Republic of Iraq
Ministry of Education
General Directorate of Curricula

Series of Maths Books for Intermediate Stage

Mathematics

Second Intermediate

Dr. Ameer Abdulmageed Jassim
Dr. Tariq Shaban Rajab
Dr. Ahmad M. Abdul Hadi
Dr.Taghreed Hur Majeed
Dr. Hussein Sadeq Kadhim
Marwah Flayyih Hasan

Revised by

Specialized Committee in Ministry of Education

Seventh Edition **2025**

This series (Maths for Intermediate stage) has been edited by a special team of specialists in Ministry of Education/ General Directorate of curricula with participation of specialists from universities professors in Ministry of higher Education according to international standards to achieve the goals of designing modern syllabus which helps the students to be:
Successful learners long life
Self-esteem individuals
Iraqi citizens feeling proud

Scientific supervisor

Marwah Flayyih Hasan

Art supervisor

Dr.Amel Ibrahim Muhsin

المديرية العامة للمناهج
قسم التصميم الطباعي

الموقع والصفحة الرسمية للمديرية العامة للمناهج

manahjb@yahoo.com
Info@manahj.edu.iq
manahjb
manahj

INTRODUCTION

The Maths subject is considered one of the basic courses that helps student to acquire educational abilities to develop his thinking and solving problems and it helping to deal with difficult situations in his life.

As a starting point of attention by the Ministry of Education represented by the General Directorate of curricula to develop the curricula in general and specially of Maths in order to go a long with the technological and scientific development in different fields of life. A plan has set up to edit the series of Maths books for the three stages. Primary stage has been achieved and the work started to continue the series by editing the books of intermediate stage.

The series of new Iraqi Maths Books as a part of General frame work of curricula that reinforces the basic values as Iraqi identity, forgiveness, respecting different opinions, social justice and offering equal chance for creativity and it also reinforces abilities of thinking and learning, self-efficiency, action and citizen ship efficiency.

The series of Iraqi Maths books has been built on student- centered learning according to international standards.

The series of Iraqi maths books for intermediate stage has been built on six items: learn, make sure of your understanding, solve the exercises, solve the problems, think and write. The Maths book for second intermediate stage contains four basic fields: The numbers and the operations, Algebra, Geometry and Measurement, Statistics and Probabilities for each field.

The maths books have distinguished by presenting material in modern styles that attract and help the student to be active through presenting drills, exercises and environmental problems in addition there are extra exercises at the end of the book that are different from the exercises and drills in the lessons because they are objective so the student can answer through multiple choices and that prepare the student to participate the international competitions.

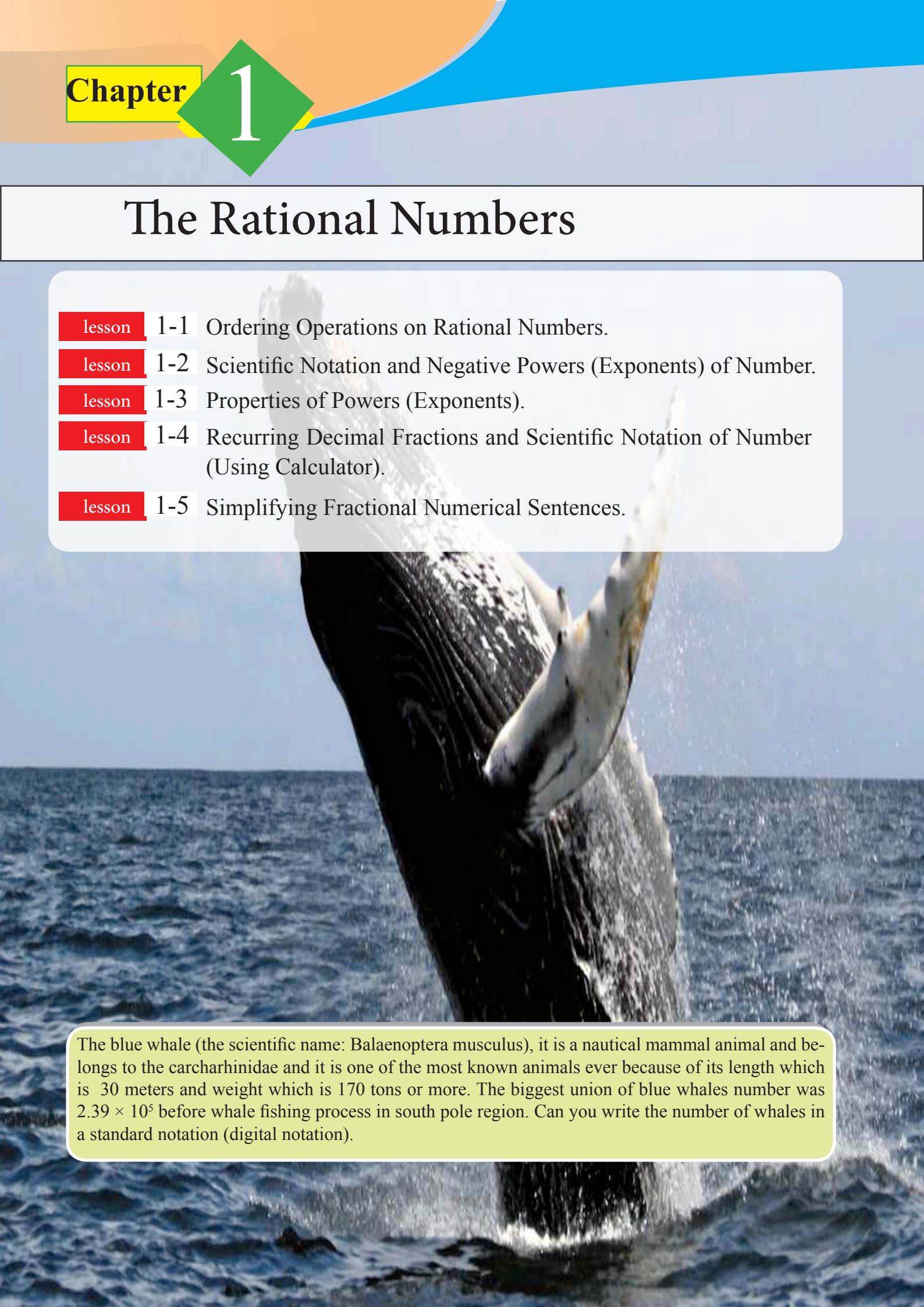
This book is an expansion for the series of developed Maths books for primary stage and it is also considered as support for the developed syllabus in maths and it also has a teachers book so we hope in applying them, the student will gain scientific and practical skills and develop their interest to study Maths.

We hope God help us to serve our country and our sons

Authors

Chapter 1

The Rational Numbers



The blue whale (the scientific name: Balaenoptera musculus), it is a nautical mammal animal and belongs to the carcharhinidae and it is one of the most known animals ever because of its length which is 30 meters and weight which is 170 tons or more. The biggest union of blue whales number was 2.39×10^5 before whale fishing process in south pole region. Can you write the number of whales in a standard notation (digital notation).

Pretest

Express the following numbers in form of rational number:

1 $9 = \ldots\ldots$ 2 $2.3 = \ldots\ldots$ 3 $0.29 = \ldots\ldots$ 4 $1.4 = \ldots\ldots$ 5 $6.1 = \ldots\ldots$

Compare the following rational numbers by using the symbols (>, < , =):

6 $\frac{11}{25} \square \frac{6}{10}$ 7 $\frac{5}{6} \square \frac{8}{15}$ 8 $\frac{7}{16} \square \frac{25}{64}$

Arrange the following rational numbers from smallest to largest:

9 $\frac{4}{5}, \frac{3}{12}, \frac{15}{25}$ 10 $\frac{3}{8}, \frac{5}{7}, \frac{3}{21}$

Arrange the following rational numbers from largest to smallest:

11 $\frac{12}{27}, \frac{6}{9}, \frac{10}{18}$ 12 $\frac{8}{14}, \frac{10}{25}, \frac{4}{28}$

Find the result of the following:

13 $\frac{3}{5} \div \frac{1}{6} = \ldots\ldots$ 14 $(-3.6) + (-2.2) = \ldots\ldots$ 15 $(7.1) + (-5.9) = \ldots\ldots$

16 $\frac{5}{9} - \frac{3}{8} = \ldots\ldots$ 17 $(4.1) \times (-2) = \ldots\ldots$ 18 $\frac{3}{19} \times \frac{6}{10} = \ldots\ldots$

Find the estimated percentage for each of the following:

19 $\frac{1}{9}$ 20 $\frac{8}{26}$ 21 $\frac{11}{74}$ 22 $\frac{5}{47}$

23 **Complete the following table showing the type of the proportion:**

A	1	2	4	5	8	10
B	1000	500			125	
C	1000		1000			

Estimate the following roots:

24 $\sqrt{50}$ 25 $\sqrt{13}$ 26 $\sqrt[3]{145}$ 27 $\sqrt[3]{33}$

Lesson [1-1] Ordering Operations on Rational Numbers

Idea of the lesson:
* Using ordering operations on rational numbers to simplify numerical sentence.

Vocabulary:
* Ordering Operations
* Numerical Sentence

Learn

Aws and Aeman practise tennis, where Aws practice tennis two days per a week which is $\frac{5}{6}$ hours the first day, and $\frac{3}{4}$ hours the second day, and Aeman practises tennis one day per a week which is $\frac{2}{3}$ hours. How much more is the period of the hours that Aws practises than Aeman in a week?

[1-1-1] Simplifying Numerical Sentence Contains Addition and Subtraction or Multiplication and Division Rational Numbers

You have previously learned how to find the result of adding and subtracting ratational numbers and also multiplying and dividing two rational numbers. And now you will learn how to simplify a numerical sentence containing two operations (addition and subtraction) or (multiplication and division) for rational numbers.

Example (1) **How much more is the period of the hours that Aws practises than Aeman practises in a week?**

We write the numerical sentence which represents the problem:

$$\frac{5}{6}+\frac{3}{4}-\frac{2}{3}=\frac{5\times 2+3\times 3}{12}-\frac{2}{3}$$ *Firstly we apply addition operation*

$$=\frac{19}{12}-\frac{2}{3}=\frac{19-8}{12}=\frac{11}{12}$$ *Secondly we apply subtraction operation*

Aws practises $\frac{11}{12}$ hour more than Aeman in a week.

Example (2) **Use addition and subtraction of rational numbers to write each expression in simplest form:**

i) $\frac{1}{3}-\frac{8}{9}+\frac{12}{6}=\frac{1\times 6-8\times 2+12\times 3}{18}=\frac{6-16+36}{18}=\frac{26}{18}=\frac{13}{9}$

ii) $2\frac{2}{7}+1\frac{1}{5}-\frac{6}{35}=\frac{16}{7}+\frac{6}{5}-\frac{6}{35}=\frac{16\times 5+6\times 7-6}{35}$

$$=\frac{80+42-6}{35}=\frac{116}{35}$$

iii) $7.34-3.08+1.9=4.26+1.9=6.16$

iv) $3.16+0.092-0.07-5.005=3.252-5.075=-1.823$

v) $0.07-0.006+0.503-0.08=0.064+0.503-0.08=0.567-0.08=0.487$

Example (3) **Use multiplication and division of rational numbers to write each expression in simplest form:**

i) $-3\frac{1}{2} \times 1\frac{2}{3} \div 3\frac{7}{2} = -\frac{7}{2} \times \frac{5}{3} \div \frac{13}{2} = -\frac{35}{6} \div \frac{13}{2} = -\frac{35}{6} \times \frac{2}{13} = -\frac{35}{39}$

ii) $\frac{48}{9} \div \frac{-12}{3} \times \frac{-8}{13} = \frac{48}{9} \times \frac{3}{-12} \times \frac{-8}{13} = \frac{4}{-3} \times \frac{-8}{13} = \frac{32}{39}$

iii) $4.5 \times 3.65 \div (-1.125) = 16.425 \div (-1.125) = -14.6$

iv) $7.29 \div 0.9 \times (-0.03) = 8.1 \times (-0.03) = -0.243$

[1-1-2] Using Ordering Operations to Simplifies Numerical Sentence Containing Rational Numbers

You have previously learned using the ordering operations on integers to simplify numerical sentences. And now you will learn how to use ordering operations to simplify numerical sentences that contain rational numbers by following these steps:

1) Put the operatins that have priority in the brackets.
2) Start with the operations between the brackets.
3) Multiply and divide from left to the right.
4) Add and subtract from left to the right.

Example (4) **Use ordering operations on rational numbers to write each expression in simplest form:**

i) $\frac{-1}{5} \times \frac{25}{-3} + \frac{3}{2} \times \frac{8}{21} = (\frac{-1}{5} \times \frac{25}{-3}) + (\frac{3}{2} \times \frac{8}{21}) = \frac{5}{3} + \frac{4}{7} = \frac{35+12}{21} = \frac{47}{21}$

ii) $\frac{7}{12} \div \frac{21}{-4} - \frac{9}{11} \div \frac{-27}{44} = (\frac{7}{12} \times \frac{-4}{21}) - (\frac{9}{11} \times \frac{44}{-27}) = \frac{-1}{9} + \frac{4}{3} = \frac{-1+12}{9} = \frac{11}{9}$

iii) $\frac{4}{5} \div \frac{8}{15} \times \frac{2}{-7} + \frac{5}{14} = (\frac{4}{5} \times \frac{15}{8}) \times \frac{2}{-7} + \frac{5}{14} = (\frac{3}{2}) \times \frac{2}{-7} + \frac{5}{14} = \frac{-3}{7} + \frac{5}{14} = -\frac{1}{14}$

iv) $\frac{12}{7} - \frac{2}{11} \times \frac{22}{-6} \div \frac{4}{15} = \frac{12}{7} - (\frac{2}{11} \times \frac{22}{-6} \div \frac{4}{15}) = \frac{12}{7} - (\frac{2}{-3} \div \frac{4}{15}) = \frac{12}{7} - (\frac{2}{-3} \times \frac{15}{4})$
$= \frac{12}{7} + \frac{5}{2} = \frac{59}{14}$

v) $4.01 \times 1.2 + 11.5 \times 0.6 = (4.01 \times 1.2) + (11.5 \times 0.6) = 4.812 + 6.9 = 11.712$

vi) $6.4 \div 0.8 - 12.5 \div 0.5 = (6.4 \div 0.8) - (12.5 \div 0.5) = 8 - 25 = -17$

vii) $0.09 \times 16 + 5.4 \div 0.6 = (0.09 \times 16) + (5.4 \div 0.6) = 1.44 + 9 = 10.44$

Make sure of your understanding

Use addition and subtraction of rational numbers to write each expression in simplest form:

1 $\frac{1}{5} - \frac{2}{7} + \frac{3}{5} =$ .....

2 $4\frac{1}{3} + 3\frac{2}{5} - \frac{8}{15} =$ .....

3 $\frac{2}{9} - 1\frac{1}{8} + \frac{4}{3} - \frac{5}{4} =$ .....

4 $\frac{6}{5} + \frac{9}{2} - \frac{3}{8} + \frac{7}{15} =$ .....

Questions1-6 are similar to example 2

5 $9.14 - 6.07 + 2.5 =$ .....

6 $1.12 + 0.82 - 0.02 + 4.002 =$ .....

Use the multiplication and division of rational numbers to write each expression in simplest form:

7 $1\frac{1}{3} \times (-2\frac{5}{6}) \div 2\frac{4}{9} =$ .....

8 $\frac{54}{7} \div \frac{-9}{14} \times \frac{-5}{24} =$ .....

Questions 7-10 are similar to example 3

9 $6.1 \times 3.25 \div (-0.25) =$ .....

10 $0.81 \div 0.9 \times (-0.05) =$ .....

Use ordering operations on rational numbers to write each expression in simplest form:

11 $\frac{-2}{5} \times \frac{15}{-6} + \frac{1}{3} \times \frac{12}{5} =$ .....

12 $\frac{8}{13} \div \frac{-16}{39} - \frac{7}{15} \div \frac{-21}{10} =$ .....

13 $\frac{2}{9} \div \frac{4}{27} \times \frac{1}{-4} + \frac{7}{16} =$ .....

14 $\frac{12}{7} - \frac{-2}{17} \times \frac{34}{-5} \div \frac{8}{15} =$ .....

Questions 11-16 are similar to example 4

15 $9.02 \times 1.4 - 8.03 \times 0.7 =$ .....

16 $4.9 \div 0.7 + 4.8 \div 0.12 =$ .....

Solve the Exercises

Use addition and subtraction of rational numbers to write each expression in simplest form:

17 $\frac{1}{6} - \frac{3}{4} + \frac{2}{3} =$ .....

18 $\frac{3}{7} - 1\frac{2}{7} + \frac{5}{2} - \frac{9}{14} =$ .....

19 $2.04 - 8.09 + 3.05 =$ .....

Use multiplication and division of rational numbers to write each expression in simplest form:

20 $2\frac{1}{6} \times 3\frac{3}{2} \div (-5\frac{3}{2}) =$ .....

21 $0.01 \times 6.4 \div (-0.04) =$ .....

Use ordering operations of rational numbers to write each expression in simplest form:

22 $\frac{5}{-3} \times \frac{24}{45} + \frac{1}{3} \times \frac{-15}{6} =$ .....

23 $\frac{1}{7} \div \frac{5}{42} \times \frac{1}{-5} - \frac{1}{12} =$ .....

24 $-4.09 \times 3.2 - 5.03 \times 0.8 =$ .....

Solve the problems

25 **Football:** The world cup is the most important international competition in football which is held by (FIFA). The champion was held for the first time in 1930 every 4 years, it has been held 20 times. And still being held where the Brazillian team won the cup 5 times, and the Italian team won 4 times. What is the difference between the winning ration of both of the teams to the total number for organizing the championship?

26 **Running:** The runner who won the first place in the competition of runnning covered 2000m in the single round and 400m in the period of 44.2sec, and the runner who won the second place covered the single round in 46.5sec. What is the difference of the total time for covering the distance of both runners?

27 **Weather:** The temperature in the northern pole region was -48c° at 10 am and the temperature started increasing gradually until 4pm in the rate of 0.5c° per hour, and then started decreasing in the rate of 1.5c° per hour. What was the temperature at 6pm?

Think

28 **Challenge:** Use ordering operations on rational numbers to write each expression in simplest form:

i) $\frac{1}{3}\left(\frac{-6}{5}+2\frac{1}{10}\right)\times\frac{4}{5}\div2\frac{8}{5}=$ ..... ii) $\frac{1}{2}\left(\frac{8}{9}-3\frac{1}{3}\right)\div3\frac{2}{3}\div\frac{-15}{7}=$ .....

29 **Correct the mistake:** Jamal simplified the numerical sentence and wrote it in simplest form:

$$\frac{3}{7}\div\frac{9}{28}\times\frac{1}{-2}\div\frac{4}{-9}=6$$

Determine the mistake of Jamal and correct it.

30 **Numerical sense:** What is the number that should be written in the square to make the following numerical sentence correct?

i) $\frac{1}{-2}\times\frac{1}{2}+\frac{3}{2}\times\frac{1}{\square}=0$ ii) $\frac{1}{5}\div\frac{2}{5}-\frac{1}{3}\div\frac{\square}{3}=0$

Write

The expression in simplest form by using the ordering operations:

$$-1.03\times4.2-7.07+0.6\times2.5=\text{.....}$$

Lesson [1-2] Scientific Notation and Negative Powers (Exponents) of Number

Idea of the lesson:
*How to calculate expressions that contain negative powers.
*The expressing of a number in a scientific notation.

Vocabulary:
*Exponents (power).
*Scientific Notation.
*Digital Notation.

Learn

The giant panda or the chinese bear, is a huge animal that belongs to the Bear family which is originated from China, it is known with its thick black and white fur which protects it from the cold. The panda eats an amount of 10^{-1} of its weight everyday. Find the number of the kilograms that the 110kg weight panda eats?

[1-2-1] Negative Powers of Number

You have previously learned how to write the number in positive powers (exponents)
$10^1 = 10$ ، $10^2 = 100$ ، $10^3 = 1000$ ، .....

And in the same way you will learn how to write the number in negative powers (exponents)

$10^{-1} = \frac{1}{10}$, $10^{-2} = \frac{1}{10^2} = \frac{1}{100}$, $10^{-3} = \frac{1}{10^3} = \frac{1}{1000}$, ...

With noticing: Any number (except zero) with exponent (0) equals (1), and the number (1) with any positive or negative power equals (1), and the number (-1) with any even exponent equals (1) and with odd power equals (-1).

$8^0 = 1$, $(-3)^0 = 1$, $1^5 = 1$, $1^{-7} = 1$, $(-1)^2 = 1$, $(-1)^{-2} = 1$, $(-1)^3 = -1$, $(-1)^{-3} = -1$.

