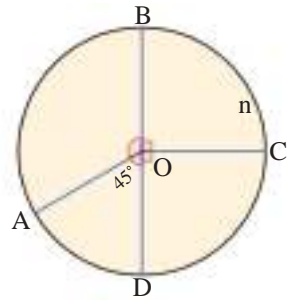
- a) 180 b) 90 c) 225 d) 135

4 The measure of \widehat{BC} is:

- a) 90 b) 42 c) 45 d) 135

5 The length of the chord AB in the nearby figure is:

- a) 12 b) 10 c) 6 d) 8



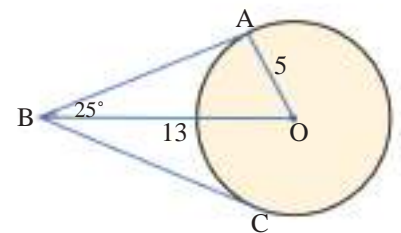
Look at the nearby figure and choose the correct answer for the questions (6 - 7):

6 The measure of $\angle AOB$ is:

- a) 115° b) 120° c) 65° d) 90°

7 The length of the segment BC is:

- a) 10 b) 14 c) 12 d) 5

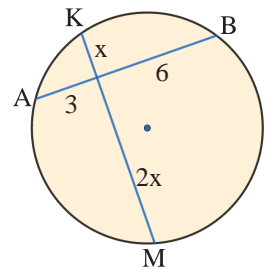


Multiple Choice

[5 - 5]

Triangle and Circle, Line Segments and Circle

Look at the nearby figure and choose the correct answer for questions (1- 2) :



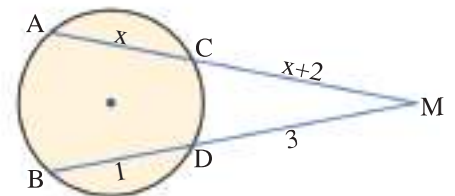
1 The value of x is:

- a) 2 b) 6
c) 9 d) 3

2 The length of \overline{MK} is:

- a) 12 b) 9 c) 5 d) 4

Look at the nearby figure and choose the correct answer for questions (3_5)



3 The value of x is:

- a) 2 b) 3 c) 1 d) 4

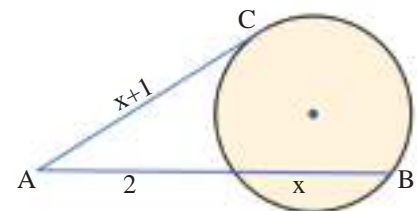
4 The length of \overline{BM} is:

- a) 4 b) 6 c) 5 d) 2

5 The length of \overline{AM} is:

- a) 4 b) 2 c) 6 d) 3

Look at the nearby figure and choose the correct answer for the questions (6_8):



6 The value of X is:

- a) 1 b) $\sqrt{2}$ c) $\sqrt{3}$

d) 0

7 The length of the tangent is:

- a) $\sqrt{2} + 1$ b) $\sqrt{3} + 1$ c) 4 d) $\sqrt{5} + 1$

8 The length of \overline{AB} is:

- a) $6 + \sqrt{3}$ b) $2 + \sqrt{3}$ c) $5 + \sqrt{3}$ d) $4 + \sqrt{3}$

Multiple Choice

[5 - 6]

Angles and Circle

Look at the nearby figure and choose the correct answer for questions (1- 3) :

1 The measure of the angle (w) is:

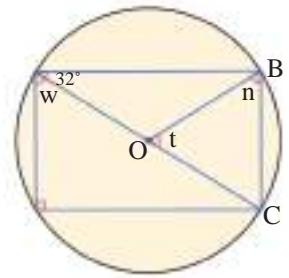
- a) 45° b) 30°
c) 90° d) 32°

2 The measure of the angle (t) is:

- a) 45° b) 64° c) 32° d) 48°

3 The measure of the angle (n) is:

- a) 45° b) 64° c) 32° d) 84°



Look at the nearby figure and choose the correct answer for questions (4_6)