Example (1) **Find the number of the kilograms that the Panda bear eats everyday:**

$10^{-1} = \frac{1}{10}$

The panda eats tenth of its weight everyday

$110 \times \frac{1}{10} = 11\text{kg}$

So the panda eats 11kg everyday

Example (2) **Calculate the negative exponents for each of the following:**

i) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

ii) $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

iii) $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

iv) $(-2)^{-2} = -(\frac{1}{(-2)^2}) = \frac{1}{4}$

v) $(-2)^{-3} = (\frac{1}{-2})^3 = \frac{1}{-2} \times \frac{1}{-2} \times \frac{1}{-2} = -\frac{1}{8}$

Use the ordering operations to calculate each of the following:

vi) $\frac{1}{2} + 2^{-3} - (-2)^{-4} = \frac{1}{2} + \frac{1}{2^3} - (\frac{1}{(-2)^4})$

$= \frac{1}{2} + \frac{1}{8} - \frac{1}{16}$

$= \frac{8 + 2 - 1}{16} = \frac{9}{16}$

vii) $4^{-3} - (-6)^0 - 8^{-2} = \frac{1}{4^3} - 1 - \frac{1}{8^2}$

$= \frac{1}{64} - 1 - \frac{1}{64}$

$= -1$

[1-2-2] Scientific Notation of Number

You have previously learned how to write the large numbers in scientific notation as an integer multiplied by 10 with positive exponents $10^5 \times 14 = 1400000$, now you will learn using the scientific notation of the number to write the numbers that their absolute value are very big or very small.

And to write the decimal number from the scientific notation to the digital notation we will notice the following:

1) If the number is multiplied by number 10 with positive exponents then the decimal point will move to the right.
2) If the number is multiplied by number 10 with negative exponents then the decimal point will move to the left.
3) The number of decimal places by which the decimal point moves, is the absolute value of the exponent.

Example (3) **Write the following numbers in the digital notations:**

i) $7.3 \times 10^3 = 7300$ — *Decimal point moves (3) places to the right*

ii) $2.64 \times 10^5 = 264000$ — *Decimal point moves (5) places to the right*

iii) $9.17 \times 10^{-2} = 0.0917$ — *Decimal point moves (2) places to the left*

vi) $52.6 \times 10^{-4} = 0.00526$ — *Decimal point moves (4) places to the left*

To change the decimal numbers from digital notation to scientific notation, we will notice the following:

1) Move the decimal point so that it would be located to the right of the first non zero place from the left .
2) Count the places by which the decimal point is moved.
3) Find the power of 10, if the absolute value of the original number was between 0 and 1, then the exponent will be negative, and if it was greater than 1, then the exponent will be positive.

Example (4) **Write the numbers in the scientific notation:**

i) $72300 = 7.23 \times 10000$ — *Decimal point moved (4) places to the left*

$= 7.23 \times 10^4$ — Since $72300 > 1$, *So the exponent will be positive*

ii) $802000 = 8.02 \times 100000$ — *Decimal point moved (5) places to the left*

$= 8.02 \times 10^5$ — Since $802000 > 1$, *So the exponent will be positive*

iii) $0.0025 = 2.5 \times 0.001$ — *Decimal point moved (3) places to the right*

$= 2.5 \times 10^{-3}$ — Since $0 < 0.0025 < 1$, *So the exponent will be negative*

iv) $0.000371 = 3.71 \times 0.0001$ — *Decimal point moved (4) places to the right*

$= 3.71 \times 10^{-4}$ — Since $0 < 0.00037 < 1$, *So the exponent will be negative*

Make sure of your understanding

Calculate the negative exponents for each of the following:

1 $4^{-2} =$ ..... 2 $2^{-3} =$ ..... 3 $3^{-4} =$ .....

4 $(-2)^{-2} =$ ..... 5 $(-2)^{-3} =$ ..... 6 $(-5)^{-4} =$ .....

7 $(-1)^{-5} =$ ..... 8 $(-7)^{-1} =$ ..... 9 $(-9)^{0} =$ .....

Questions 1-9 are similar to example 1

Use ordering operations to calculate each of the following:

10 $\frac{1}{3} + 3^{-2} - (-3)^{-3} =$ ..... 11 $(2)^{-4} - (-6)^{0} - 5^{-1} =$ .....

12 $(-4)^{-2} + 1^{3} - (-6)^{-2} =$ ..... 13 $(8)^{0} - (-3)^{2} - 1^{-4} =$ .....

Questions 10-13 are similar to example 2

Write the following numbers in the digital notation:

14 $4.2 \times 10^{4} =$ ..... 15 $5.14 \times 10^{6} =$ .....

16 $2.16 \times 10^{-3} =$ ..... 17 $15.8 \times 10^{-5} =$ .....

Questions 14-17 are similar to example 3

Write the following numbers in the scientific notation:

18 72300 = ... 19 802000 = ...

20 0.0025 = ... 21 0.000371 = ...

Questions 18-21 are similar to example 4

Solve the Exercises

Calculate the negative exponents for each of the following:

22 $7^{-2} =$ ..... 23 $(-3)^{-2} =$ ..... 24 $1^{-4} =$ ..... 25 $(-1)^{0} =$ .....

Use ordering operations to calculate each of the following:

26 $\frac{1}{4} + 5^{-2} - (-1)^{-3} - \frac{1}{2} =$ .....

27 $6^{-2} - 4^{-2} - (-8)^{-2} - \frac{1}{36} =$ .....

Write the following numbers in the digital notation:

28 $6.3 \times 10^{3} =$ .....

29 $23.7 \times 10^{-2} =$ .....

Write the following numbers in the scientific notation:

30 52100 = ...

31 0.0035 = ...

Solve the problems

32 **Spider:** The spiders spin silk fibre and this fibre has functions depending on its usage. It makes mucoid attachment discs which work as hard wedges that help in the attachment of the spider web on different places and surfaces. And the diameter of the silk fibre is 3×10^{-3} mm .
Write the diameter of the silk fibre in the digital notation form.

33 **Whales:** Whales cover a distance $5^3 \times 2^5$ km in each trip of its migration, but the gray whale cover a distance $2^4 \times 3 \times 5^3$ km more than the other whales in each trip. Write the distance that the gray whale covers in each trip, in the digital notation form.

34 **Space:** Earth is the 3rd planet in the solar system which is considered as a part of the Milky Way Galaxy, it has an elliptic shape and is considered very small compared to the sun, the approximate distance between the earth and the sun is 140,000000000m. Write the distance in the scientific notation form.

Think

35 **Challenge:** Write the following numbers in the digital notation:

i) $\frac{1}{4} \times 10^{-5} =$ ..... ii) $\sqrt{\frac{1}{25}} \times 10^{-3} =$ ..... iii) $\sqrt{\frac{9}{16}} \times 10^{-4} =$ .....

36 **Correct the mistake:** Samah used the ordering operations on rational numbers, and wrote it as following:

$(-3)^{-2} - (-1)^0 + 1^{-2} - \frac{1}{9} = 1$

Determine the mistake of Samah and correct it.

37 **Numerical sense:** Is the number $\sqrt{0.09} \times 10^{-2}$ lies between the two numbers 0.004 , 0.005 ? explain your answer.

Write

The rusult of sumation of two numbers in scientific notation:

$2.17 \times 10^{-3} + 3.83 \times 10^{-3} =$ .....

Lesson [1-3] Properties of Powers (Exponents)

Idea of the lesson:
*Multiplying two powers with the same base.
*Dividing two powers with the same base.
*Raising a power to a power.

Vocabulary:
*Base
*Exponent

Learn

Lion: lion is one of the big cats from mammal animals, and a member of the Felidae family, if an adult lion needs 2^3kg of meat in every meal, so how many kilograms do 2^4 lions need in one meal?

[1-3-1] Multiplying two Powers with the same Base

You have previously learned the number with positive and negative powers, now you will learn the rules of the powers:
1) Multiplying two powers with the same base,
$a^n \times a^m = a^{n+m}$, for $a \in Q$, $n, m \in Z$, $a \neq 0$
So when multiplying two powers with the same base, the result will be the same base raised to the addition result of the two exponents.

Example (1) **Find the number of kilograms which 2^4 lions need in one meal:**

$2^3 \times 2^4 = 2^{3+4}$
$= 2^7$

Put the same base and add the exponents

So 2^4 lions need 2^7 kilograms of meat.

Example (2) **Simplify and write the multiplication result as a single power:**

i) $5^6 \times 5^2 = 5^{6+2} = 5^8$

ii) $7^9 \times 7^{-3} = 7^{9-3} = 7^6$

iii) $3^{-2} \times 3^{-5} = 3^{-2-5} = 3^{-7}$

iv) $8 \times 2^{-7} = 2^3 \times 2^{-7} = 2^{-4}$

v) $11^2 \times 11^{-6} \times 11^{-5} = 11^{2-6-5} = 11^{-9}$

vi) $(\frac{1}{2})^{-3} \times (\frac{1}{2})^{-3} \times (\frac{1}{2})^6 = (\frac{1}{2})^{-3-3+6} = (\frac{1}{2})^0 = 1$

vii) $12 \times 8 \times 3^{-4} = (2^2 \times 3) \times 2^3 \times 3^{-4} = 2^2 \times 2^3 \times 3 \times 3^{-4}$

$= 2^{2+3} \times 3^{1-4} = 2^5 \times 3^{-3}$

viii) $4^{-3} \times (\frac{1}{4})^5 = (\frac{1}{4})^3 \times (\frac{1}{4})^5 = (\frac{1}{4})^{3+5} = (\frac{1}{4})^8 = 4^{-8}$

[1-3-2] Dividing Two Powers with the same Base

2) Dividing a power by a power with the same base:

$\frac{a^n}{a^m} = a^{n-m}$, for $a \in Q$, $n, m \in Z$, $a \neq 0$.

So when dividing a power by a power with the same base, the result will be the same base raising to subtraction result of two exponents, (denominator exponent from numerator exponent).

Example (3) **Simplify and write the dividing results as a single power:**

i) $\frac{7^6}{7^4} = 5^{6-4} = 7^2$

ii) $\frac{3^{-5}}{3^{-9}} = 3^{-5-(-9)} = 3^{-5+9} = 3^4$

iii) $\frac{2^{-7}}{16} = \frac{2^{-7}}{2^4} = 2^{-7-4} = 2^{-11}$

iv) $\frac{54}{12} = \frac{2 \times 3^3}{2^2 \times 3} = 2^{1-2} \times 3^{3-1} = 2^{-1} \times 3^2$

vi) $\frac{(\frac{1}{8})^3}{(\frac{1}{8})^5} = \frac{8^{-3}}{8^{-5}} = 8^{-3+5} = 8^2$

vi) $\frac{(-5)^2 \times 5^0 \times (-5)^7}{5^6} = 5^2 \times 5^0 \times (-5^7) \times 5^{-6}$

$= -5^{2+0+7-6} = -5^3$

[1-3-3] Raising a Power to a Power

3) Raising a power to another power with same base

$(a^n)^m = a^{nm}$, for $a \in Q$, $n, m \in Z$, $a \neq 0$

So when raising a power to another power with the same base, the result will be the same base raising to the multiplication result of the first exponent multiplying by the second exponent.

Example (4) **Simplify and write the results as a single power:**

i) $(7^3)^4 = 7^{3 \times 4} = 7^{12}$

ii) $(9^{-5})^6 = 9^{-5 \times 6} = 9^{-30}$

iii) $(10^3 \times 10^{-8})^{-2} = (10^{-5})^{-2}$

$= 10^{-5 \times -2} = 10^{10}$

iv) $(72)^2 = (8 \times 9)^2 = (2^3 \times 3^2)^2$

$= (2^3)^2 \times (3^2)^2 = 2^6 \times 3^4$

vi) $\frac{(-7)^3 \times 7^8 \times (-49)^2}{7^{-4} \times (-7^3)^5} = \frac{(-7)^3 \times 7^8 \times (-7^2)^2}{7^{-4} \times (-7^{3 \times 5})} = \frac{(-7^{3+8}) \times (7^{2 \times 2})}{7^{-4} \times (-7^{15})}$

$= \frac{(-7^{11}) \times 7^4}{-7^{-4+15}} = \frac{-7^{11+4}}{-7^{11}}$

$= \frac{-7^{15}}{-7^{11}} = 7^{15-11} = 7^4$

Make sure of your understanding

Simplify and write the multiplication result as a single power:

1 $2^4 \times 2^2 =$ .....

2 $9^0 \times 9^{-5} =$ .....

3 $7^{-3} \times 7^{-5} =$ .....

4 $16 \times 2^{-8} =$ .....

5 $13^{-4} \times (-13)^3 \times 13^6 =$ .....

6 $(\frac{1}{3})^{-2} \times (\frac{1}{3})^5 \times (\frac{1}{3})^{-1} =$ .....

7 $32 \times 3^4 \times 24 =$ .....

8 $9^{-6} \times (\frac{1}{9})^7 =$.....

Questions 1-8 are similar to example 2

Simplify and write the division result as a single power:

9 $\frac{5^3}{5^7} =$.....

10 $\frac{2^{-6}}{2^{-1}} =$.....

11 $\frac{3^{-3}}{27} =$.....

12 $\frac{6 \times 3^{-2}}{2^4 \times 3} =$.....

13 $\frac{(\frac{1}{25})^4}{(\frac{1}{25})^8} =$.....

14 $\frac{(-6)^3 \times 6^{-6} \times (-6)^0}{6^9} =$.....

Questions 9-14 are similar to example 3

Simplify and write the result as a single power:

15 $(5^2)^6 =$ .....

16 $(8^{-3})^4 =$.....

17 $(10^{-7} \times 10^6)^{-3} =$.....

18 $(3^2)^{-2} =$.....

19 $\frac{(-1)^3 \times 1^8}{((-1)^4)^6} =$.....

20 $\frac{(-3)^2 \times 3^5}{((-3)^3)^{-1}} =$.....

Questions 15-20 are similar to example 4

Solve the Exercises

Simplify and write the multiplication result as a single power:

21 $6^{-7} \times 6^3 =$ .....

22 $7^{-1} \times 7^{-4} =$ .....

23 $27 \times 3^{-6} =$.....

24 $8^{-7} \times (\frac{1}{8})^7 =$.....

25 $(\frac{1}{5})^{-3} \times (\frac{1}{5})^7 \times (\frac{1}{5})^{-2} =$.....

Simplify and write the division result as a single power:

26 $\frac{7^{-2}}{7^7} =$.....

27 $\frac{6^{-4}}{6^{-3}} =$.....

28 $\frac{48 \times 3^{-4}}{3^3 \times 2} =$.....

29 $\frac{(\frac{1}{49})^{-3}}{(\frac{1}{49})^6} =$.....

Simplify and write the result as a single power:

30 $(3^4)^{-3} =$ .....

31 $(6^{-2})^{-5} =$ .....

32 $\frac{(-2)^2 \times 2^6 \times (-4)^5}{2^{-4} \times ((-2)^5)^3}$

Solve the problems

33 **Cheetah:** The hunter cheetah which lives in the continent of Asia and Africa, belongs to the Felidae, it is from the mammals that has possibility of extinction, and is the fastest animal on earth ever where its maximum speed reaches $\frac{2}{10^{-3}}$ m/min. Write the number of the meters which the cheetah spends during 2^3min.

34 **Production:** Iraq produced $(\frac{3}{10^{-2}})^3$ barrels of raw petrol in the period of 9 months in one of the years. Write the production of the petrol in a digital notation form.

35 **Rubic's cube:** Is a puzzle which represents a three dimentional cube that consists of 6 faces, each face has 3^2 of stickers having the same color, and the Rubic's cube faces have the main colors which are red, white, green, blue, orange, and yellow. The inventor of the Rubic's cube is the sculptor Rno Rubic. Write the number of the stickers in 6 cubes as a number of single power form.

Think

36 **Challenge:** Simplify and write the result as a single power:

i) $\frac{(-10)^5 \times 10^{-4} \times (-10)^{-2}}{10^{-7} \times (-10^{-6})^2} = ...$

ii) $\sqrt{\frac{1}{64}} \times 2^{-4} \times (-4)^7 = ...$

37 **Correct the mistake:** Jameel simplified the following exponent and wrote the result as follows: $(\frac{1}{9})^{-7} \times (\frac{1}{3})^5 \times (\frac{1}{27})^{-1} = 3^{-14}$

Determine the mistake of Jameel and correct it.

38 **Numerical sense:** Is the number $\frac{16 \times 2^{-6}}{2^0 \times 2}$ lies between the two numbers 3^{-1} , 3^{-2} ? explain your answer.

Write

The rusult as a fraction form:

$36^{-5} \times (-2)^8 \times (-3)^8 = ...$

Lesson [1-4] Recurring Decimal Fractions and Scientific Notation of Number (Using Calculator)

Idea of the lesson:
*Using calculator for simplifying numerical sentence containing:
Recurring decimal fraction and scientific notation for the number.

Vocabulary:
*Calculator
*Recurring Decimal Fraction

Learn

Giraffe is one of the mammals that eats herbs and tree leaves,fruits and vegetables, and it is the tallest animal ever. Girraffes drink water twice a week and if one of the Giraffes drinks $\frac{15}{2}$ liters in the first time and $\frac{17}{3}$ liters in the second time. Write the number of the liters that Girraffe has drunk the whole week in the form of decimal fraction.

[1-4-1] Using Calculator to Simplify Numerical Sentence Contain

You have previously learned how to simplify the numerical sentence by using the ordering operations on rational numbers, now you will learn how to simplify a numerical sentence that contains recurring decimal fractions (using calculator) as follow:
If the numerical sentence was written as fraction forms only, you can apply the operations as you have previously learned and writting the result as a finite or recurring decimal fraction.

$$\frac{10}{3}+\frac{2}{5}=\frac{50+6}{15}=\frac{56}{15}=3.7333...=3.7\overline{3}$$

And if the numerical sentence was written as a recurring decimal fractions, it will be left to the later stages.

Example (1) **Write the number of liters that the giraffe drunk for the whole week in the decimal form:**

$$\frac{15}{2}+\frac{17}{3}=\frac{15\times 3+17\times 2}{6}$$ *Add the fractions*

$$=\frac{45+34}{6}$$ *Simplify the result to simplest form*

$$=\frac{79}{6}=13.1666...=13.1\overline{6}$$ *Write the result as a recurring decimal fraction*

Example (2) **Use the calculator to write the addition and subtraction result in a decimal fraction form:**

i) $\frac{8}{5}+\frac{13}{6}=\frac{48+65}{30}=\frac{113}{30}=3.7666...=3.7\overline{6}$

ii) $\frac{9}{2}+\frac{21}{8}-\frac{5}{12}=\frac{57}{8}-\frac{5}{12}=\frac{171-10}{24}=\frac{161}{24}=6.708333...=6.708\overline{3}$

iii) $\frac{6}{13}-\frac{11}{3}-\frac{10}{6}=\frac{36-286-130}{78}=-\frac{380}{78}=-4.8717948717948...=-4.\overline{871794}$

Example (3) **Use the calculator to write the multiplication and division result in a decimal fraction form:**

i) $\frac{5}{3} \times \frac{12}{15} = \frac{5 \times 12}{3 \times 15} = \frac{4}{3} = 1.333... = 1.\overline{3}$

ii) $\frac{8}{9} \div \frac{48}{-45} = \frac{8}{9} \times \frac{-45}{48} = \frac{8 \times -45}{9 \times 48} = \frac{-5}{6} = -0.8333... = -0.\overline{8}3$

iii) $\frac{6}{5} \times \frac{13}{12} \div \frac{39}{4} = \frac{13}{10} \div \frac{39}{4} = \frac{13}{10} \times \frac{4}{39} = \frac{13 \times 4}{10 \times 39} = \frac{2}{15} = 0.1333... = 0.1\overline{3}$

iv) $\frac{17}{6} \div \frac{34}{42} \times \frac{-7}{6} = \frac{17}{6} \times \frac{42}{34} \times \frac{-7}{6} = \frac{7}{2} \times \frac{-7}{6} = -\frac{49}{12} = -4.08333... = -4.08\overline{3}$

[1-4-2] Using Calculator to Simplify Numerical Sentence that Contains Scientific Notation of Numbers

You have previously learned how to write the number as a scientific notation from standard (digital) notation, and vice versa. And now you will learn how to simplify a numerical sentence that contains multiplication and division numbers in scientific notation form (using calculator) as follows:

1) Do first applying operations on the decimal fractions in ordering (using calculator).

2) Find the power of ten result.

3) Move the decimal point as required for result.

Example (4) **Use the calculator to write the multiplication and division result in scietific notation of the number:**

i) $(3.1 \times 10^4)(7.6 \times 10^{-3})$

$= (3.1 \times 7.6)(10^4 \times 10^{-3})$ *Put the operations that have priority between the brackets*

$= 23.56 \times 10$ *Find the result of multiplying the decimal fraction and the power of ten result*

$= 2.356 \times 10^2$ *Move the decimal point so that the number become in one place*

ii) $(1.05 \times 10^{-5})(0.9 \times 10^{-4})$

$= (1.05 \times 0.9)(10^{-5} \times 10^{-4})$

$= 0.945 \times 10^{-9}$

$= 9.45 \times 10^{-10}$

iii) $(12.5 \times 10^{-3})(9.08 \times 10^7) = (12.5 \times 9.08)(10^{-3} \times 10^7) = 113.5 \times 10^4 = 1.135 \times 10^6$

iv) $\frac{7.29 \times 10^6}{0.09 \times 10^4}$

$= (7.29 \div 0.09)(10^6 \times 10^{-4})$ *Put the operations that have priority between the brackets*

$= 81 \times 10^2$ *Find the result of dividing the decimal fraction and the power of ten result*

$= 8.1 \times 10^3$ *Move the decimal point so that the number become in one place*

v) $\frac{2.25 \times 10^{-7}}{0.005 \times 10^{-2}} = (2.25 \div 0.005)(10^{-7} \times 10^2) = 450 \times 10^{-5} = 4.5 \times 10^{-3}$

Make sure of your understanding

Use the calculator to write the addition and subtraction result as a decimal fraction form:

1 $\frac{2}{3} - \frac{5}{8} = ...$

2 $\frac{1}{3} + \frac{5}{6} = ...$

3 $\frac{7}{2} + \frac{9}{4} - \frac{13}{12} = ...$

4 $\frac{21}{5} - \frac{17}{10} - \frac{3}{11} = ...$

Questions 1-4 are similar to example 2

Use the calculator to write the multiplication and division result as a decimal fraction form:

5 $\frac{4}{7} \times \frac{21}{12} = ...$

6 $\frac{6}{11} \div \frac{42}{-22} = ...$

7 $\frac{15}{9} \times \frac{36}{75} \div \frac{12}{35} = ...$

8 $\frac{13}{5} \div \frac{39}{15} \times \frac{-11}{9} = ...$

Questions 5-8 are similar to example 3

Use the calculator to write the multiplication and division result as a scietific notation for number:

9 $(2.7 \times 10^{3})(6.6 \times 10^{-7}) = ...$

10 $(5.08 \times 10^{-6})(0.8 \times 10^{-3}) = ...$

11 $\frac{6.25 \times 10^{8}}{0.5 \times 10^{3}} = ...$

12 $\frac{1.69 \times 10^{-6}}{0.13 \times 10^{-2}} = ...$

Questions 9-12 are similar to example 4

Solve the Exercises

Use the calculator to write the addition and subtraction result in a decimal fraction form:

13 $\frac{1}{3} + \frac{2}{9} = ...$

14 $\frac{2}{9} + \frac{1}{6} - \frac{7}{6} = ...$

Use the calculator to write the multiplication and division result as a decimal fraction form:

15 $\frac{1}{8} \times \frac{56}{3} = ...$

16 $\frac{24}{25} \times \frac{5}{4} \div \frac{18}{35} = ...$

Use the calculator to write the multiplication and division result as a scietific notation for number:

17 $(1.3 \times 10^{-4})(9.1 \times 10^{-6}) = ...$

18 $\frac{2.256 \times 10^{-2}}{0.16 \times 10^{4}} = ...$

Solve the problems

19 **Whale:** Whale is the biggest living animal ever, either nautical or terrestrial, it belongs to the mammals, the female whale gave birth to 4 baby whales, the length of two of them according to their mums length is $\frac{4}{15}$, $\frac{1}{5}$ Meter. Write the total length of the babies in a decimal fraction.

20 **Vegtable salad:** Sarah made a dishe of vegtable salad and added 4 strawberries where each of them weight is 1.5×10^{-2}kg, and 12 grains of olive which each of them weight is 0.06×10^{-1}kg . Write the total weight that is added to the salad in a scientific notation form.

21 **Education:** The ratio of the final success in the first and second term in one of the secondary schools is $\frac{19}{20}$, what is the ratio of the success in the second term if the ratio of the success in the first term was $\frac{19}{25}$?
Write the number in a decimal fraction form.

Think

22 **Challenge:** Use the calculator to write the result as a decimal fraction form:

i) $\frac{3}{7} \times \frac{35}{6} + \frac{10}{13} \times \frac{26}{-6} =$ .....

ii) $\frac{5}{22} \div \frac{20}{33} - \frac{7}{13} \div \frac{42}{26} =$ .....

23 **Correct the mistake:** Khulood used the caculator and wrote the expression result as follows: $(6.7 \times 10^{3})(2.8 \times 10^{-7}) = 1.876 \times 10^{-4}$

Determine the mistake of Khulood and correct it.

24 **Numerical sense:** Is the number $\frac{23.5 \times 10^{-3}}{2.35 \times 10^{-2}}$ lies between the two numbers 0 , 2 ? explain your answer.

Write

The rusult as a decimal fraction form:

$\frac{9}{4} \div \frac{63}{-12} \times \frac{-2}{3} =$ .....

Lesson [1-5] Simplifying Fractional Numerical Sentences

Idea of the lesson:
*Simplifying a fractional numerical sentence containing roots and absolute values, powers and scientific notation of the number.

Vocabulary:
*Root
*Perfect square

Learn

On one of the flights of the Iraqi Airlines from Babhdad to Istanbul, the company sold a first class ticket in a price 5.5×10^5 Iraqi dinars and the economy class in a price 46.5×10^4 Iraqi dinars. If the plane has 180 chairs 12 of them for the first class and the rest is for the economical classes. What is the amount of money that the company gets if all tickets were sold?

[1-5-1] Simplifying Numerical Sentence Containing Powers and Scientific Notation Number

You have previously learned how to simplify a numerical sentence that contains multiplication and division only for the numbers with scientific notation, now you will learn how to simplify a numerical sentence that contains addition and subtraction as well as multiplication and division to the numbers with scientific notation.
And to find the addition or subtraction result of two terms containing a scientific notation of number we will follow one of these methods:

First method: Equalizing the powers of ten for the two terms and taking them out as a common factor, then applying operation of addition or subtraction of the two decimal fractions after putting them between the brackets and writting their results which are multiplied by the power of ten, then adjusting the power of ten as required in the scientific notation.

Second method: From the scientific notation of the number writing the digital notation, then doing the addition or subtraction operation, after that returning the number to the scientific notation or as the question requires.

Example (1) **Find the amount of money that the company gets if the tickets of the plane were sold:**

First method:

$46.5 \times 10^4 = 4.65\times 10^5$ *Equalizing powrers of ten to both of the terms*

$12 \times 5.5 \times 10^5 = 66 \times 10^5$ *First class tickets price*

$168 \times 4.65 \times 10^5 = 781.2 \times 10^5$ *Economy class tickets price*

$66\times10^5 + 781.2 \times 10^5 = (66 + 781.2) \times 10^5$ *Taking out the powers of ten as a common factor*

$= 847.2 \times 10^5 = 8.472 \times 10^7$ *The total amount for the tickets in IQD*

Second method:

$12 \times 5.5 \times 10^5 = 66 \times 10^5 = 6\ 600\ 000$ *First class tickets price*

$168 \times 46.5 \times 10^4 = 7812 \times 10^4 = 78\ 120\ 000$ *Economy class tickets price*

$6\ 600\ 000 + 78\ 120\ 000 = 84\ 720\ 000$ *The total amount for the tickets in IQD*

$= 8.472 \times 10^7$ *Writing the number in scientific notation*

Note: *You can use calculator in applying operations on decimal fractions.*

Example (2) **Simplify the following fractional numerical sentences and write the result in the scientific notation:**

i) $4.1\times10^{-5} + 0.61 \times 10^{-4} = 4.1\times10^{-5} + 6.1\times 10^{-5} = (4.1 + 6.1) \times 10^{-5}$

$= 10.2 \times10^{-5} = 1.02 \times10^{-4}$

ii) $7.6 \times10^{-4} - 0.23\times 10^{-3} = 7.6 \times10^{-4} - 2.3\times \times10^{-4} = (7.6- 2.3) \times10^{-4} = 5.3\times10^{-4}$

iii) $2.3 \times10^{-2} + 0.176 \times 10^{3} = 0.023 + 176 = 176.023 = 1.76023 \times 10^{2}$

iv) $300.1 \times10^{-2} - 0.005\times 10^{4} = 3.001 - 50 = -46.999 = -4.6999 \times 10$

v) $(6.3 \times10^{3})(8.2 \times 10^{-7}) = (6.3 \times 8.2)\ (10^{-3} \times10^{-7}) = 51.66 \times10^{-4} = 5.166 \times10^{-3}$

[1-5-2] Simplifying Fractional Numerical Sentence that contains Roots , Absolute Values and Powers

You have previously learned how to simplify a numerical sentence that contains rational numbers by using the ordering operations, and now you will learn how to simplify a fractional numerical sentence that contains roots and absolute values and powers of numbers, and in the same way you are going to follow the previously steps you have learned in simplifying the numerical sentence which is:

1) Simplifying each term in simplest form by removing the roots, absolute values of the nember, and simplifying the powers.

2) Using ordering operations for simplifying the expression, with giving preference to the operations between brackets.

Example (3) **Simplify the following fractional numerical sentences and write the result in simplest form:**

i) $\frac{4}{3} - \frac{\sqrt{25}}{\sqrt[3]{27}} - \frac{-2}{\sqrt{16}} = \frac{4}{3} - \frac{5}{3} - \frac{-2}{4}$ *Finding root values and writing the fractions in simplest form*

$= (\frac{4}{3} - \frac{5}{3}) - \frac{-1}{2}$ *Putting operations which have preference between prackets*

$= -\frac{1}{3} + \frac{1}{2} = \frac{-2+3}{6} = \frac{1}{6}$ *Adding the two fractions to find the result*

ii) $\frac{1}{23}(\frac{|-15|}{7} - \frac{\sqrt[3]{-8}}{-4}) - \frac{1}{28} = \frac{1}{23}(\frac{15}{7} - \frac{1}{2}) - \frac{1}{28} = \frac{1}{23}(\frac{30-7}{14}) - \frac{1}{28}$

$= (\frac{1}{23} \times \frac{23}{14}) - \frac{1}{28} = \frac{1}{14} - \frac{1}{28} = \frac{2-1}{28} = \frac{1}{28}$

iii) $\frac{\sqrt[3]{-125}}{36} \times \frac{|-6|}{10} + \frac{9}{4} \div \frac{18}{\sqrt[3]{-8}} = (\frac{-5}{36} \times \frac{6}{10}) + (\frac{9}{4} \div \frac{-18}{2}) = \frac{-1}{12} - \frac{1}{4} = -\frac{1}{3}$

iv) $\frac{1}{3^2} \times 3^4 \div (-3)^{-1} + 3^{-2} = (3^{-2} \times 3^4) \div (-3^{-1}) + 3^{-2}$

$= \frac{3^2}{-3^{+1}} + \frac{1}{3^2} = -3 + \frac{1}{9} = \frac{-27+1}{9} = \frac{-26}{9}$

v) $(-1)^{-2} - (-8)^0 + \frac{\sqrt{25}}{3^2} \div \frac{25}{-12} = 1 - 1 + (\frac{5}{3^2} \times \frac{-12}{25}) = 0 + (\frac{-4}{15}) = -\frac{4}{15}$

Make sure of your understanding

Simplify the following fractional numerical sentences and write the result in scientific notation:

1 $7.3\times10^{4} + 3.6\times10^{4} =$ .....

2 $1.5\times10^{-3} - 5.42\times10^{-3} =$ .....

3 $5.2\times10^{3} + 0.61\times10^{5} =$ .....

4 $2.4\times10^{-6} + 0.21\times10^{-4} =$ .....

Questions 1-8 are similar to example 2

5 $7.6\times10^{-5} - 0.23\times10^{-4} =$ .....

6 $7.4\times10^{-2} + 0.176\times10^{2} =$ .....

7 $(9.9\times10^{2})(81.8\times10^{-2}) =$ .....

8 $(5.3\times10^{4})(7.2\times10^{-9}) =$ .....

Simplify the following fractional numerical sentences and write the result in simplest form:

9 $\frac{2}{5} - \frac{\sqrt{36}}{\sqrt[3]{-8}} - \frac{3}{\sqrt{25}} =$ .....

10 $\frac{1}{14}\left(\frac{|-11|}{2} - \frac{\sqrt[3]{-64}}{3}\right) - \frac{1}{4} =$ .....

Questions 9-14 are similar to example 3

11 $\frac{\sqrt[3]{-216}}{7} \times \frac{|-21|}{12} + \frac{-5}{\sqrt{16}} =$ .....

12 $\frac{\sqrt[3]{-27}}{15} \div \frac{6}{|-5|} + \frac{-7}{\sqrt{144}} =$ .....

13 $\frac{1}{5^{-2}} \times 5^{-4} \div (-5)^{-3} + 5^{-1} =$ .....

14 $(-1)^{-4} - (-9)^{0} + \frac{\sqrt{49}}{2^{3}} \div \frac{7}{-12} =$ .....

Solve the Exercises

Simplify the following fractional numerical sentences and write the result in scientific notation:

15 $2.8\times10^{5} + 1.2\times10^{5} =$ .....

16 $6.4\times10^{2} + 0.73\times10^{4} =$ .....

17 $4.6\times10^{-3} - 0.56\times10^{-5} =$ .....

18 $(8.2\times10^{-6})(9.6\times10^{2}) =$ .....

Simplify the following fractional numerical sentences and write the result in simplest form:

19 $\frac{1}{7} - \frac{\sqrt{64}}{\sqrt[3]{-27}} - \frac{-6}{\sqrt{49}} =$ .....

20 $\frac{\sqrt[3]{-27}}{4} \times \frac{|-8|}{9} + \frac{5}{14} \div \frac{-15}{\sqrt{49}} =$ .....

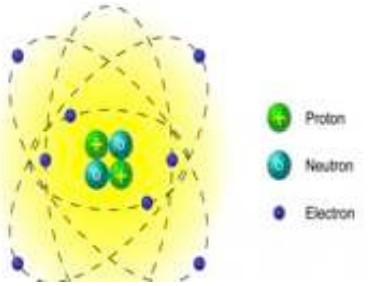
21 $\frac{1}{7^{-3}} \times 7^{-5} \div (-7)^{-2} + 7^{-1} =$ .....

Solve the problems

22 **Algae:** Algaes grow on surfaces of stagnant ponds and the rocks on the coast. If the mass of the algae on the first rock was 2.6×10^{-4} kg, and the second rock was 1.7×10^{-5} kg. What is the total mass of algae on both rocks?

23 **Light:** The amount of the distance that the light covers in one year is 9.46×10^{12} km. What is the amount of the distance that the light covers in 4.5×10^{3} years?

24 **Atom:** The atom consists of proton which has positive electrical charge and that is exactly equal to the electron's charge, but the electron's charge is negative. The mass of proton is approximately 1.67×10^{-27} kg, and the mass of the electron is approximately 9.11×10^{-13} kg.
What is the difference between the two masses?



Think

25 **Challenge:** Simplify the following fractional numerical sentences and write the result in simplest form:

i) $\frac{\sqrt[3]{-27}}{3^{-2}} \times \frac{|(-3)^{-3}|}{27} - \frac{8}{5} \div \frac{-2^3}{\sqrt{25}} = ...$ ii) $\frac{7}{16} \div \sqrt{\frac{49}{64}} \times \sqrt[3]{\frac{-8}{27}} \div \frac{15}{-3} = ...$

26 **Correct the mistake:** Sundus used the caculator and wrote the result of expression in scientific notation:

$(5.4 \times 10^{-4})(3.6 \times 10^{-5}) = 19.44 \times 10^{-5}$

Determine the mistake of Sundus and correct it.

27 **Numerical sense:** Is the number $\frac{3.2 \times 10^{-3}}{6.4 \times 10^{-4}}$ lies between the two numbers 0 , 10 ? explain your answer

Write

The rusult as a decimal fraction form:

$$(-1)^{-4} - (-5)^0 + \frac{5^{-4}}{5^{-3}} \div (\frac{1}{2})^0 = ...$$

Chapter Test

Using ordering operations on rational numbers to write each expression in simplest form:

1 $\frac{-8}{22} \times \frac{11}{-4} + \frac{1}{2} \times \frac{11}{4} =$ .....

2 $\frac{9}{5} \div \frac{-22}{25} - \frac{9}{4} \times \frac{-16}{3} =$ .....

3 $\frac{7}{4} - \frac{-5}{12} \times \frac{48}{15} \div \frac{8}{21} =$ .....

4 $6.03 \times 1.5 - 4.03 \times 0.9 =$ .....

Use ordering operations to calculate each of the following:

5 $\frac{1}{3} + 3^{-2} + (-1)^{-4} \times \frac{1}{6} =$ .....

6 $(-3)^{-3} - (8)^{0} + 1^{3} + \frac{1}{9} =$ .....

7 $4^{-2} - 2^{4} - (-5)^{2} - \frac{1}{5} =$ .....

8 $(-4)^{0} - (-6)^{-2} + 1^{-4} - \frac{1}{6} =$ .....

Write the following numbers in digital notaion:

9 $3.4 \times 10^{4} =$ .....

10 $2.14 \times 10^{-5} =$ .....

11 $8.05 \times 10^{-6} =$ .....

Write the following numbers in scientific notaion:

12 $720000 =$ .....

13 $0.00012 =$ .....

14 $0.0000066 =$ .....

Simplify and write the multiplication result as a single power:

15 $5^{-6} \times 5^{2} =$ .....

16 $(-1)^{-4} \times 1^{-3} =$ .....

17 $9^{-5} \times (\frac{1}{3})^{5} =$ .....

18 $(\frac{1}{2})^{-1} \times (\frac{1}{2})^{-3} \times (\frac{1}{2})^{6} =$ .....

19 $12^{-3} \times (-3)^{4} \times (3)^{5} =$ .....

Simplify and write the result as a single power:

20 $\frac{5^{-4}}{5^{9}} =$ .....

21 $(7^{-3})^{-2} =$ .....

22 $\frac{50 \times 5^{-6}}{5^{3} \times 8} =$ .....

23 $\frac{(-3)^{4} \times 3^{3} \times (-9)^{5}}{3^{-6} \times (-3^{4})^{3}} =$ .....

24 $\frac{(-8)^{3} \times 2^{-2} \times ((-4)^{-2})}{2^{-7} \times (2^{3})^{3}} =$ .....

Use the calculator to write the result as a decimal fraction form:

25 $\frac{1}{5} + \frac{3}{10} =$ .....

26 $\frac{2}{3} - \frac{5}{6} =$ .....

27 $\frac{2}{7} - \frac{1}{2} + \frac{2}{3} =$ .....

28 $\frac{1}{6} \times \frac{48}{3} =$ .....

29 $\frac{2}{-7} \div \frac{18}{35} =$ .....

30 $\frac{3}{5} \times \frac{7}{15} \div \frac{14}{35} =$ .....

Use the calculator to write the multiplication and division result in scietific notation of the number:

31 $(3.5 \times 10^{-6})(8.1 \times 10^{-3}) =$ .....

32 $\frac{7.29 \times 10^{7}}{0.09 \times 10^{3}} =$ .....

Simplify the following fractional numerical sentences and write the result in simplest form:

33 $\frac{\sqrt[3]{-27}}{5} \times \frac{|-35|}{9} + \frac{2}{28} \times \frac{-7}{\sqrt[3]{8}} =$ .....

34 $\frac{6}{\sqrt{49}} \div \frac{|-54|}{-7} - \frac{\sqrt[3]{-125}}{3} \times \frac{-15}{6} =$ .....

Chapter 2

The Real Numbers

The chameleon is a reptile of the reptiles. It is found in almost every part of the world. It is known by its ability to change its colour. The largest length of chameleon male is 68.5cm . This number is a rational number and can be written as fraction.

Pretest

Use addition and subtraction of rational numbers to write each expression in simplest form:

1 $\frac{1}{3} - \frac{2}{5} + \frac{4}{2} = \ldots\ldots$

2 $3\frac{1}{2} + 1\frac{2}{4} - \frac{7}{2} = \ldots\ldots$

3 $\frac{2}{7} - 2\frac{1}{6} - \frac{5}{12} = \ldots\ldots$

4 $3.12 - 3.07 + 1.5 = \ldots$

5 $0.14 + 0.92 - 0.76 = \ldots$

6 $5.12 - 0.37 - 6.96 = \ldots$

Use multiplication and division of rational numbers to write each expression in simplest form:

7 $2\frac{1}{4} \times 3\frac{1}{2} \div 1\frac{1}{4} = \ldots$

8 $\frac{22}{5} \div \frac{-11}{15} \times \frac{-1}{3} = \ldots$

9 $2.5 \times 4.05 \div (-1.2) = \ldots$

Use ordering operations to calculate each of the following:

10 $\frac{1}{5} + 5^{-2} - (-5)^{-3} = \ldots\ldots$

11 $3^{-3} - (-3)^{0} - 3^{-2} = \ldots\ldots$

12 $(-2)^{-2} + 1^{3} - (-4)^{-2} = \ldots\ldots$

13 $(9)^{0} - (-4)^{2} - 1^{-5} = \ldots\ldots$

Write the following numbers in digital notation:

14 $5.3 \times 10^{3} = \ldots\ldots$

15 $2.04 \times 10^{5} = \ldots\ldots$

16 $1.17 \times 10^{-4} = \ldots\ldots$

17 $61.4 \times 10^{-6} = \ldots\ldots$

Write the following numbers in scientific notation:

18 $65300 = \ldots\ldots$

19 $108000 = \ldots\ldots$

20 $0.0043 = \ldots\ldots$

21 $0.000276 = \ldots\ldots$

Simplify and write the multiplication and division result as a single power:

22 $3^{5} \times 3^{2} = \ldots\ldots$

23 $7^{0} \times 7^{-4} = \ldots\ldots$

24 $6^{-1} \times 6^{-5} = \ldots\ldots$

25 $\frac{5^{4}}{5^{7}} = \ldots\ldots$

26 $\frac{2^{-4}}{8} = \ldots\ldots$

27 $\frac{6 \times 3^{-3}}{2^{5} \times 3} = \ldots\ldots$

Use the calculator to write the addition and subtraction result in decimal fraction form:

28 $\frac{1}{4} - \frac{5}{2} = \ldots\ldots$

29 $\frac{1}{5} + \frac{2}{3} = \ldots\ldots$

30 $\frac{1}{2} + \frac{1}{3} - \frac{13}{6} = \ldots\ldots$

Use the calculator to write the multiplication and division result in decimal fraction form:

31 $\frac{2}{7} \times \frac{28}{6} = \ldots\ldots$

32 $\frac{7}{3} \div \frac{56}{-21} = \ldots\ldots$

33 $\frac{15}{9} + \frac{81}{60} - \frac{27}{8} = \ldots\ldots$

Write one example for each property of the following properties:

34 For each $a, b, c \in Q$ if $a \leq b$ then $a + c \leq b + c$

35 For each $a, b, c \in Q$ if $a > b$ and $c < 0$ then $ac < bc$

36 For each $a, b, c \in Q$ if $a \geq b$ and $c < 0$ then $\frac{a}{c} \leq \frac{b}{c}$

Lesson [2-1] Concept of Real Numbers and Representing them on a Number Line

Idea of the lesson:
* Identifying the concept of real numbers and representing them on a number line and Comparing, arranging.