4 The measure of the angle (h) is:

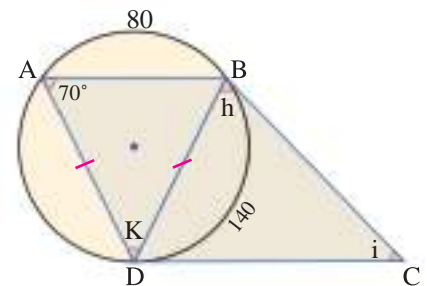
- a) 70° b) 72° c) 90° d) 80°

5 The measure of the angle (i) is:

- a) 39° b) 70° c) 40° d) 45°

6 The measure of the angle (k) is:

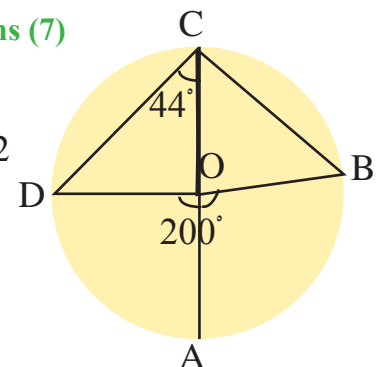
- a) 70° b) 30° c) 40° d) 78°



Look at the nearby figure and choose the correct answer for questions (7)

7 The measure of \widehat{ABC} is:

- a) 56 b) 112 c) 65 d) 82



Multiple Choice

[6 - 1]

Design a Survey Study and analysis its results

Choose the correct answer for each of the following:

- 1 Which one of the measurements of the central tendency (if it is found) is suitable to the following data: 8, 8, 12, 11, 15, 15, 16, 21, 23, 27, 31, 70?
a) Range b) Mode c) Median d) Arithmetic mean
- 2 Which of the central tendency measurements (if it is found) for the following data 2, 3, 4, 5, 6, 7?
a) Range b) Mode c) Median d) Arithmetic mean
- 3 Which of the central tendency measurements (if it is found) 18, 1, 3, 16, 23, 3, 2?
a) Range b) Mode c) Median d) Arithmetic mean
- 4 The range of the following data 24, 18, 32, 24, 22, 18 is:
a) 18 b) 32 c) 14 d) 50
- 5 Which measurements is not from the measurements of the central tendency?
a) Range b) Mode c) Median d) Arithmetic mean
- 6 The extreme value of these data 4, 30, 3, 5, 5, 6, 5, 3 is:
a) 3 b) 5 c) 5 d) 30
- 7 The median will be the best for the measurements of the central tendency for the data which:
a) Have extreme value with great gaps in the middle b) Don't have extreme value there are not great gaps in the middle c) Have extreme value and there are not great gaps in the middle d) Don't have extreme value there are not great gaps in the middle

Multiple Choice

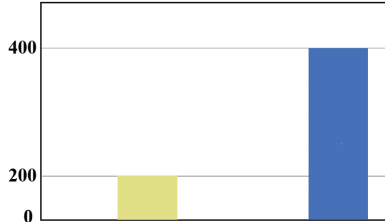
[6 - 2]

Data and misleading statics

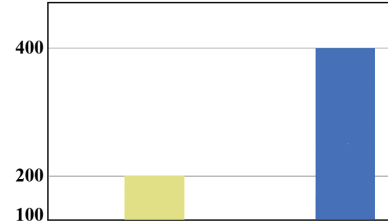
Choose the correct answer for each of the following:

1 Which graphic diagram is best in representing certain data:

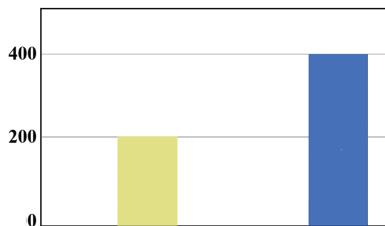
a)



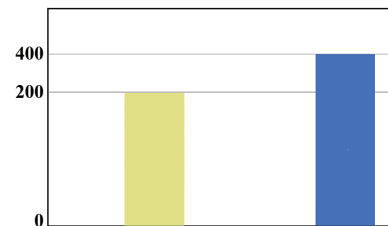
b)



c)



d)



2 The graphic diagram will be misleading if:

- a) Starts from zero and the intervals are not equal b) Does not start from zero and the intervals are not equal c) Does not start from zero and the intervals are equal d) Starts from zero and the intervals are equal

3 In a questionnaire includes 6 teachers about the work, 4 of them prefer to work in the morning, the person who does the questionnaire had written the following (2 teachers out of 3 prefer to work in the morning) why does this announcement consider misleading?

- a) The sample is so large b) The sample should include building workers c) The sample is so small d) The sentence should be (a teacher should be separated out of 2 teachers)

4 In a trade of shop offer kind the cheese to 12 persons to evaluate before viewing it, 6 persons were admired it, According to that the product said the product is good because the ratio whose prefer it was 6 to 3

- a) Data is not misleading because the ratio 6 to 3 is high ratio .
b) Data is not misleading because the people who prefer it of the rest
c) Misleading data , although people whose prefer it double of rest .
d) Misleading data because the sample was medium size .