Vocabulary:
*Rational number
*Irrational number
*Real number

Learn

Ahmed painted a painting representing one of the archeological landmarks, and at the top of the front of building appears the right-angled triangle, the length of its two right sides are 2m, 3m. Find the length of hypotenuse, and show if it is rational or not?

[2-1-1] Concept of Real Numbers

You have previously learned the rational numbers that can be written in form of fraction or finite decimal numbers or recurring decimal number for example: $4\frac{1}{2} = 4.5$, $2\frac{5}{3} = \frac{11}{3}$, $\frac{4}{3} = 1.\overline{3}$, $\sqrt{1.69} = 1.3$

And irrational numbers are written as infinite and non recurring decimal number, and every natural number that is not written in form of perfect-square, its square root is irrational number.

For example: $\sqrt{5} = 2.2360679...$, $\sqrt{3} = 1.7320508...$.

As well the irrational number cannot be written in form of fraction $\frac{a}{b}$ where $b \neq 0$.

The set of real numbers (R) consists of the union of set of rational numbers (Q) and set of irrational numbers (H) so $R = Q \cup H$.

Example (1) **Find the length of the hypotenuse, and show if it is rational number or not?**

Assume that the length of the hypotenuse is the variable x, and by using the Pythagorean theorem then:

$x^2 = 2^2 + 3^2 \Rightarrow x^2 = 4 + 9 \Rightarrow x^2 = 13 \Rightarrow x = \sqrt{13}$

By using the calculator then:

$x = \sqrt{13} = 3.6055512...$

We notice that the square root for the number 13 is unrecurring decimal number and infinite that's why it is an irrational number (real number).

Example (2) **Classify the number whethre it is rational number or irrational number or unreal number:**

i) $\sqrt{9} = 3$ integer , rational number, real number

ii) $\sqrt{8} = 2.828427...$ real number, irrational number

iii) $\frac{0}{5} = 0$ integer , rational number, real number

iv) $\sqrt{\frac{4}{9}} = \frac{2}{3}$ real number, rational number

v) $\sqrt{-17}$ unreal number

vi) $\frac{4}{0}$ unknown , that is why it is not a number

vii) $\sqrt{\frac{16}{5}} = 2.23606...$ real number, irrational number

viii) $\frac{10}{3} = 3.33333...$ real number, rational number

ix) $-3\frac{2}{3} = -3.4$ real number, rational number

x) $-\sqrt{2} = -1.414213...$ real number, irrtional number

[2-1-2] Representing The Real Numbers on a Number Line

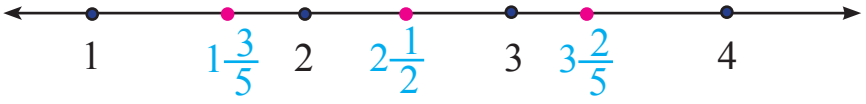
The real numbers has density property that means there exist another real number between each two real numbers; and you can't see this property in natural numbers and integer. The real numbers could be represented on a number line, and we need to approximate the irrational numbers into nearest tenth to represent them on a number line.

Example (3) **Find a real number between these two numbers $1\frac{3}{5}$, $3\frac{2}{5}$ and represent it on a number line:**

$(3\frac{2}{5} + 1\frac{3}{5}) \div 2$ *Add the two numbers and divide it by 2*

$= 4\frac{5}{5} \div 2$

$= 5 \div 2 = 2\frac{1}{2}$



Therefor the number $2\frac{1}{2}$ is one of real numbers that lies between the two numbers $1\frac{3}{5}$, $3\frac{2}{5}$

Example (4) **Estimate the following square roots into nearest tenth then represent them on a number line:**

i) $\sqrt{5} \approx 2.2$

ii) $-\sqrt{3} \approx -1.7$

[2-1-3] Comparison and Arrangement of Real Numbers

You have previously learned the comparison and arrangement of rational numbers. And in the same way you are going to compare and arrange the real numbers after writting them in form of decimal approximate to nearest tenth (tenth).

Example (5) **Compare between the following real numbers by using the symbols (< , > , =):**

i) $\sqrt{5}$ $<$ $2\frac{1}{2}$, $\sqrt{5} = 2.236...$, $2\frac{1}{2} = 2.5$ $\Rightarrow$ $2.236... < 2.5$

ii) $\sqrt{3}$ $>$ $\sqrt{1.69}$, $\sqrt{3} = 1.732...$, $\sqrt{1.69} = 1.3$ $\Rightarrow$ $1.732... > 1.3$

Example (6)

i) Arrange the following real numbers from smallest to largest $\sqrt{3}$, $\sqrt{11}$, $\sqrt{7}$

$\sqrt{7} = 2.645 \ldots$, $\sqrt{11} = 3.316\ldots$, $\sqrt{3} = 1.732\ldots$ $\Rightarrow$ $\sqrt{3}$, $\sqrt{7}$, $\sqrt{11}$

ii) Arrange the following real numbers from largest to smallest $-\sqrt{8}$, $-2\frac{1}{2}$, -2.9

$-\sqrt{8} = -2.828 \ldots$, $-2\frac{1}{2} = -2.5$, -2.9 $\Rightarrow$ $-2\frac{1}{2}$, $-\sqrt{8}$, -2.9

Make sure of your understanding

Classify the number whether it is rational number or irrational number or unreal number:

1 $\sqrt{16}$ 2 $\sqrt{11}$ 3 $\frac{0}{2}$ 4 $\sqrt{\frac{9}{25}}$ 5 $\sqrt{-27}$

6 $\frac{8}{0}$ 7 $\sqrt{\frac{36}{7}}$ 8 $\frac{20}{6}$ 9 $-5\frac{3}{2}$ 10 $-\sqrt{6}$

Questions 1-10 are similar to examples (1-2)

Estimate the following square roots into nearest tenth then represent them on a number line:

11 $\sqrt{7} \approx$ ..... 12 $-\sqrt{5} \approx$ ..... 13 $\sqrt{\frac{8}{25}} \approx$ .....

Questions 11-13 are similar to example 4

Compare between the following real numbers by using the symbols (< , > , =):

14 $\sqrt{3}$ ☐ $3\frac{1}{3}$ 15 $-\sqrt{8}$ ☐ $\sqrt{2.25}$ 16 $\frac{0}{12}$ ☐ $\frac{0}{5}$

Questions 14-16 are similar to example 5

17 **Arrange the following real numbers from smallest to largest:** $2.236..., \sqrt{13}, \sqrt{2}$

Questions 17,18 are similar to example 6

18 **Arrange the following real numbers from smallest to largest:** $-\sqrt{6}, -3\frac{1}{4}, -2.21$

Solve the Exercises

Classify the number whether it is rational number or irrational number or unreal number:

19 $\sqrt{25}$ 20 $\sqrt{17}$ 21 $\frac{0}{-6}$

22 $\frac{13}{0}$ 23 $\sqrt{\frac{49}{5}}$ 24 $\frac{20}{6}$

Estimate the following square roots into nearest tenth then represent it on a number line:

25 $\sqrt{8} \approx$ ..... 26 $-\sqrt{2} \approx$ .....

Compare between the following real numbers by using the symbols (< , > , =):

27 $\sqrt{13}$ ☐ $3\frac{1}{5}$ 28 $-\sqrt{12}$ ☐ $\sqrt{6.25}$

29 **Arrange the following real numbers from largest to smallest:** $-\sqrt{14}, -3\frac{1}{5}, -3.06$

Solve the problems

30 **Tahrir square:** Tahrir square is one of the main squares in middle of Baghdad city and it is designed in circular shape, the measure of its circumference is 176m and its diameter is 56m.
Show whether the product of division of circumference by the diameter shows rational number or irrational number?

31 **Dolphins:** Three dolphins dive under the water with the following depths:

$-10 \times \sqrt{\frac{36}{144}}$ m , $-10 \times \sqrt{\frac{1}{8}}$ m , $-10 \times \sqrt{\frac{9}{25}}$ m

Arrange the depths of the dolphins according to the level of water surface from the nearest to water surface to the furthest depth in decimal numbers.

32 **Painting:** A picture is painted on a piece of rectangular glass, its dimensions are 40cm , 60cm. Find the diameter of the painting. does it represent rational number or irrational number?

Think

33 **Challenge:** You have previously learned that the circumference of a circle divided by its diameter equals π and its irrational number. What is the diameter of a circle having the circumference 12cm? Explain your answer.

34 **Correct the mistake:** Shayma said that the number $\sqrt{\frac{16}{48}}$ is rational number while the number $\sqrt{\frac{16}{49}}$ is irrational number. Determine Shayma's mistake and correct it.

35 **Numerical sense:** Is the number $\sqrt{\frac{9}{4}}$ lies between the two numbers $\sqrt{2}$ and $\sqrt{3}$?

Write

The result of summation of the two real numbers after approximated them to the nearest tenth : $\sqrt{5} + \sqrt{7} \simeq$ .....

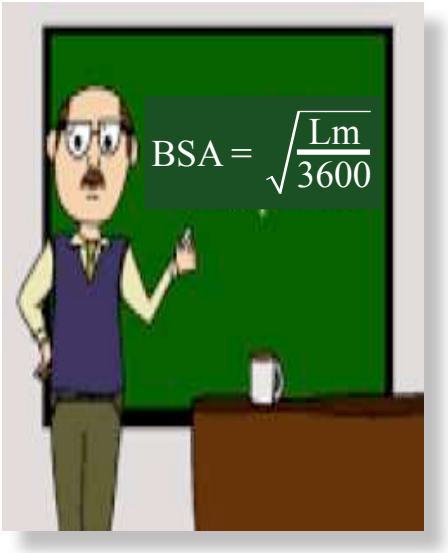
Lesson [2-2] Properties of Real Numbers

Idea of the lesson:
Identifying of properties of real numbers.

Vocabulary:
*Commutative pro.
*Associative pro.
*Distributive pro.
*Identity element.
*Additive inverse.
*Multiplicative inverse.

Learn

The value of root $\sqrt{\frac{Lm}{3600}}$ represents the surface area of human body, where L represents the height of the human in centimeter, and M represents the mass of human in kilograms. And the surface area for human body used to calculate the dose of medicine that should be taken. Find the surface area for a man his mass is 80kg and his height is 170cm.



[2-2-1] Commutative Property, Associative Property and Distributive Property

You have previously learned the properties of integer and rational numbers. In same way you will identify the properties of real numbers which is; commutative property, distribution property, associative property, identity element, additive inverse and multiplicative inverse.

i) $a+b = b+a$, $ab = ba$, $\forall\ a,b \in R$ Commutative pro.
ii) $a+(b+c) = (a+b)+c$, $a(bc) = (ab)c$, $\forall\ a,b,c \in R$ Associative pro.
iii) $a(b+c) = ab + ac$, $\forall\ a,b,c \in R$ Distributive pro.

The symbol $\forall$ means for all

Note: Addition operation does not distribute on multiplication operation.

Example (1) **Find the surface area for a human body that has weigh 80 kg and height 170cm.**

The formula that represents the human body surface area is:

$$BSA = \sqrt{\frac{Lm}{3600}}$$

Where (BSA) is the short form for (Body Surface Area).

Now substitute the variables by the given values:

$$BSA = \sqrt{\frac{170 \times 80}{3600}} \Rightarrow BSA = \sqrt{\frac{(100+70) \times 80}{3600}}$$ *Write 170 in a form of (100 + 70)*

$$\Rightarrow BSA = \sqrt{\frac{100 \times 80 + 70 \times 80}{3600}}$$ *Use the distributive property*

$$\Rightarrow BSA \approx 1.943\ cm^2$$

Example (2) **Write an example for each property of the following properties:**

i) $a+b=b+a$, $ab=ba$, $\forall\ a,b \in R \Rightarrow \sqrt{3}+\sqrt{2}=\sqrt{2}+\sqrt{3}$; $\sqrt{2}$,$\sqrt{3} \in R$

ii) $a+(b+c)=(a+b)+c$, $a(bc)=(ab)c$, $\forall\ a,b,c \in R$

$\Rightarrow \sqrt{3}+(\sqrt{2}+\sqrt{5})=(\sqrt{3}+\sqrt{2})+\sqrt{5}$, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5} \in R$

iii) $a(b+c)=ab+ac$, $\forall\ a,b,c \in R \Rightarrow \sqrt{7}(\sqrt{3}+\sqrt{5})=\sqrt{7}\sqrt{3}+\sqrt{7}\sqrt{5}$,

$\sqrt{7}$,$\sqrt{3}$,$\sqrt{5} \in R$

Activity: Ask the students to give an example to second part for each property.

[2-2-2] Identity Element , Additive Inverse and Multiplicative Inverse

iv) Number 0 is the identity element for addition operation $a+0=0+a=a$, $\forall\ a \in R$

Number 1 is the identity element for multiplication operation $1\times a=a\times 1=a$, $\forall\ a \in R$

v) Additive inverse $a+(-a)=(-a)+a=0$, $\forall\ a, -a \in R$

vi) Multiplicative inverse $a\times\frac{1}{a}=\frac{1}{a}\times a=1$, $\forall\ a \in R, a\neq 0$

Example (3) **Write an example for each property of the following properties:**

iv) $a+0=0+a=a$, $\forall\ a \in R \Rightarrow \sqrt{8}+0=0+\sqrt{8}=\sqrt{8}$, $\sqrt{8} \in R$

$1\times a=a\times 1=a$, $\forall\ a \in R \Rightarrow 1\times\sqrt{13}=\sqrt{13}\times 1=\sqrt{13}$, $1, \sqrt{13} \in R$

v) $a+(-a)=(-a)+a=0$, $\forall\ a, -a \in R \Rightarrow \sqrt{5}+(-\sqrt{5})=(-\sqrt{5})+\sqrt{5}=0$, $\sqrt{5}$,$-\sqrt{5} \in R$

vi) $a\times\frac{1}{a}=\frac{1}{a}\times a=1$, $\forall\ a \in R, a\neq 0 \Rightarrow \sqrt{17}\times\frac{1}{\sqrt{17}}=\frac{1}{\sqrt{17}}\times\sqrt{17}=1$, $\sqrt{17} \in R$

Example (4) **Find the additive inverse for the following real numbers:**

i) $6\sqrt{5}+2$, ii) $\sqrt{3}-\sqrt{11}$, iii) $-2\sqrt{8}+\frac{1}{2}$

i) $6\sqrt{5}+2+(-6\sqrt{5}-2)=(6\sqrt{5}-6\sqrt{5})+(2-2)=0$, $-6\sqrt{5}-2$ is the additive inverse of $6\sqrt{5}+2$

ii) $\sqrt{3}-\sqrt{11}+(-\sqrt{3}+\sqrt{11})=(\sqrt{3}-\sqrt{3})+(-\sqrt{11}+\sqrt{11})=0$

$-\sqrt{3}+\sqrt{11}$ is the additive inverse of $\sqrt{3}-\sqrt{11}$

iii) $-2\sqrt{8}+\frac{1}{2}+(2\sqrt{8}-\frac{1}{2})=(-2\sqrt{8}+2\sqrt{8})+(\frac{1}{2}-\frac{1}{2})=0$

$2\sqrt{8}-\frac{1}{2}$ is the additive inverse of $-2\sqrt{8}+\frac{1}{2}$

Find the multiplicative inverse for the following real numbers: iv) $\sqrt{\frac{1}{7}}$, v) $-2\sqrt{3}+3$, vi) $-6\frac{2}{5}$

vi) $\sqrt{\frac{1}{7}}=\frac{1}{\sqrt{7}}\Rightarrow\frac{1}{\sqrt{7}}\times\sqrt{7}=1$, $\sqrt{7}$ is the multiplicative inverse of $\sqrt{\frac{1}{7}}$

v) $(-2\sqrt{3}+3)\times\frac{1}{-2\sqrt{3}+3}=1$, $\frac{1}{-2\sqrt{3}+3}$ is the multiplicative inverse of $-2\sqrt{3}+3$

iv) $\frac{-32}{5}\times\frac{5}{-32}=1$, $\frac{5}{-32}$ is the multiplicative inverse of $\frac{-32}{5}$

Make sure of your understanding

Write an example for each property of the following properties:

1. $a+b = b+a$, $ab = ba$, $\forall\ a,b \in R$
2. $a+(b+c) = (a+b)+c$, $a(bc) = (ab)\,c$, $\forall\ a,b,c \in R$
3. $a(b+c) = ab+ac$, $\forall\ a,b,c \in R$

Questions 1-3 are similar to example 2

Find the additive inverse for the following real numbers:

4. $4\sqrt{2}\ -1$
5. $\sqrt{5}\ +\sqrt{7}$
6. $-9\sqrt{11}+\frac{1}{3}$
7. $-5\sqrt{3}\ +8$
8. $\frac{1}{\sqrt{11}}\ -6$
9. $-4\frac{2}{3}$

Questions 4-9 are similar to example 4

Find the multiplicative inverse for the following real numbers:

10. $\sqrt{\frac{1}{5}}$
11. $3\sqrt{7}\ -7$
12. $-11\frac{2}{3}$
13. $-6\sqrt{\frac{1}{3}}$
14. $\sqrt{2}\ -\frac{1}{3}$
15. $8\frac{1}{2}-5\frac{5}{2}$

Questions 10-15 are similar to example 4

Solve the Exercises

Write an example for each property of the following properties:

16. $a+0 = 0+a = a$, $\forall\ a \in R$
17. $a+(-a) = (-a)+a = 0$, $\forall\ a, -a \in R$

Find the additive inverse for the following real numbers:

18. $-6\sqrt{13}\ -5$
19. $-\sqrt{1}\ -\sqrt{3}$

Find the multiplicative inverse for the following real numbers:

20. $-\sqrt{\frac{1}{8}}$
21. $-5\sqrt{2}\ -5$

Estimate the multiplicative inverse for the following square roots by approximate to the nearest tenth:

22. $-\sqrt{\frac{1}{7}}$
23. $\sqrt{\frac{1}{7.3}}$

Solve the problems

24 **TV screen:** Marwa bought a television. The length of the screen is 48cm and the width is 36cm, find the additive inverse for the diagonal of the television screen.

25 **Diving:** : Three divers dive under the water by the following depths

$-18 \times \sqrt{\frac{36}{81}}$ m , $-24 \times \sqrt{\frac{1}{16}}$ m , $-30 \times \sqrt{\frac{9}{25}}$ m .

Arrange the multiplicative inverse for the depths of the divers according to the surface of water from the furthest depth to the nearest depth to surface of water.

26 **The Shark:** The multiplicative inverse of length for two sharks is $\sqrt{\frac{2.25}{81}}$ m and $\sqrt{\frac{6.25}{36}}$ m.

Compare between their real lengths.

Think

27 **Challenge:** Arrange the additive inverse for the estimated value for the following square roots by approximate it to the nearest tenth from smallest to the largest: $\sqrt{7}$, $\sqrt{3}$, $\sqrt{5}$

28 **Correct the mistake:** Yaseen said that $(-5\sqrt{7} + 1)$ is the additive inverse for number $\frac{1}{-5\sqrt{7} + 1}$ because $\frac{1}{-5\sqrt{7} + 1} \times (-5\sqrt{7} + 1) = 1$, Determine Yaseen's mistake and correct it.

29 **Numerical sense:** Is the product of any real number by its inverted is equal 1?

Write

The property that express the following example:

$\sqrt{3} + (-\sqrt{3}) = (-\sqrt{3}) + \sqrt{3} = 0$, $\sqrt{3}$, $-\sqrt{3} \in R$

Lesson [2-3] Simplifying the Numerical Sentences which contain Square Roots

Idea of the lesson:
*Simplifying the Numerical Sentences which contain Square Roots.

Vocabulary:
* Displacer dominator.

Learn

Spider: The female of spider is building the net of a spider house by using 400 thousand strings and the length of each string is 20cm . Bassam painted a spider net on a rectangle shaped glass its length is $5\sqrt{18}$ cm and the width is $3\sqrt{2}$ cm.
What is the perimeter of the art painting?

[2-3-1] Simplifying the Numerical Sentences by using the Properties (Commutative , Accusative and Distributive)

You have previously learned simplifying numerical sentences that contains rational numbers by using numbers properties, by the same way you are going to learn the simplifying of the numerical sentences that contain square roots by using the properties of (Commutative, Accusative and Distributive).

i) $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, $\forall$ a,b ≥ 0

ii) $\sqrt{a} \cdot \sqrt{a} = a$, $\forall$ a ≥ 0

iii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, $\forall$ a ≥ 0 , b > 0

Example (1) **Find the perimeter of the art painting in shape of rectangle whose dimentions are $3\sqrt{2}$ cm , $5\sqrt{18}$ cm, that Bassam painted it.**

The variables represent : P for perimeter, L for length , W for width.

$P = 2 (L + W)$

$= 2 (5\sqrt{18} + 3\sqrt{2})$ *Use the property of distribution*

$= 2 \times 5 \sqrt{18} + 2 \times 3\sqrt{2}$

$= 10 \times \sqrt{9}\sqrt{2} + 6\sqrt{2}$ *Replace $\sqrt{18}$ with $\sqrt{9}\sqrt{2}$*

$= 10 \times 3\sqrt{2} + 6\sqrt{2}$ *Replace $\sqrt{9}$ with 3*

$= 30\sqrt{2} + 6\sqrt{2}$ *Simplify the expression by adding the similar terms*

$= 36\sqrt{2}$

Therefor the perimeter of the art painting is $36\sqrt{2}$ cm

Example (2) **Simplify the following numerical sentences by using the properties of (Commutative, Accusative and Distributive)**

i) $5\sqrt{12} - 7\sqrt{32} = 5\sqrt{3 \times 4} - 7\sqrt{2 \times 16}$

$= 5\sqrt{3}\sqrt{4} - 7\sqrt{2}\sqrt{16}$

$= 5\sqrt{4}\sqrt{3} - 7\sqrt{16}\sqrt{2}$

$= (5 \times 2)\sqrt{3} - (7 \times 4)\sqrt{2}$

$= 10\sqrt{3} - 28\sqrt{2}$

ii) $\sqrt{5}(\sqrt{10} + \sqrt{3}) = \sqrt{5}\sqrt{10} + \sqrt{5}\sqrt{3}$

$= \sqrt{50} + \sqrt{15}$

$= \sqrt{25 \times 2} + \sqrt{15}$

$= \sqrt{25}\sqrt{2} + \sqrt{15}$

$= 5\sqrt{2} + \sqrt{15}$

[2-3-2] Simplifying the Numerical Sentences by using Properties (Identity Element, Additive Invers and Multiplicative Inverse)

You will learn the simplifying of the numerical sentences that contain square roots by using the properties of (Identity Element ,Additive Inverse andMultiplicative Inverse), and in case of fraction that its denominator square root you will use the operation of displacer denominator (this means changing the fractions denominator to a number without root by multiplying the fraction by the identity element of multiplication (the number 1) , and then replace it by an appropriate fraction).

Example (3) **Simplify the Numerical Sentences by using properties(Identity Element, Additive Inverse and Multiplicative Inverse)**

$$\frac{3+\sqrt{7}}{\sqrt{7}} = \frac{3+\sqrt{7}}{\sqrt{7}} \times 1$$

Multiply the fraction with number 1

$$= \frac{3+\sqrt{7}}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$$

Replace the number 1 with $\frac{\sqrt{7}}{\sqrt{7}}$ (multiplying the numerator and the denominator)

$$= \frac{\sqrt{7}\,(3+\sqrt{7})}{\sqrt{7}\,\sqrt{7}}$$

Use the distributive property

$$= \frac{3\sqrt{7}+\sqrt{7}\,\sqrt{7}}{7}$$

Simplify the expression, where $\sqrt{7}\,.\,\sqrt{7} = 7$

$$= \frac{3\sqrt{7}+7}{7}$$

Example (4) **Simplify the following numerical sentences by using the properties of real numbers:**

i) $\sqrt{8}\,(1-\sqrt{2}\,) = \sqrt{8} - \sqrt{8}\,\sqrt{2}$
$= \sqrt{4}\,\sqrt{2} - \sqrt{16}$
$= 2\sqrt{2} - 4$

ii) $\sqrt{3}\,(\sqrt{3}+\sqrt{27}\,) = \sqrt{3}\,\sqrt{3}+\sqrt{3}\,\sqrt{27}$
$= 3 + \sqrt{3}\,\sqrt{3}\,\sqrt{9}$
$= 3 + 9 = 12$

iii) $6\sqrt{125} + 2\sqrt{5} - 4\sqrt{5}$
$= 6\sqrt{25}\,\sqrt{5} - 2\sqrt{5}$
$= 30\sqrt{5} - 2\sqrt{5} \quad = 28\sqrt{5}$

iv) $\frac{1}{2}\sqrt{7} + \frac{1}{3}\sqrt{7} - \frac{1}{6}\sqrt{7}$
$= \sqrt{7}\,(\frac{1}{2}+\frac{1}{3}-\frac{1}{6})$
$= \sqrt{7}\,(\frac{3+2-1}{6}) = \frac{2}{3}\sqrt{7}$

v) $\frac{\sqrt{5}-8}{2\sqrt{5}} = \frac{\sqrt{5}-8}{2\sqrt{5}} \times 1$
$= \frac{\sqrt{5}-8}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$
$= \frac{\sqrt{5}\,\sqrt{5}-8\sqrt{5}}{2\sqrt{5}\,\sqrt{5}}$
$= \frac{5-8\sqrt{5}}{10}$

vi) $\frac{9\sqrt{44}}{4\sqrt{3}} \div \frac{8\sqrt{11}}{\sqrt{3}}$
$= \frac{9\sqrt{44}}{4\sqrt{3}} \times \frac{\sqrt{3}}{8\sqrt{11}}$
$= \frac{18\sqrt{11}}{4} \times \frac{1}{8\sqrt{11}}$
$= \frac{9}{16}$

Make sure of your understanding

Simplify the following numerical sentences by using the properties of (Commutative, Accusative and Distributive):

1. $4\sqrt{3} - 9\sqrt{3}$
2. $\sqrt{7}\,(\sqrt{7} + \sqrt{2})$
3. $3\sqrt{12} - 5\sqrt{8}$
4. $\sqrt{6}\,(2\sqrt{3} + 6\sqrt{2})$
5. $\sqrt{27}\,(5 - \sqrt{3})$
6. $\sqrt{2}\,(\sqrt{6} - 3\sqrt{2})$

Questions 1-6 are similar to examples 1,2

Simplify the numerical sentences by using (Identity Element , Additive Inverse and Multiplicative Inverse):

7. $\dfrac{\sqrt{7} - 2}{\sqrt{3}}$
8. $\dfrac{8 + \sqrt{7}}{4\sqrt{5}}$
9. $\dfrac{\sqrt{8} - 1}{2\sqrt{8}}$
10. $\dfrac{\sqrt{12} - 2}{2\sqrt{3}}$
11. $\dfrac{\sqrt{20} + \sqrt{5}}{\sqrt{5}}$
12. $\dfrac{12 - 6\sqrt{3}}{\sqrt{27}}$

Questions 7-12 are similar to example 3

Simplify the following numerical sentences by using the properties of real numbers:

13. $\sqrt{3}\,(5 - \sqrt{3})$
14. $\sqrt{7}\,(\sqrt{7} + \sqrt{28})$
15. $4\sqrt{32} + 3\sqrt{2} - 5\sqrt{2}$
16. $\frac{1}{4}\sqrt{11} + \frac{1}{5}\sqrt{11} - \frac{1}{2}\sqrt{11}$
17. $\dfrac{\sqrt{24} - 5}{2\sqrt{3}}$
18. $\dfrac{7\sqrt{3}}{\sqrt{2}} \div \dfrac{14\sqrt{3}}{\sqrt{6}}$

Questions 13-18 are similar to example 4

Solve the Exercises

Simplify the following numerical sentences by using the properties of (Commutative, Accusative and Distributive):

19. $\sqrt{2}\,(1 + 7\sqrt{2})$
20. $\sqrt{5}\,(\sqrt{20} - \sqrt{5})$
21. $\sqrt{10}(3\sqrt{5} - 7\sqrt{2})$
22. $\sqrt{54}(2 - \sqrt{2})$

Simplify the numerical sentences by using (Identity Element , Additive Inverse and Multiplicative Inverse):

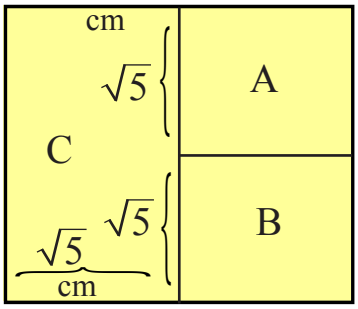
23. $\dfrac{9 - 2\sqrt{5}}{3\sqrt{5}}$
24. $\dfrac{\sqrt{6} - 5}{\sqrt{3}\,\sqrt{2}}$

Simplify the following numerical sentences by using the properties of real numbers:

25. $\sqrt{2}\,(7 + \sqrt{3}) - 4\sqrt{6}$
26. $3\sqrt{8} + 3(\sqrt{6} - \sqrt{72})$
27. $\dfrac{8\sqrt{6}}{\sqrt{8}} \div \dfrac{12\sqrt{3}}{\sqrt{98}}$

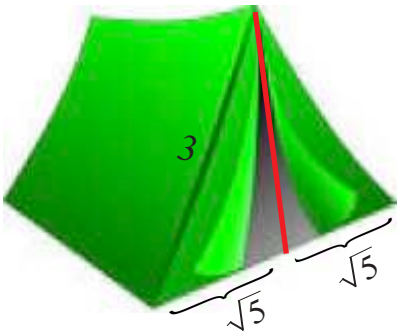
Solve the problems

28 **Geometry:** The near by figure consists of three parts: Part A represents a square and it is identical to part B, the part C represents a rectangle. Find the total area for the figure.



29 **Fire engine:** A fire engine stops about 20m far from a residental building, it lifted up the fire ladder at an angle with its all length 35m and it reached one of the building floors. What is the height of the floor that the ladder reached to it on considered the fire engine with the building represent a right-angled triangle?
Find the result by approximate to nearest tenth.

30 **A tent:** The shape of the front part for a tent is isosceles triangle, and every tent needs two poles one for the front part and the second one to the backward part to fix the tent, if the length of the front base for the tent is $2\sqrt{5}$ m, and the length of the oblique edge is 3m. what is the length of the tent poles that we need it to compose 4 tents?



Think

Challenge: Simplify the following numerical sentences by using the properties of real numbers:

31 $\dfrac{1-7\sqrt{2}}{\sqrt{2}} - \dfrac{1-2\sqrt{3}}{\sqrt{3}}$

32 $\dfrac{12-8\sqrt{3}}{6\sqrt{5}} \div \dfrac{4\sqrt{2}}{\sqrt{20}}$

33 **Correct the mistake:** Muneer said that the result of simplifying for the numerical sentence $\frac{1}{2}\sqrt{8} + \frac{1}{3}\sqrt{18} - \frac{2}{5}\sqrt{50}$ is $2\sqrt{2}$, Determine Muneer's mistake and correct it.

34 **Numerical sense:** What is the length probability for each side of the two right sides in a right-angled triangle with the length of hypotenuse is $\sqrt{5}$ cm?

Write

A numerical sentence which contains real roots then simplifies it by using distributive property.

Lesson [2-4] Application for Pythagorean Theorem

Idea of the lesson:
*Finding the two square roots for the number (negative and positive).
*Converse of Pythagoraean Theorem.
*Representing the irrational numbers on a number line.

Vocabulary:
* Principal Square Root.

Learn

An ancient inscription represents the daily life of Sumerian people (2400 – 2850 B.C.).
We see that the mixing of Sculpture with beginnings of mosaic art and collection of colorful stones. The painting board is squared shape and its area is $9m^2$, find the length of side for the painting board.

[2-4-1] Finding the Positive and Negative Squre Roots for Number

You have previously learned how to find the positive square root for a number, and the square root for a number is the opposite operation to squared that root: $7^2 = 49$ which mean $7 = \sqrt{49}$, and now you will learn that for every positive number there are two square roots one of them is positive and the other one is negative: the positive square root for the number 64 is 8 because $8\times8=64$ and the negative square root is -8 because $-8\times-8 = 64$ and it could be written in form of : $\pm\sqrt{64} = \pm 8$ which means $+\sqrt{64} = +8$, $-\sqrt{64} = -8$, when you use the calculator you can see just the positive square root, and it is called the principal square root for the number.Therefore $\sqrt{a} \geq o \ \forall \ a \geq o$.

Example (1) **Find the length of the side for a squared painting board which has $9m^2$ area.**

$L = \sqrt{9}$ The variable L represents the length of side of the squared painting board.

$= 3$ The length of a side for the painting board is 3m

We see that $3 \times 3 = 9 \Rightarrow +\sqrt{9} = +3$

$-3 \times -3 = 9 \Rightarrow -\sqrt{9} = -3$

But the negative square root is neglected because there is no real length in negative.

Example (2) **Find both negative and positive square roots for the following numbers:**

i) 36 $\Rightarrow$ $\sqrt{36} = 6$, $6\times6 = 36$; $-\sqrt{36} = -6$, $-6\times(-6) = 36$

ii) 81 $\Rightarrow$ $\sqrt{81} = 9$, $9\times9 = 81$; $-\sqrt{81} = -9$, $-9\times(-9) = 81$

iii) $\frac{9}{16}$ $\Rightarrow$ $\sqrt{\frac{9}{16}} = \frac{3}{4}$, $\frac{3}{4}\times\frac{3}{4} = \frac{9}{16}$; $-\sqrt{\frac{9}{16}} = -\frac{3}{4}$, $-\frac{3}{4}\times-(\frac{3}{4}) = \frac{9}{16}$

iv) 6.25 $\Rightarrow$ $\sqrt{6.25} = 2.5$, $2.5\times2.5 = 6.25$; $-\sqrt{6.25} = -2.5$, $-2.5\times(-2.5) = 6.25$

[2-4-2] The Converse of Pythagorean Theorem

You have previously learned about Pythagorean theorem which describes the relationship between the longitudinal side and hypotenuse in the right-angled triangle(the square of the hypotenuse length is equal the sum of square length sides). Now you will learn the converse of Pythagorean theorem (in a triangle if the square length of one of its sides equals to the sum of the other two sides square length, so that triangle is a right-angled).

Example (3) **If the measurements of the three sides of the given triangle are: 5cm,4cm,3cm. Determine if the triangle is a right-angled or not?**

Write the square of each triangle sides $3^2 = 9$, $4^2 = 16$, $5^2 = 25$.

Since $16 + 9 = 25$, so that the triangle satisfies the converse of Pythagorean theorem

This means that the triangle is right-angled.

[2-4-3] Representing The Irrational Numbers on a Number Line

You have previously learned how to represent the rational numbers on a number line, and also you learnt in the first lesson from this chapter how to represent the square roots for the numbers on a number line after writing its value approximating to the nearest tenth. Now you will learn how to represent the irrational number on a number line properly and without approximating.

Example (4) **Represent the number $\sqrt{41}$ on a number line properly.**

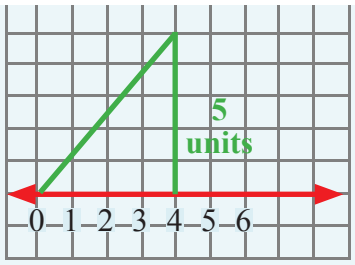
1st step: Find two squared numbers that its sum is 41.

The length of the hypotenuse for a right-angled triangle with the length of sides $5,4$ is $\sqrt{41}$

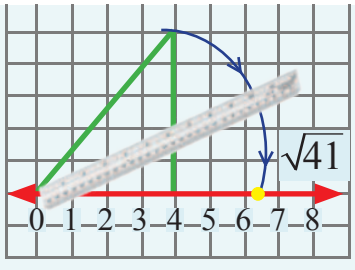
$41 = 16 + 25$

$41 = 4^2 + 5^2$

2nd step: Draw on a squared line paper (graph) a right-angled triangle with the length of sides $4,5$ unit.



3rd step: Open the compass as the length of the hypotenuse then put the vertex of the compass by the number zero, and draw an arc cuts a number line in the point that represents the number $\sqrt{41}$.



Make sure of your understanding

Find the both positive and negative square roots for the following numbers:

1. 16
2. 25
3. 49
4. 64
5. $\frac{9}{25}$
6. $\frac{36}{81}$
7. 1.21
8. 2.25

Questions 1-8 are similar to examples 1,2

Determine if each triangle with the given side lengths is a right-angled triangle. check your answer:

9. 40 cm, 9 cm, 41 cm
10. 4 cm, 7 cm, 5 cm
11. 6 cm, 8 cm, 10 cm
12. 17 cm, 13 cm, 12 cm

Questions 9-12 are similar to example 3

Find the length of the right side by approximating the result to the nearest tenth, If you know that the length of the hypotenuse and length of the right side for each triangle as given bellow.

13. 12 cm, 6 cm, ...
14. 10 cm, 20 cm, ...

Represent the following numbers on a number line properly:

15. $\sqrt{5}$
16. $\sqrt{13}$
17. $\sqrt{20}$
18. $\sqrt{29}$
19. $\sqrt{34}$
20. $\sqrt{52}$
21. $\sqrt{45}$
22. $\sqrt{65}$

Questions 15-22 are similar to example 4

Solve the Exercises

Find the both positive and negative square roots for the following numbers:

23. 4
24. 36
25. 81
26. $\frac{9}{4}$
27. $\frac{25}{64}$
28. 10.24

Determine if each triangle with the given side lengths is a right-angled triangle. check your answer:

29. 2 cm, 3 cm, $\sqrt{13}$ cm
30. 20 cm, 15 cm, 625 cm

Find the length of the right side by approximating the result to the nearest tenth, If you know that the length of the hypotenuse and length of the right side for each triangle as given bellow.

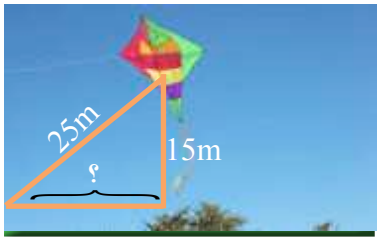
31. 9cm, 6 cm, ...
32. 10 cm, 5 cm, ...

Represent the following numbers on a number line properly:

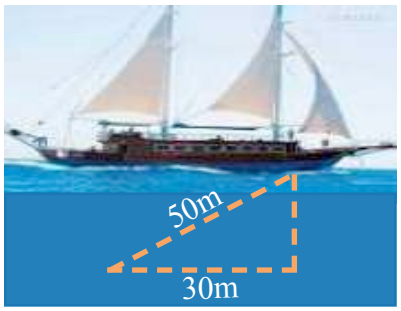
33. $\sqrt{2}$
34. $\sqrt{10}$
35. $\sqrt{29}$
36. $\sqrt{41}$

Solve the problems

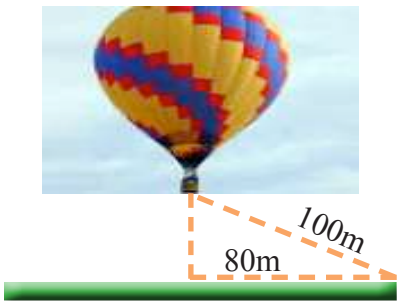
37 **The kite:** Kite is one of the famous game among many of children. In the given figure if the perpendicular height from the surface of the earth is 15m and the length of the string that is connected to the kite from a side and in the other side it connected to child's hand is 25m. What is the type of the triangle that's made by the height of the kite and the string and the horizontal distance. Find the length of the horizontal distance.



38 **Sailing Boat:** The given figure is for a sailing boat sails on the surface of the water. Find the depth of the water by approximating to the nearest tenth, if you know that the length of the hypotenuse and the right side as given in the figure.



39 **Balloon:** Is a big balloon which connected to its end with a basket to carry people or goods and it has different uses as a transportation. the near by figure represents an air balloon. Find its height from the surface of the earth.



Think

Challenge: Find the both positive and negative square roots for the following numbers:

40 $\frac{\sqrt{32}}{\sqrt{2}}$ 41 $\frac{\sqrt{243}}{\sqrt{3}}$ 42 $\frac{\sqrt{625}}{\sqrt{16}}$

43 **Correct the mistake:** Sundus said that the triangle that its sides are 1.5cm, 1.1cm ,1.8cm represents a right-angled triangle. Determine Sundus's mistake and correct it.

Numerical sense: Is it possible to draw a right-angled triangle with the given side's length, as following? explain your answer:

44 3 cm , 5 cm , $\sqrt{34}$ cm 45 1.5 cm, 2.5 cm, 3.5 cm

Write

An equation that you can find the unknown length side for a right-angled triangle, the length of hypotenuse is 10cm, and right side is 6cm.

Lesson [2-5] Coordinate Plane

Idea of the lesson:
*Representing the values table in coordinate plane.
*Finding distance between two points.

Vocabulary:
* Ordered pair
* Coordinate plane
* Origin
* X-axis
* Y-axis
* Values table
* The quadrants

Learn

Tiger: Tiger is a huge wild animal from the mammals meat eater. From panther family. The female tiger can give birth form one pup to eight pups. The mother tiger takes care of them for 6 weeks then they will depend on themselves. The table below shows the amount of the milk that the pups need over a one day. Represent the table in the coordinate plane.