Multiple Choice

[6 - 3]

Permutation and Combination

Choose the correct answer for each of the following:

- 1 The numerical value of the expression $(5! - 3!)(0!)$ equals:
a) 2 b) 0 c) 114 d) None
- 2 The value of C_1^{51} equals:
a) 1 b) 51 c) 50 d) None
- 3 The value of P_0^{100} equals:
a) 100 b) 100! c) 0 d) 1
- 4 Number the ways of forming a quadruple committee consists of 5 members, each one has a special duty:
a) P_4^5 b) $5!$ c) $4!$ d) C_4^5
- 5 The value of $\frac{n!}{(n-2)!}$ equals:
a) $n!$ b) $(n-2)!$ c) $n(n-1)!$ d) $n(n-1)$
- 6 Number the ways of choosing 5 questions out of the test paper which contains 7 questions:
a) 7 b) 5 c) $2!$ d) 21
- 7 The value of $\frac{(8-3)!}{(3+2)!}$ equals:
a) $4!$ b) $3!$ c) $2!$ d) $1!$
- 8 The value of $C_0^n + P_0^n$ equals:
a) 1 b) 2 c) 0 d) 0

Multiple Choice

[6 - 4]

Experimental Probability and theoretical Probability

Choose the correct answer for each of the following:

1 If E_1 , E_2 are two disjoint events, then $P(E_1 \text{ or } E_2)$ equals:

- a) $P(E_1) - P(E_2)$ b) $P(E_1) \times P(E_2)$ c) $P(E_1) + P(E_2)$ d) $\frac{P(E_1)}{P(E_2)}$

2 Ahmed had scored 20 goals out of 25 attempts, which percentage for the experimental probability that Ahmed will record in the next attempt?

- a) 50% b) 60% c) 70% d) 80%

3 Tamara had released the disc spinner once, which percentage for the theoretical probability that the pointer will indicate to white number ?



- a) 35% b) 30% c) 12.5% d) 20%

4 By throwing the two dices once, the probability of getting two number their sum is 3 or the result of multiplying them is 3, is:

- a) $\frac{1}{3}$ b) $\frac{1}{9}$ c) $\frac{2}{3}$ d) 1

5 E_1 , E_2 are two disjoint event, if $P(E_1 \text{ or } E_2) = \frac{5}{6}$ and $P(E_2) = \frac{2}{3}$, then $P(E_1)$ equals:

- a) $\frac{1}{3}$ b) $\frac{1}{6}$ c) $\frac{1}{4}$ d) $\frac{1}{5}$

6 By throwing the two dices, the probability of getting two number their sum is 13, is:

- a) 3 b) 2 c) 1 d) 0

Multiple Choice

[6 - 5]

Compound Events

Choose the correct answer for each of the following:

- 1 E_1, E_2 are two independent events, where $P(E_1) = 0.3$ and $P(E_2) = 0.9$, the probability of happening E_1, E_2 together is:
a) 1.2 b) 0.6 c) 0.27 d) 0.3
- 2 Mustafa had thrown a dice and a coin, the probability of appearing a number greater than 5 on the dice and the writing (tail) on the coin is :
a) $\frac{2}{3}$ b) $\frac{1}{3}$ c) $\frac{1}{12}$ d) 3
- 3 A box has 5 red balls and 4 green balls.
 E_1 represent pulling a red ball, E_2 represents pulling a green ball without returning the red one, the probability of happening them together is :
a) $\frac{10}{9}$ b) $\frac{5}{18}$ c) $\frac{19}{18}$ d) $\frac{1}{18}$
- 4 E_1, E_2 are two dependent events, the probability of happening them together is:
a) $P(E_1 \text{ and } E_2) = P(E_1) + P(E_2 \text{ after } E_1)$ b) $P(E_1 \text{ and } E_2) = P(E_1) + P(E_2 \text{ before } E_1)$
c) $P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2 \text{ after } E_1)$ d) $P(E_1 \text{ and } E_2) = P(E_1) \times P(E_1 \text{ after } E_2)$
- 5 The relation $P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2)$ between the two events E_1, E_2 , where they are:
a) No relation between them b) Independent c) dependent d) Another thing
- 6 E_1, E_2 are two disjoint events, $P(E_2) = 0.45$ $P(E_1) = 0.15$ the probability of happening E_1 or E_2 is:
a) 0.0675 b) 3 c) 0.6 d) 0.3

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