Number of pups	2	4	6	8
Amount in (liter)	1	2	3	4

[2-5-1] Representing Values Table in the Coordinate Plane

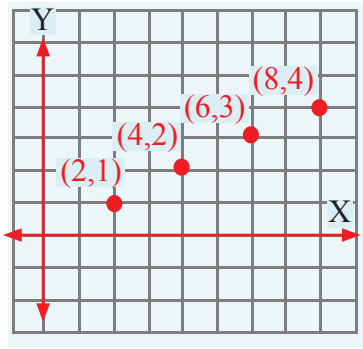
You have learned previously about the coordinate plane that consists of two perpendicular lines in a point which is called origin, the horizontal line is known as x-axis and the vertical line known as y-axis and the plane are divided into four quadrants. Also you learned how to represent the ordered pairs in coordinate plane. And now you are going to learn how to represent the values table in coordinate plane includes rational numbers and that by writing the ordered pair firstly from the table then representing it on the coordinate plane.

Example (1)

Represent the following values table in the coordinate plane.

Number of pups	2	4	6	8
Amount in (liter)	1	2	3	4

Write the ordered pairs: (2,1) , (4,2) ,(6,3) , (8,4)
Represent each pair with a point in the coordinate plane.
All the ordered paires lie in the first quadrant.

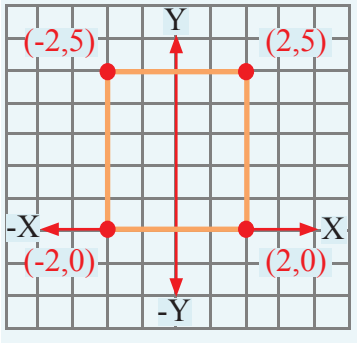


Example (2)

Represent the following values table in the coordinate plane, and determine the geometrical shape that it represented.

X	-2	2	-2	2
Y	5	5	0	0

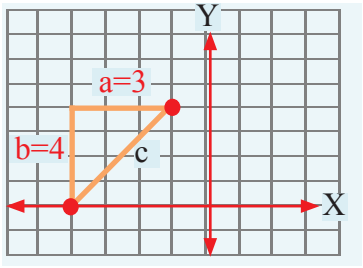
Write the ordered pairs: (-2,5) , (2,5) ,(-2,0) , (2,0)
Represent each pair with a point in the coordinate plane,
connect between the points that represented by the values table.
The figure is a rectangle.



[2-5-2] Finding the Distance between two Points in the Coordinate Plane

You have previously learned from the first part of this lesson how to write the ordered pair from the vlues table and then representing them in the coordinate plane, and now you will learn how to find the distance between two points in the coordinate plane and also finding approximate dimension between the cities by using Pythagoras theorem.

Example (3) **Represent the two ordered pairs (-4,0) , (-1,4) in the coordinate plane then find the distance between them.**

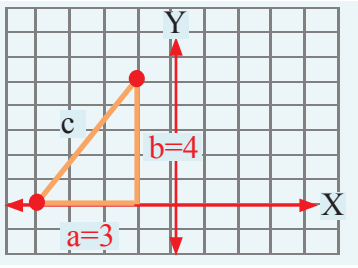


First: Represent the two points in the coordinate plane.
Second: Draw a right-angled triangle as shown in near by the figure.
Third: Find the length of the two perpendicular sides.

b = 4 units, and a = 3 units

Fourth: Use Pythagorean theorem to find the length of the hypotenuse C

$$c^2 = a^2+b^2 \Rightarrow c = \sqrt{a^2+b^2}$$
$$\Rightarrow c = \sqrt{9+16}$$
$$\Rightarrow c = \sqrt{25} \Rightarrow c = 5$$



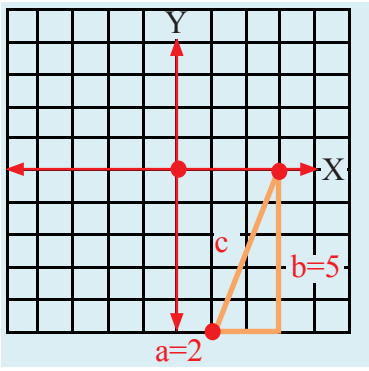
Therefore the distance between the two points is 5 units.

Note: To get a right-angled triangle draw as following:

1) From intersection of the two drawn lines, from two points which are parallel to each of axis.

2) From intersection at the two perpendicular lines which draw from two points on each axis.

Example (4) **Represent each pair from the ordered pairs {(3,0),(1,-5)},{(0,0),(-3,3)} in the coordinate plane then find the distance between them approximate to the nearest tenth.**



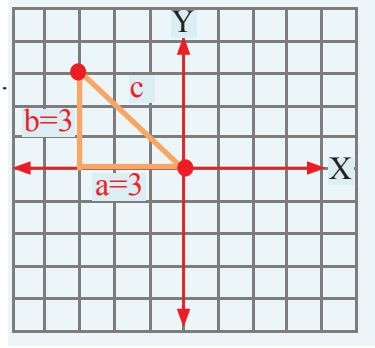
First: Represent both points in the coordinate plane.
Second: Draw right-angled triangles as shown in the near by figure.
Third: Find the length of each side from the perpendicular sides for each triangle.

The triangle in the fourth quadrant: a = 2, b = 5 units
The triangle in the second quadrant: a = 3, b = 3 units

Fourth: Use the Pythagorean theorem to find the length of hypotenuse c
The triangle in the fourth quadrant:

$$c^2 = a^2+b^2 \Rightarrow c = \sqrt{a^2+b^2}$$
$$\Rightarrow c = \sqrt{4+25}$$
$$\Rightarrow c = \sqrt{29} \Rightarrow c \approx 5.4$$

Therefore the distance between the two points is approximately 5.4 units.



The triangle in the second quadrant:

$$c^2 = a^2+b^2 \Rightarrow c = \sqrt{a^2+b^2}$$
$$\Rightarrow c = \sqrt{9+9}$$
$$\Rightarrow c = \sqrt{18} \Rightarrow c \approx 4.2$$

Therefore the distance between the two points is approximatly 4.2 units.

Make sure of your understanding

Represent each of the following tables in the coordinate plane:

1

X	0	-4	-3.5	5
Y	-1.5	1.5	-4	3.5

2

X	0	-2.5	0	2.5
Y	0	1.5	-3	0

3

X	-4	7	-3.5	0
Y	0	2.5	-4.5	-2.5

4

X	2	3	4	5
Y	0	0	0	0

Questions 1-4 are similar to examples 1,2

Represent each pair from the ordered pairs in the coordinate plane, then find the distance between them with approximate to the nearest tenth if it's not represent integer number:

5 $\{(1 , 0) ، (4 , 4)\}$

6 $\{(0 , 0) ، (-5 , 4)\}$

7 $\{(-4 , -1) ، (-1 , -6)\}$

8 $\{(-3 , -6) ، (0 , 0)\}$

9 $\{(7 , 2) ، (3 , 5)\}$

10 $\{(-2 , 0) ، (0 , 3)\}$

Questions 5-10 are similar to examples 3,4

Solve the Exercises

Represent each table from the following tables in the coordinate plane and determine the geometrical shape that the values table represents it:

11

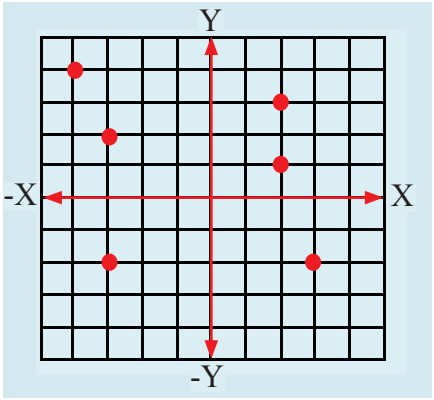
X	2	-2	-1	-5
Y	3.5	3.5	-4.5	-4.5

12

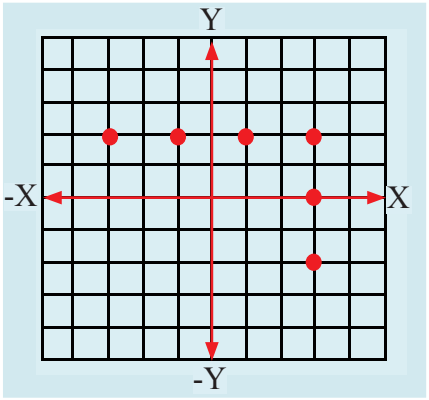
X	4	-4	0	0
Y	0	0	2	-5

Write the ordered pairs from the figure, then arrange them in the values table:

13



14



Represent each pair from the two ordered pairs in the coordinate plane then find the distance between them with approximate to the nearest tenth if it doesn't represent an integer:

15 $\{(6 , 0) ، (0 , 3)\}$

16 $\{(-5 , 0) ، (0 , 5)\}$

17 $\{(2 , -4) ، (4 , 0)\}$

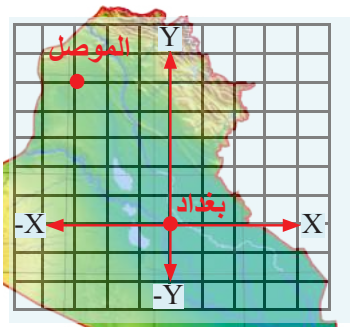
18 $\{(5 , 1) ، (2 , 5)\}$

19 $\{(0 , 0) ، (-4 , -4)\}$

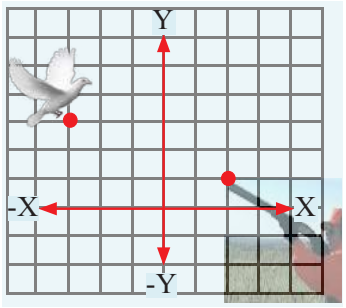
20 $\{(-5 , -1) ، (-1 , -6)\}$

Solve the problems

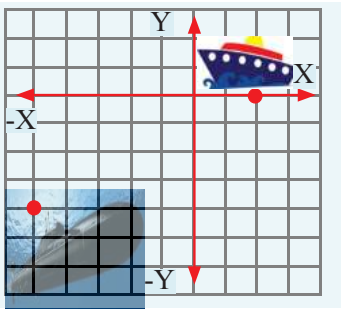
21 **Map:** Each unit in the map represents 77km from the real distance. Mosul city located at point (-3,5) and Baghdad city located at the origin. What is the approximate distance between Baghdad and Mosul?



22 **Hunting:** Each unit in the figure represents 50m of the real distance. If a hunter stops at point (2,1) and fixed his gun to a bird at point (-3,3), what is the approximate distance between the bird and the hunter?



23 **Submarine:** Each unit in the figure represents 5km of the real distance. A submarine under the water stops at point (-5,-4) wants to attack a target at point (2, 0). What is the approximate distance between the target and the submarine?



Think

Challenge: Represent each table from the following tables in the coordinate plane and determine the geometrical shape that the values table represents it:

24

X	3	-3	-3	3
Y	3	3	-3	-3

25

X	1	-5	1	-2
Y	0	0	3	3

26 **Correct the mistake:** Khalid said that the (Y) coordinate plane for a point located in X-axis represent the distance between the point and origin. Determine Khalid's mistake and correct it.

27 **Numerical sense:** Does the following values table represent a hexagonal figure when representing it in the coordinate plane? and why?

X	4	0	-4	-4
Y	4	4	-4	0

Write

Name of the geometrical shape that the following values table represented it in the coordinate plane.

X	1	1	6	6
Y	3	1	3	1

Chapter Test

Classify the following number whether it is rational or irrational or unreal numbers:

1 $-\sqrt{49}$ 2 $\sqrt{13}$ 3 $\frac{0}{-6}$ 4 $\sqrt{\frac{9}{25}}$ 5 $\sqrt{-16}$

Estimate the following square roots in to nearest tenth then represent it on a number line:

6 $\sqrt{15} \approx$..... 7 $-\sqrt{32} \approx$..... 8 $\sqrt{\frac{16}{21}} \approx$..... 9 $\sqrt{7.3} \approx$.....

Compare between the following real numbers by using the symbols (< , > , =) :

10 $\sqrt{17}$ ☐ $4\frac{1}{5}$ 11 $-\sqrt{9}$ ☐ $-\sqrt{6.25}$ 12 $\frac{0}{\sqrt{7}}$ ☐ $\frac{0}{\sqrt{5}}$

13 **Arrange the following real numbers from smallest to largest:** $2.236..., \sqrt{2.25}, \sqrt{2}$

14 **Arrange the following real numbers from largest to smallest :** $-\sqrt{11}, -3\frac{1}{4}, -3.33$

Write an example for each property of the following properties:

15 $1 \times a = a \times 1 = a , \forall a \in R$ 16 $a \times \frac{1}{a} = \frac{1}{a} \times a = 1 , \forall a \in R , a \neq 0$

Find the additive inverse for the following real numbers:

17 $5\sqrt{11}$ -7 18 $-\sqrt{1} - \sqrt{2}$ 19 $\sqrt{12} - \frac{1}{16}$ 20 $\frac{3}{\sqrt{5}} - \frac{4}{\sqrt{5}}$

Find the multiplicative inverse for the following real numbers:

21 $\sqrt{\frac{1}{12}}$ 22 $-6\sqrt{3}$ -7 23 $\sqrt{5} - \frac{1}{5}$ 24 $-3\frac{1}{2} - 1\frac{4}{3}$

Estimate the multiplicative inverse for the following square roots by apprpximate to the nearest tenth:

25 $\sqrt{\frac{1}{7}}$ 26 $-\sqrt{\frac{1}{8}}$ 27 $\sqrt{\frac{11}{25}}$ 28 $\sqrt{\frac{1}{5.6}}$

Simplify the following numerical sentences by using the properties of real numbers:

29 $\frac{6 - 2\sqrt{3}}{3\sqrt{5}}$ 30 $\frac{\sqrt{8} - 7\sqrt{2}}{\sqrt{2}}$ 31 $\frac{6\sqrt{8}}{\sqrt{6}} \div \frac{12\sqrt{3}}{\sqrt{27}}$ 32 $\frac{3\sqrt{8}}{\sqrt{45}} \times \frac{\sqrt{125}}{\sqrt{32}}$

33 $\sqrt{3}(9+\sqrt{3}) - 2\sqrt{27}$ 34 $\sqrt{7}(\sqrt{7} - \sqrt{14}) - 9\sqrt{2}$ 35 $\frac{1}{\sqrt{5}}\sqrt{11} + \frac{1}{5}(\sqrt{50} - \sqrt{55})$

Find both positive and negative square root for the following numbers:

36 9 37 225 38 $\frac{25}{36}$ 39 1.21 40 10.24

Determine if the triangle with given sides is a right-angled triangle. Explain your answer:

41 3cm , 5cm , 6cm 42 7cm , 5cm , $\sqrt{74}$ cm 43 2cm , 1.5cm , 2.5cm

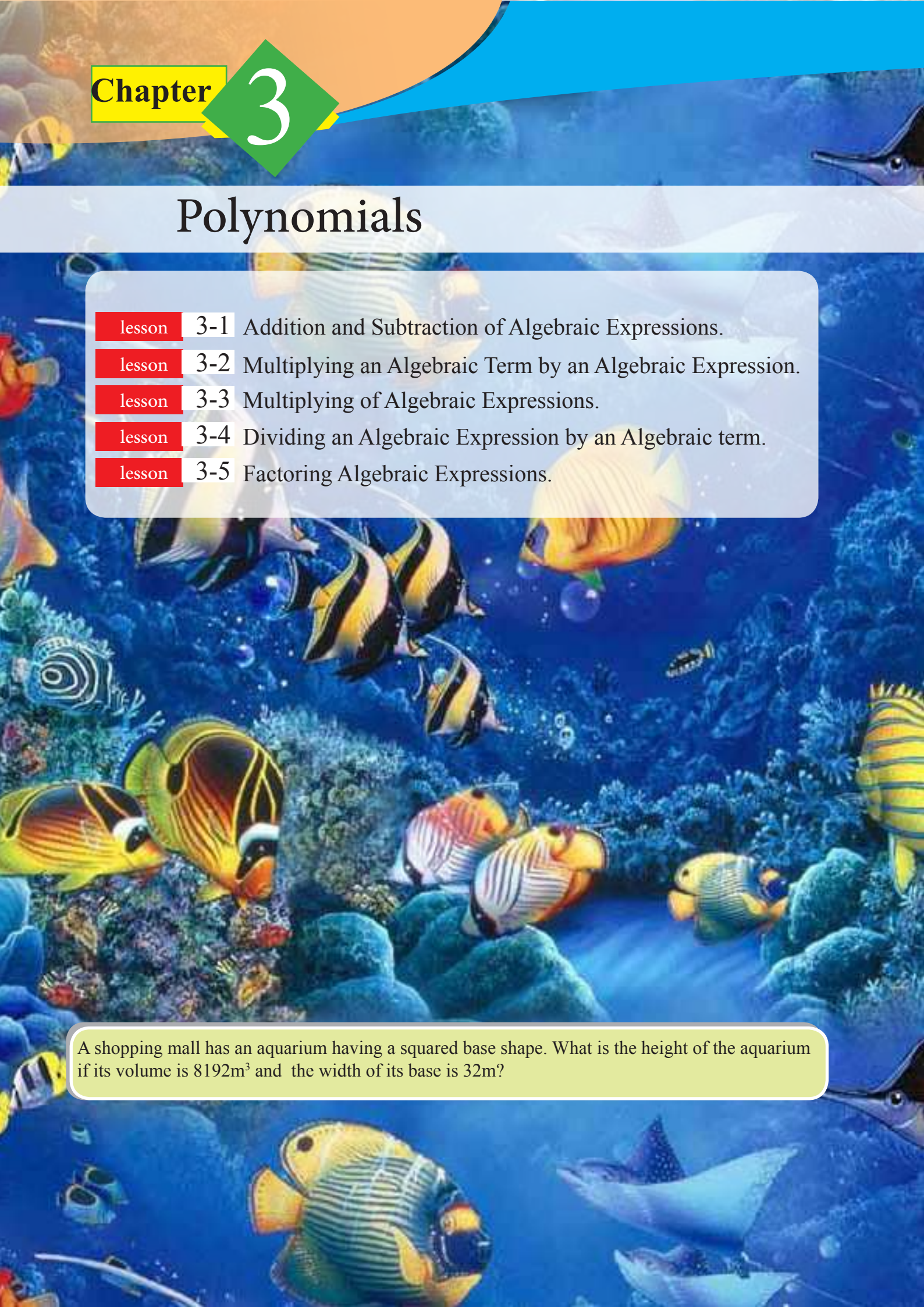
Represent each pair from the ordered pairs in the coordinate plane then find the distance between them approximate to the nearest tenth if it does not represent an integer:

44 $\{(3 , 0) , (0 , 3)\}$ 45 $\{(-4 , 0) , (0 , 4)\}$ 46 $\{(-1 , 5) , (-4 , 2)\}$

47 $\{(4 , -1) , (1 , -5)\}$ 48 $\{(0 , 0) , (-3 , -3)\}$ 49 $\{(-2 , -3) , (-4 , 1)\}$

Chapter 3

Polynomials



A shopping mall has an aquarium having a squared base shape. What is the height of the aquarium if its volume is $8192m^3$ and the width of its base is 32m?

Pretest

Write the coefficient (C) and the variable (V) of the following algebraic terms:

1 $|-2|xy$, Coefficient.....Variable .....

2 $30r^2v^2$ Coefficient.....Variable .....

3 $\sqrt{16}\,h^3k^2$, Coefficient.....Variable .....

4 $\frac{1}{2}z^2y$, Coefficient.....Variable .....

5 $10w^3h$, Coefficient.....Variable .....

6 w^5z^2, Coefficient.....Variable.....

Find the addition result of the following algebraic terms:

7 $5h^2k + 10h^2k + \frac{1}{2}h^2k + |-\frac{1}{5}|h^2k$

8 $x^2y^2z + \frac{1}{3}x^2y^2z + 6x^2y^2z$

9 $2r^2v + \frac{1}{4}r^2v + 16r^2v + |-\frac{1}{2}|r^2v$

10 $|-7|a^2b + 14\,a^2b + \frac{1}{28}a^2b$

11 $10xy + \frac{1}{20}xy + \frac{1}{5}xy + 4xy$

12 $7a^2b^2z + |-18|a^2b^2z + 9a^2b^2z$

13 $2x^4y^2 + \frac{1}{5}x^4y^2 + |-\frac{1}{5}|x^4y^2$

14 $12\,g^2h^2 + \frac{1}{3}g^2h^2 + \frac{1}{4}g^2h^2$

Find the subtraction result of the following algebraic terms:

15 $\frac{1}{25}h^2k - 15h^2k - 5h^2k - |-\frac{1}{5}|h^2k$

16 $16\,r^2v^2 - \frac{1}{32}r^2v^2 - 64\,r^2v^2$

17 $\frac{2}{3}xy - \frac{10}{6}xy - |-\frac{1}{3}|xy$

18 $30a^2b^2 - 5a^2b^2 - 15a^2b^2$

19 $|-15|z^2v^2 - |-35|z^2v^2 - 5z^2v^2$

20 $24w^2z - \sqrt{144}\,w^2z - 9w^2z$

Find the multiplication result of the following algebraic terms:

21 $(2x)(20y)$

22 $(6z^2)(3v^2)$

23 $(|-20|z)(r^3v^3)$

24 $10y(2x+60z+4)$

25 $30z^2(\frac{1}{3}v^2+\frac{1}{15}v^3y+\frac{2}{30}mn)$

26 $3y(20m^2n+4)$

Find the numerical value of the following algebraic expressions:

27 $\frac{1}{2}z^2y^2 + 3zy + w$; $w=2, z=4, y=3$

28 $3h^3k^2 - 5h + 4$; $h=5, k=3$

29 $|-25|r^2v^2 + \frac{1}{3}v + 5$; $r=1, v=12$

30 $3xy - 2xy + 6$; $x=2, y=10$

31 **Find the outputs for function rule to the following inputs:**

Inputs	$2x^2+4$ Function Rule	Outputs
-2		
0		
2		

Lesson [3-1] Addition and Subtraction of an Algebraic Expressions

Idea of the lesson:
*Addition of an algebraic expression.
*Subtraction of an algebraic expression.

Vocabulary:
*Addition
*Subtraction

Learn

Two loads of food one of the loads contains rice, sugar and flour in kilograms respectivly $54x^3$, $25y^5$, $30z^2$ and the other load contains the same food $36x^3$, $20y^5$, $25z^2$.
What is the sum of the loads and what is the difference between them?

[3-1-1] Addition of an Algebraic Expressions

You have previously learned the addition of similar algebraic terms and in this lesson we will learn the addition of algebraic expressions (to add two algebraic expressions) we will use commutative and associative properties in the addition of similar terms.

Example 1 **Find the sum of the first and second load together:**

$(54x^3 + 25y^5 + 30z^2)$ First load

$(36x^3 + 20y^5 + 25z^2)$ Second load

$= (54x^3 + 25y^5 + 30z^2) + (36x^3 + 20y^5 + 25z^2)$ *Sum of the two loads*

$= (54x^3 + 36x^3) + (25y^5 + 20y^5) + (30z^2 + 25z^2)$ *Using association property*

$= 90x^3 + 45y^5 + 55z^2$ *Add the similar terms*

So the sum of the loads in kilogram is $90x^3 + 45y^5 + 55z^2$

Example 2 **Find the addition result of the following algebraic expressions:**

i) $(\frac{-1}{6} w^2z + \sqrt{2}\, r^2v^2 + \sqrt{5}\, h^3k^2) + (\frac{1}{3} w^2z + \sqrt{2}\, r^2v^2 + 2\sqrt{5}\, h^3k^2)$

$= (\frac{-1}{6}w^2z + \frac{1}{3} w^2z) + (\sqrt{2}\, r^2v^2 + \sqrt{2}\, r^2v^2) + (\sqrt{5}\, h^3k^2 + 2\sqrt{5}\, h^3k^2)$ *Using association property*

$= (\frac{1}{6} w^2z) + (2\sqrt{2}\, r^2v^2) + (3\sqrt{5}\, h^3k^2)$ *Add the similar terms*

$= \frac{1}{6} w^2z + 2\sqrt{2}\, r^2v^2 + 3\sqrt{5}\, h^3k^2$ *Sum of the two expressions*

ii) $(\sqrt{2}\, xy^2 + \frac{1}{5} xy + 4\sqrt{3}\ x^2y) + (3\sqrt{2}\, xy^2 - \frac{1}{5} xy + \sqrt{3}\ x^2y)$

$= (\sqrt{2}\, xy^2 + 3\sqrt{2}\ xy^2) + (\frac{1}{5} xy - \frac{1}{5} xy) + (4\sqrt{3}\ x^2y + \sqrt{3}\ x^2y)$ *Using association property*

$= 4\sqrt{2}\ xy^2 + 0 + 5\sqrt{3}\ x^2y$ *Add the similar terms*

$= 4\sqrt{2}\ xy^2 + 5\sqrt{3}\ x^2y$ *Sum of the two expressions*

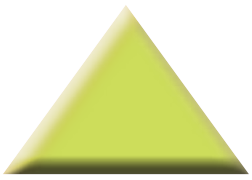
iii) $(|-15|m^2n + 12gh + \frac{1}{20}r^2v^3) + (|-45|m^2n + 10gh + \frac{2}{30}r^2v^3)$

$= (|-15|m^2n + |-45|m^2n) + (12gh+10gh) + (\frac{1}{20}r^2v^3 + \frac{1}{15}r^2v^3)$ *Using association property*

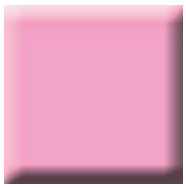
$= (15m^2n+45m^2n) + (22gh) + \frac{3+4}{20}r^2v^3$ *Add the similar terms*

$= 60m^2n + 22gh + \frac{7}{60}r^2v^3$ *Sum of the two expressions*

Example 3 **The perimeter of an equilateral triangle is $(2n^2+4y+5)$, and the perimeter of a square is $(4n^2+6y+10)$, write the algebraic expression which represents the sum of the perimeters of the square and the triangle.**



$2n^2+4y+5$



$4n^2+6y+10$

$(2n^2+4y+5) + (4n^2+6y+10)$

$= (2n^2+4n^2) + (4y+6y) + (5+10)$ *Using association property*

$= 6n^2 + 10y + 15$ *Add the similar terms*
Sum of the two expressions

So the sum of the perimeters of the square and triangle is $(6n^2 + 10y + 15)$

[3-1-2] Subtraction of an Algebraic Expressions

You have previously learned subtracting the similar algebraic terms which means subtracting any algebraic term with another add the first algebraic term with the additive inverse for second algebraic term, we will learn subtraction of algebraic expressions from another, reverse the sign of every term from the second algebraic expression (additive inverse for the algebraic expression).

Example 4 **From learn paragraph, find the difference between the two loads.**

$(54x^3 , 25y^5 , 30z^2)$ First Load

$(36x^3 , 20y^5 , 25z^2)$ Second Load

$54x^3 + 25y^5 + 30z^2$ First Load

$36x^3 + 20y^5 + 25z^2$ Second Load

$= (54x^3+25y^5+30z^2) - (36x^3 + 20y^5 + 25z^2)$ *Find the difference between the two loads*

$= (54x^3+25y^5+30z^2) + (-36x^3 - 20y^5 - 25z^2)$ *We change the sign of the algebraic terms in the second expression*

$= (54x^3 - 36x^3) + (25y^5 - 20y^5) + (30z^2 - 25z^2)$ *Using the association property*

$18x^3 + 5y^5 + 5z^2$ *Add the similar terms*

So the difference between the two loads in kilogram is $18x^3 + 5y^5 + 5z^2$

Example 5 **Find the result of subtraction of the following algebraic expressions:**

i) $(3\sqrt{2}\,xy - |-2|\,zw - \sqrt{4}\,r^2v^2)$, $(2\sqrt{2}\,xy - 10zw + 4r^2v^2)$

$= (3\sqrt{2}\,xy - |-2|\,zw - \sqrt{4}\,r^2v^2) - (2\sqrt{2}\,xy - 10zw + 4r^2v^2)$ *Subtract the second expression from the first expression.*

$= (3\sqrt{2}\,xy - 2zw - 2r^2v^2) + (-2\sqrt{2}\,xy + 10zw - 4r^2v^2)$ *Adding the additive inverse*

$= (3\sqrt{2}\,xy - 2\sqrt{2}\,xy) + (-2zw + 10zw) + (-2r^2v^2 - 4r^2v^2)$ *Add the similar terms*

$= \sqrt{2}\,xy + 8zw - 6r^2v^2$

ii) $(3x^2 - 15y - 6) - (7x^2 - 9y + 6)$

$= (3x^2 - 15y - 6) + (-7x^2 + 9y - 6)$ *Adding the additive inverse*

$= (3x^2 - 7x^2) + (-15y + 9y) + (-6 - 6)$ *Add the similar terms*

$= -4x^2 - 6y - 12$

Example 6 **A squared shape fountain has an area $(2m^2-2m-6)$ square meter, it is located in the center of a rectangular shaped garden with an area of $(3m^2-4m+5)$ square meter, what is the area of the garden surrounding the fountain?**

$=(3m^2 - 4m + 5) - (2m^2 - 2m - 6)$ *Area of the garden*

$=(3m^2 - 4m + 5) + (-2m^2 + 2m + 6)$ *Add the additive inverse*

$=(3m^2 - 2m^2) + (-4m + 2m) + (5 + 6)$ *Subtract the similar terms*

$= m^2 - 2m + 11$ *Find the expressions*

So the area of the garden surrounding the fountain in square meter is $(m^2 - 2m + 11)$

Make sure of your understanding

Find the addition of the following algebraic expressions:

1. $(20x^2y^2 + \frac{1}{2}z^3w^2 + 5)$, $(20x^2y^2 + \frac{3}{2}z^3w^2 + 10)$
2. $(\sqrt{2}\,h^3 + 2k^2y + 9)$, $(2\sqrt{2}\,h^3 + k^2y + 6)$
3. $(|-\sqrt{5}|\,m^3n^2 + \frac{1}{5}r^2v^2 + 3)$, $(5\sqrt{5}\,m^3n^2 + 2r^2v^2 + 4)$

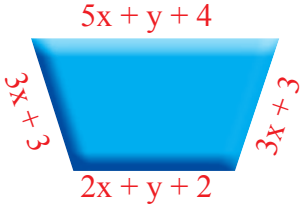
Questions 1-3 are similar to examples 1,2

Find the subtraction of the following algebraic expressions:

4. $(7m^2n^2 + \frac{1}{3}y^2 + \sqrt{7})$, $(6m^2n^2 + \frac{2}{9}y^2 + 2\sqrt{7})$
5. $(8a^3b + 10z - 4)$, $(2a^3b + 5z + 3)$
6. $(\frac{1}{25}hk + 2y - 9)$, $(5hk - y - 8)$

Questions 4-6 are similar to examples 4,5

7. **What is the perimeter for the figure?**



Question 7 is similar to example 3

Solve the Exercises

Add the following algebraic expressions:

8 $(|-12|x^2y^2z + 2ab + 4)$, $(x^2y^2z + ab - 2)$

9 $(4\sqrt{2}\,r^2v^2 + 2hk + 3)$, $(\sqrt{2}\,r^2v^2 + 5hk + 6)$

10 $(\frac{1}{10}zw + 10x + 2)$, $(\frac{2}{5}zw + 10x + 2)$

Find subtract of the following algebraic expressions:

11 $(7m^2n^2 - |3|y + \sqrt{7})$, $(7m^2n^2 + \frac{1}{3}y - 2\sqrt{7})$

12 $2\sqrt{2} + 20z - 4\sqrt{5}$, $(\sqrt{2}\,a^3b + 5z + \sqrt{5})$

13 $(\frac{1}{25}hk + 2x - 9)$, $(\frac{1}{5}hk - 10x - 8)$

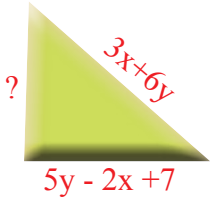
Solve the problems

14 Electrical: In one of the electrical appliances store in March washing machines were sold by an amount of $(\sqrt{2}\,x^3 + \frac{1}{2}yz + 4)$ dinar, and airconditions were sold by the expression of $(\sqrt{2}\,x^3 + 4yz + 8)$ dinar, what is the sum of the selling of the store in March?

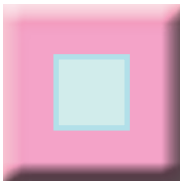
15 Stations: Two trains launched from the same stations in opposite directions one of them was $(|2|\ x^2+4y+20)$ km far from the station and the other one was $(4x^2+10y+2)$ km far from the station, find the distance between the trains.

Think

16 Challenge: If the perimeter of the triangle was $(4x+12y+8)$ meter what is the length of the unknown side?



17 Geometry: In the near by figure the area of the big square $(2x^2+24x+3)$ square meter and the area of the small square $(x^2-21x+1)$ square meter what is the difference between the area of the big and small squares?



18 Numerical sense: What is the expression which you subtract from $(7x^2-4y^2+5)$ to have the result (x^2+3)?

Write

A problem from daily life about adding and subtracting two algebraic expressions.

Lesson [3-2] Multiplying an Algebraic by an Algebraic Expression

Idea of the lesson:
*Multiplying an algebraic term by an algebraic term
*Multiplying an algebraic term by an algebraic expressin

Vocabulary:
*Algebraic Term
*Algebraic expression

Learn

Ahmad made a wooden box its base is rectangular shaped, if the length of the box was ($\sqrt{10}\ m^2n^2$) in centemeters, and its width was ($\sqrt{5}\ m^2n^2$) in centemeters .
What is the area of the base of the box?

[3-2-1] Multiplying an Algebraic Term by an Algebraic Term

You have previously learned multiplying an algebraic term by an algebraic term, if the variables were different but in this lesson you will learn multiplying an algebraic term by an algebracic term by using the following properties:
i) $a^{nm} = (a^n)^m$ ii) $a^n.a^m = a^{n+m}$ iii) $a^0 = 1$, where a is a real number and not equal to zero.

Example 1

The area of the base of the box = length × width

$A = \sqrt{10}\,m^2n^2 \times \sqrt{5}\ m^2n^2$

$= (\sqrt{10} \times \sqrt{5}\ )\ m^2 \cdot m^2 \cdot n^2 \cdot n^2$ *Multiply the coefficients and the variables*

$= (\sqrt{10} \times \sqrt{5}\ )\ m^{2+2} \cdot n^{2+2}$ *Properties of the real numbers*

$= \sqrt{50}\,m^4n^4$ *In multiplication the powers are added*

$= 5\sqrt{2}\ m^4n^4$ *So the area of the rectangle is* $5\sqrt{2}\ m^4n^4$ *Square centemeter*

Example 2 **Find the result of the multiplication of the following:**

i) $(4x^3y^4)(6x^5y^5) = (4\times6)\,(x^3\times x^5)(y^4\times y^5) = 24x^{3+5} \times y^{4+5} = 24x^8y^9$

ii) $(9h^2k)(-2h^3k) = (9\times-2)(h^2\times h^3)(k\times k) = -18h^5k^2$

iii) $(\,|-12|\,r^2v)(\,|-4|\,r^2vn) = (12\times4)\,r^{2+2}v^{1+1}n = 48r^4v^2n$

iv) $(\sqrt{36}\,z^2w^2)(6zk) = (6)(6)z^{2+1}w^2k = 36z^3w^2k$

v) $(\sqrt{7}\ g^2h^2)(\sqrt{14}\,g) = (\sqrt{7}\,)\,(\sqrt{14}\,)\,(g^2 \cdot g)\,(h^2) = 7\sqrt{2}\,g^3h^2$

vi) $(\frac{5}{7}\,a^3b^3)\,(\frac{49}{25}\,a^{-2}b^{-3}) = (\frac{\cancel{5}^{\,1}}{\cancel{7}^{\,1}} \times \frac{\cancel{49}^{\,7}}{\cancel{25}^{\,5}})\,a^{3+(-2)}b^{3+(-3)} = \frac{7}{5}\,a$ Note : $b^0 = 1$

[3-2-2] Multiplying an Algebraic Term by an Algebraic Expression

You have previously learned multiplying an algebraic term with an algebraic expression by using distribution property if the variables were different and now we will learn multiplying an algebraic term with an algebraic expression if the bases were the same or different by using distribution property also.

Example 3 **Find the result of the multiplication of the following:**

i) $-5h^2k(3h^4k^2 + 6h^2k) = -5h^2k(3h^4k^2)+(-5h^2k)(6h^2k)$ *By using associative and distributive property*

$= (-5)(3)(h^2 \times h^4) \times (k \cdot k^2) + (-5)(6) \times (h^2 \cdot h^2) \times (k \cdot k)$

$= -15h^{2+4}\, k^{1+2} + (-30)h^{2+2}\, k^{1+1}$ *In multiplication the powers are added*

$= -15h^6k^3 - 30h^4k^2$

ii) $3m^3n^4(1 - 5mn^5) = 3m^3n^4(1) - (3m^3n^4)(5mn^5)$ *By using associative and distributive property*

$= (3)(m^3 n^4) - (5)(3)\, m^3 \cdot m \cdot n^5 \cdot n^4$

$= 3m^3 n^4 - 15m^{3+1} n^{5+4}$ *In multiplication the powers are added*

$= 3m^3 n^4 - 15m^4 n^9$

iii) $\frac{1}{3}x^2y(\frac{1}{2}xy^2z + 4x^{-2}yz) = \frac{1}{3}x^2y(\frac{1}{2}xy^2z) + (\frac{1}{3}x^2y)(4x^{-2}yz)$

$= (\frac{1}{3})(\frac{1}{2})(x^2 \cdot x)(y \cdot y^2)(z) + (\frac{1}{3})(4)(x^2 \cdot x^{-2})(y \cdot y)(z)$ *By using associative and distributive property*

$= \frac{1}{6}x^3y^3z + \frac{4}{3}zy^2$ *In multiplication the powers are added*

iv) $\sqrt{5}(5z^2w^2 + \sqrt{5}\, zw + 2)$ *By using associative and distributive property*

$= \sqrt{5}(5)z^2w^2 + (\sqrt{5})\sqrt{5}\, zw + 2\sqrt{5}$

$= 5\sqrt{5}\, z^2w^2 + 5\, zw + 2\sqrt{5}$ *In multiplication the powers are added*

v) $\sqrt{2}\, x^3y(3\sqrt{2}\, x^{-2}y^{-1} - \sqrt{2}\, x^{-3}y^2)$ *By using associative and distributive property*

$= (\sqrt{2})(3\sqrt{2})\, x^3x^{-2} \cdot yy^{-1} - (\sqrt{2})(\sqrt{2})\, x^3x^{-3}yy^2$

$= 6xy^0 - 2x^0y^3$ *In multiplication the powers are added*

$= 6x - 2y^3$

Example 4 **A rectangular shaped stadium with a length of $(4x^2)$ meters and width of $(2x^3-4xy-3)$ meters. What is the area of the stadium?**

area of the rectangle = (length) × (width)

$A = 4x^2 \times (2x^3 - 4xy-3)$

$= 4x^2(2x^3)-(4x^2)(4xy)-(4x^2)(3)$ *By using distribution property*

$= 8x^5 - 16x^3y - 12x^2$

So the area of the stadium is $(8x^5 - 16x^3y - 12x^2)$ square meter

Make sure of your understanding

Find the result of the multiplication of the following:

1. $(\frac{\sqrt{3}}{4} m^2n)(2m^3)$
2. $(20x^{-5}yz)(10xy)$

Questions 1-2 are similar to example 2

3. $(8r^3v^2)(|-5|r^2v+6r^2v^2)$
4. $\sqrt{2}\, x^2y^2(\sqrt{2}\, xy^5-y^5)$
5. $\sqrt{7}\, z^2w(z^{-1}wy+2\sqrt{7}\, z^4w^2y)$
6. $\frac{1}{4}h^2k\,(\frac{\sqrt{16}}{4} h^{-2}kr + 6h^3k^{-1}r^2 + \sqrt{8}\,)$

Questions 3-6 are similar to example 3

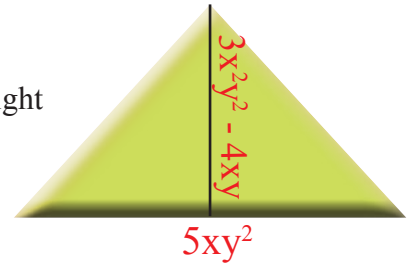
Solve the Exercises

Find the result of the multiplication of the following:

7. $(m^4n)(m^3n^{-4})$
8. $\sqrt{5}\, a^2b^2c^3(2\sqrt{5}\, a^4)$
9. $-7r^4vy^3(5rv^4y^2)$
10. $\sqrt[3]{-27}\, x^2y^2\,(\sqrt[3]{-8}\, xyz)$
11. $(y-2)(xy)$
12. $(|-3|z)wx^4z^{-3}$
13. $(8w-4)(7w^{-5})$
14. $-3xy^2z\,(5x^4y^2+ 4xy^2z - 6x^2y^2z^3)$
15. $\frac{1}{2}ab^2c\,(2a^{-1}b^{-2}c^{-1})$
16. $(5a^2b + 4a^3b - \frac{1}{2}abc^2)\,8a^{-2}b$
17. $hk\,(6h^2k^2 - 7h^2y + 2zy)$
18. $12x^6y^7\,(1-\frac{1}{2}-x^3y)$
19. $\frac{-1}{3} r^2\,(r^2 - \sqrt{7}\ r^2vy)$
20. $4ab\,(a^2b - \sqrt[3]{64}\, abc)$
21. $x^{-4}(x-x^3y^5 + \sqrt{2}\, x^{-2})$
22. $5m^{-3}(2m+nz^4 + 4)$
23. $|-3|r^2v^2\,(|4|rv + 25)$
24. $\frac{1}{6}yz\,(36y^2z^2 + 6yz + 36)$
25. $\sqrt{9}\ gh(2g^4h^2 + 3gh + 5)$
26. $|-8|z^6w^5\,(|-2|z^{-6}w^{-5} + \frac{1}{4}z^{-4}w^{-5})$

Solve the problems

27 **Geometry:** A triangle its base length is $(5xy^2)$ cm and its height is $(3x^2y^2-4xy)$ cm, what is the area of the triangle?



28 **Technology:** Ahmad subscribed with internet services with a speed of $4x^5y^2$, while Mohammed's speed is more than Ahmad speed with $5xy^{-2}$. what is the speed of Anwar that represents the product of Ahmad and Mohammed speed?

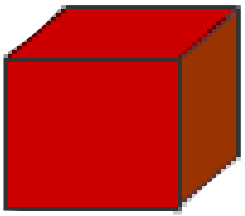
29 **Biology:** If the number of a bee kingdom is $10w^3z$ and it doubled with $5z^2$, so what's the number of the bees?

Think

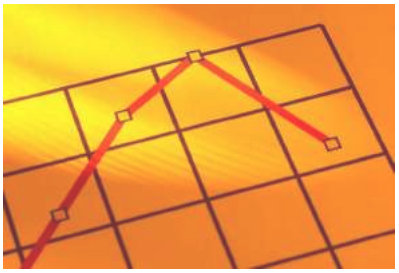
30 **Correct the mistake:** Ali wrote the result of multiplying $\sqrt{2}\ h^2k^2\ (\sqrt{2}\ h^2k^2 - \sqrt{4}\ h^3k)$ as $4h^3k^4-4h^5k^3$ find Ali's mistake and correct it.

31 **Challenge:** What is the algebraic term that if we multiply it by $\frac{5}{2}x^2y + 5x - 4$ the result will be $10x^4y^4 + 20x^3y^3 - 16x^2y^3$?

32 **Challenge:** A quadrilaterals prism shaped box with an area of base is z^2 square unit and height $(5z^2+5)$ unit, what is the volume of the quadrilaterals prism?



33 **Statistics:** If number of the population of Diyala was $2x^{-3}y^3$, while the population of Baghdad $20x^{-2}y^2$ times more than Diyala's population. So what's number of the population of Baghdad?



Write

Product of an algebraic term by an algebraic expression, and find the result of multiplying .

Lesson [3-3] Multiplying of Algebraic Expressions

Idea of the lesson:
*Multiplying two expressions,each expression from two terms.
*Multiplying two expressions, the first of the two terms and the second of three terms.

Vocabulary:
* Two terms
*Three terms
*Vertical multiplication
*Horizontal multiplication

Learn

A farmer planted a rectangular shaped land for vegetables which has a length $(\sqrt{5}\ x^2y+x)$ meters and width $(\sqrt{25}\ x^2y+4)$ meters.
What is the area of the land?

[3-3-1] Multiplying two Algebraic Expressions each of them with by Two Terms

We have previously learned that multiplying an algebraical term with algebraical expression you will learn in this lesson multiplying an algebraical expression with another algebraical expression and each one consists of two terms by using distribution property, there is two types of multiplication vertical and horizontal.

Example 1

Area of rectangular = (length) × (width)

$A= (\sqrt{5}\ x^2y+x)(\sqrt{25}\ x^2y+4)$ — *Area of rectangular*

$= (\sqrt{5}\ x^2y \times \sqrt{25}\ x^2y) + \sqrt{5}\ x^2y(4) + x(\sqrt{25}\ x^2y) + (4)(x)$ — *By using distribution property*

$= (\sqrt{5}\ x^2y \times 5x^2y) + (4\sqrt{5}\ x^2y) + (5x^3y) + (4x)$ — *By using horizontal multiplaction*

$= 5\sqrt{5}\ x^4y^2 + 4\sqrt{5}\ x^2y + 5x^3y + 4x$ — *Area of the land in square meters*

Example 2

Find the result of multiplication (horizontal) of the following :

i) $(\frac{1}{2}x^2+y^2)(x^3+2y^2)$ — *By using disribution property*

$= (\frac{1}{2}x^2 \cdot x^3 + \frac{1}{2}x^2 \cdot 2y^2) + (y^2 \cdot x^3 + y^2 \cdot 2y^2)$ — *By using horizontal multiplication*

$= \frac{1}{2}x^5 + x^2y^2 + x^3y^2 + 2y^4$

ii) $(|-2|ab + |-5|bc)(3 + ab)$ — *By using distribution property*

$= (2\times3)ab + 2a^2b^2 + (5\times3)bc + 5ab^2c$ — *By using horizontal multiplication*

$= 6ab + 2a^2b^2 + 15bc + 5ab^2c$

Example 3 **Find the result of multiplication (vertical) of the following :**

i) $(3th^2 - 7)(5 + th^2)$

$(5 + th^2)$

$\times\ 3th^2 - 7$

$15th^2 - 3t^2h^4$ *Multiply $(3th^2)$ in the second brackets by the vertical method.*

$-35 - 7th^2$ *Multiply (-7) in the second brackets by the vertical method.*

$-35 + 8th^2 + 3t^2h^4$ *Put the similar terms* / *Add the terms*

ii) $(\frac{2}{9}z^2w^3 + 1)(3wz + 4)$

$3z^2w^3 + 4$

$\times\ \frac{2}{9}z^2w^3 + 1$

$\frac{2}{3}z^3w^4 + \frac{8}{9}z^2w^3$ *Multiply $(\frac{2}{9}z^2w^3)$ in the second brackets by the vertical method.*

$3zw + 4$ *Multiply (1) in the brackets by the vertical method.*

$\frac{2}{3}z^3w^4 + \frac{8}{9}z^2w^3 + 3zw + 4$ *Put the similar terms* / *Add the similar terms*

Example 4 **A volleyball playgroud its dimensions by meter (8y+3),(8y-6) , what is the area of the playgroud?**

area of the playgroud = (length) ×(width)

$A = (8y+3)(8y-6)$ *By using distribution property*

$= 64y^2 - 48y + 24y - 18$ *By using horizontal multiplication*

$= (64y^2 - 24y - 18)$ *The area of vollyball playgroud in square meter*

[3-3-2] Multiplying two an Algebraic Expressions the First of two Terms and the 2nd of Three Terms

You have previously learned from this lesson multiplying an algebraic expression with another algebraic expression by using distribution property by vertical and horizontal multiplication and you will learn multiplying algebraic expression which consists of two terms with algebraic expression which cosists of theree terms by using vertical and horizontal multiplication.

Example 5 **Find the horizontal multiplication result of the followings.**

i) $(-2x^2 - 8)(x^3 + x - 2)$ *By using distribution property*

$= -2x^2(x^3 + x - 2) - 8(x^3 + x - 2)$ *By using horizontal multiplication*

$= -2x^5 - 2x^3 + 4x^2 - 8x^3 - 8x + 16$

$= -2x^5 - 10x^3 + 4x^2 - 8x + 16$

ii) $(\sqrt{7}\,y^2 - 5z)(\sqrt{7}\,y^2 + 3z - \sqrt{7}\,w)$ *By using distribution property*

$= (\sqrt{7}\,y^2)(\sqrt{7}\,y^2 + 3z - \sqrt{7}\,w) - (5z)(\sqrt{7}\,y^2 + 3z - \sqrt{7}\,w)$ *By using horizontal multiplication*

$= 7y^4 + 3\sqrt{7}\,zy^2 - 7wy^2 - 5\sqrt{7}\,zy^2 - 15z^2 + 5\sqrt{7}\,zw$

$= 7y^4 - 2\sqrt{7}\,zy^2 - 7wy^2 - 15z^2 + 5\sqrt{7}\,zw$

Example 6

Find the result of multiplication (vertical) of the following :

i) $(z + 7)(z^2 - 2z + 3)$

$z^2 - 2z + 3$ *Multiply (z) by the second bracket*

$\times \quad z + 7$ *Multiply (7) by the second bracket*

$z^3 - 2z^2 + 3z$

$7z^2 - 14z + 21$ *Add the similar terms*

$z^3 + 5z^2 - 11z + 21$

ii) $(x + y)(x - xy + y^2)$

$x - xy + y^2$ *Multiply (x) by the second bracket*

$\times \quad x + y$ *Multiply (y) by the second bracket*

$x^2 - x^2y + xy^2$

$xy - xy^2 + y^3$ *Add the similar terms*

$x^2 + xy - x^2y + y^3$

Make sure of your understanding

Find the result of the horizontal multiplication for the following:

1 $(3x-2)(4x+1)$

2 $(\sqrt{5}\,x-5)(\frac{\sqrt{25}}{5}x - 6)$

3 $(2a^2b-\frac{5}{8})(\frac{3}{5}\,ab^3-\frac{1}{6}b)$

4 $(-4m^3n^2 - 6mn^3)(6mn - 3m)$

Questions 1-4 are similar to examples 2,5

Find the result of the vertical muliplcation for the following:

5 $(2x-5)(-4x^3+5y-7)$

6 $(2x^2y-3)(xy^2 - 3z - 15w)$

7 $(4a-4y)(y^2+5z-6)$

8 $(x^2-2x)(5x^2+3x-4)$

Questions 5-8 are similar to examples 3,6

Solve the Exercises

Find the result of the horizontal multiplication for the following:

9 $(x^2y-5z)(y+4)$

10 $(\frac{1}{2}w^2+4)(\frac{16}{\sqrt{4}}z^2+4)$

11 $(3m-5)(|10|\,m^2-3n)$

12 $(x+2y)(2x+1)$

13 $(ab+3c)(2a+c)$

14 $(\sqrt{5}\,m-3x)(m-2x)$

15 $(\sqrt{7}\,y-z)(\sqrt{7}\,yw+z)$

16 $(6m+5)(2x^2-3x-5)$

17 $(h^2k+1)(rv+5)$

Find the result of the vertical muliplcation for the following:

18 $(3xy-3)(4y+3z)$

19 $(\frac{1}{2}m^2+2m)(4m^2-8m+5)$

20 $(\sqrt{2}\,x^2+3y-1)(\sqrt{2}\,x^2-5y)$

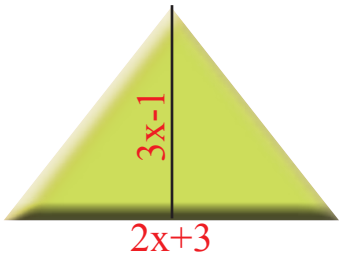
21 $(3x-5)(6x^2+12x-8)$

22 $(|-24|+6y)(|-2|\,zw+2y)$

23 $(\sqrt{6}\,x^2-3y)(4y^2+10y+2)$

Solve the problems

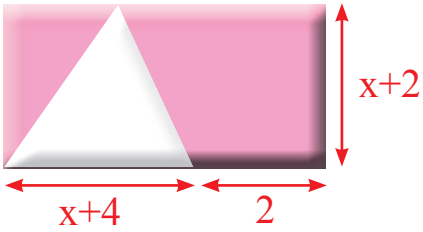
24 **Geometry:** What is the area of the triangle if the length of its base was (2x+3) meter and height (3x-1) meter?



25 **Geometry:** A passage with a width x surrounds a rectangular garden which its length is 8 meters and widht 6 meters. What is the total area of the garden and the passage?

26 **Molds:** A cake mold is made of aluminum with a rectangular shaped base, what is the area of the base if you knew it's dimensions in centimeters as in the figure?

27 Find the area of the shaded region in the near by figure?



Think

28 **Challenge:** Find the result of the following:

i) $(y^m + y^n)(y^{m-1} - y^{n-1} + y^n)$, $n, m \in z$

ii) $(3 - z)(3^2 + 3z + z^2)$

iii) $(\frac{1}{2}t - \frac{1}{3})^3$

iv) $(x + y)(x - y)$

29 **Correct the mistake:** Suaad wrote the result of the expression $(2x+6)^2$ in the form of $4x^2+12x-36$ discover Suaad's mistake and correct it.

30 **Opened Problem:** Write an expression which contains two terms, and an expression contains three terms and then find the result of their multiplication.

Write

Two expressions each one contains two terms and find the result of their multiplication by horizontal and vertical method.

Lesson [3-4] Dividing an Algebraic Expression by an Algebraic Term

Idea of the lesson:
*Dividing an algebraic expression by an algebraic term.

Vocabulary:
*Division
*Algebraic term

Learn

In arrow games the arrow takes off horizontally according to the law $x = \frac{5h^2n}{h}$, where x represents the speed of the arrow and h represents the height of the arrow in meters and n represents the time in seconds find the speed of the arrow if the values was h=5, n=2.

[3-4-1] Dividing an Algebraic Term by an Algebraic Term

You have previously learned division of powers which means $\frac{a^n}{a^m} = a^{n-m}$ where (a) is a real number and the denominator must not equal to zero. In this lesson you will study division the algebraic expressions which is an algebraic term dividing by an algebraic term where dividing the coefficient of the first term by the coefficient of second term then subtract the powers that have the same base.

Example 1 **For calculating the speed of the arrow** $x = \frac{5h^2n}{h}$

$x = \frac{5h^2 n}{h}$ *Divide the term on (h)*

$x = 5h^{2-1} n$ *Replace the value h,n*

$x = 5h\,n$

$x = 5(5)(2)$

$x = 50$ m/s

So the speed of the arrow during its taking off is 50 meters per second.

Example 2 **Find the division of the following where the denominator must not equal to zero:**

i) $\frac{8x^7 y^4}{6x^5 y^3} = \frac{4}{3}x^{7-5}y^{4-3} = \frac{4}{3}x^2 y$

Divide the coefficient by coefficient then subtract the powers

ii) $\frac{-5h^6 k}{25h^2} = \frac{-1}{5} h^{6-2}k = \frac{-1}{5} h^4 k$

iii) $\frac{\sqrt{16}\, r^2 v^2}{4rv} = \frac{4}{4}r^{2-1} v^{2-1} = r\,v$

iv) $\frac{\frac{1}{3}zw}{3z^3} = \frac{1}{3}\times\frac{1}{3}z^{-2}w = \frac{1}{9}z^{-2}w = \frac{w}{9z^2}$

v) $\frac{12a^5 b^2}{4a^4b^2} = \frac{12}{4} ab^{2-2} = 3a$

vi) $\frac{-32m^5n^2}{-8m^3n} = \frac{-32}{-8} m^{5-3}n^{2-1} = 4m^2n$

vii) $\frac{|-30|\, x^5 y^8 z^5}{10x^5y^6} = \frac{30}{10}x^{5-5}y^{8-6}z^5$

$= 3x^0y^2z^5 = 3y^2z^5$

[3-4-2] Dividing an Algebraic Expression by an Algebraic Term

You have learned dividing an algebraic term by an algebraic term in the previous section and in this section you will learn dividing an algebraic expression by an algebraic term where the denominator is not equal to zero (partial fraction method).

Example 3 **Find the result of dividing an algebraic expression by using partial fraction method where the denominator is not equal to zero for each of the followings:**

i) $\frac{12x^3+24x^2}{6x} = \frac{12x^3}{6x} + \frac{24x^2}{6x}$

$= 2x^{3-1}+4x^{2-1} = 2x^2 + 4x$

ii) $\frac{x^6y^2 - x^3y^5 - 3x^2y^7}{x^2y} = \frac{x^6y^2}{x^2y} - \frac{x^3y^5}{x^2y} - \frac{3x^2y^7}{x^2y} = x^{6-2}y^{2-1} - x^{3-2}y^{5-1} - 3x^{2-2}y^{7-1}$

$= x^4y - xy^4 - 3x^0y^6 = x^4y - xy^4 - 3y^6$

iii) $\frac{12z^5w^2+9z^4w^5+15z^2w^7}{3z^2w} = \frac{12z^5w^2}{3z^2w} + \frac{9z^4w^5}{3z^2w} + \frac{15z^2w^7}{3z^2w}$

$= 4z^{5-2}w^{2-1}+3z^{4-2}w^{5-1}+5z^{2-2}w^{7-1} = 4z^3w + 3z^2w^4 + 5w^6$

iv) $\frac{5a^9b^6 - 25a^3b^4}{5a^5b^4} = \frac{5a^9b^6}{5a^5b^4} - \frac{25a^3b^4}{5a^5b^4} = a^{9-5}b^{6-4} - 5a^{3-5}b^{4-4}$

$= a^4b^2 - 5a^{-2}b^0 = a^4b^2 + \frac{5}{a^2}$

v) $\frac{\sqrt{15}x^7 - \sqrt{20}x^4}{\sqrt{10}x^3} = \frac{\sqrt{15}x^7}{\sqrt{10}x^3} - \frac{\sqrt{20}x^4}{\sqrt{10}x^3} = \frac{\sqrt{5}\times\sqrt{3}}{\sqrt{5}\times\sqrt{2}}x^{7-3} - \frac{2\sqrt{5}}{\sqrt{5}\times\sqrt{2}}x^{4-3} = \frac{\sqrt{3}}{\sqrt{2}}x^4 - \sqrt{2}x$

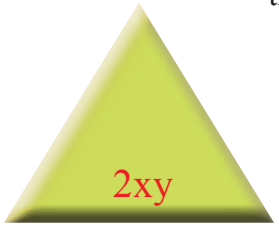
vi) $\frac{64r^4v^2 - 16r^2v^2 - 32}{8rv} = \frac{64r^4v^2}{8rv} - \frac{16r^2v^2}{8rv} - \frac{32}{8rv} = 8r^3v - 2rv - \frac{4}{rv}$

vii) $\frac{\sqrt{49}x^2y^2 + \sqrt[3]{8}x^5y^6 - 7xy}{14xy} = \frac{\sqrt{49}x^2y^2}{14xy} + \frac{\sqrt[3]{8}x^5y^6}{14xy} - \frac{7xy}{14xy}$

$= \frac{7x^2y^2}{14xy} + \frac{2x^5y^6}{14xy} - \frac{7xy}{14xy} = \frac{1}{2}xy + \frac{1}{7}x^4y^5 - \frac{1}{2}$

Example 4 **In the near by figure below if the base of the triangle 2xy and area x^2-xy+y^2, find the height in meters?**

height of triangle $= \frac{2 \text{ the area of triangle}}{\text{the base}}$



$\frac{2(x^2 - xy + y^2)}{2xy} = \frac{2x^2 - 2xy + 2y^2}{2xy}$

$= \frac{2x^2}{2xy} - \frac{2xy}{2xy} + \frac{2y^2}{2xy} = \frac{x}{y} - 1 + \frac{y}{x}$ meters

Make sure of your understanding

Find the result of the division where the denominator is not equal to zero for each of the following:

1. $\frac{15m^7n^6}{24m^6n^3}$
2. $\frac{-24x^3y^3}{36x^2y^4}$
3. $\frac{8z^5}{-12z^2}$

Questions 1-6 are similar to example 2

4. $\frac{56h^{12}k^{10}}{-21h^8k^5}$
5. $\frac{-18r^2v^6}{-15r^2v^2}$
6. $\frac{72x^5y^6}{24x^3y^4}$

Questions 7-12 are similar to example 3

7. $\frac{-84x^6y^5 + 12x^5y^5}{4x^5y^5}$
8. $\frac{36m^7 - 25m^6 + 18m^5}{6m^7}$
9. $\frac{\sqrt{5}\,h^6 - \sqrt{2}\,gh^2}{10h^3}$
10. $\frac{\sqrt{18}z^4w^6y^5 - 15z^5w^4y^6}{3x^5y^5}$
11. $\frac{-2m^5n^6 + m^7n^4}{4m^4n^5}$
12. $\frac{\sqrt[3]{-8}\,v^7 - 20v^4}{4v^3}$

Solve the Exercises

Find the result of the division where the denominator is not equal to zero for each of the following:

13. $\frac{12b^5}{4b^2}$
14. $\frac{-32m^5n^2}{-8m^3n^4}$
15. $\frac{81x^7y^8z^6}{27x^7y^7z^3}$
16. $\frac{36m^5n^5}{3m^6n^3}$
17. $\frac{49r^2v^2}{7rv}$
18. $\frac{\sqrt{36}\,h^3k^5}{6hk}$
19. $\frac{-r^4v^3 - 4r^6v^5}{3r^3v^5}$
20. $\frac{6w^5z^2 + 9w^2z^2}{3w^5z^3}$
21. $\frac{13a^9b^6c^5 - 52a^7b^4c^6}{13a^5b^3c^2}$
22. $\frac{8x^5}{12x^{-4}}$
23. $\frac{\sqrt{36}m^6 + \sqrt[3]{27}\,m^3 + 8m^2}{4m^{-2}}$
24. $\frac{72n^7 - 63n^6 - 54n^5}{9n^5}$
25. $\frac{48z^4 + 16z^3 + 8z^2}{4z^2}$
26. $\frac{12m^5n^4 + 9m^4n^2 + 12m^2n^2}{3m^2n^2}$

Solve the problems

27 **Geometry:** Cylinderical shaped can if you have the rule $h = \frac{m - 2\pi r^2}{2\pi r}$ where m is the total area for the can, r is the radius of the base r =5cm, m=280 cm^2 , find the value of h.

28 **Geometry:** If $z = \frac{n^2 - m^2}{n}$, find the value of z if you knew that n = 5, m = 4 .

29 Prove that the algebraic expression does not consists of a variable

$$\frac{25r^2v^4 - 15r^3v^2 - 5r^2v^2}{5r^2v^2} - \frac{25v^2 - 15r - 5}{5}$$

30 **Fireworks:** A fire arrow took off vertically through space according to the low of $v = \frac{h + 5t^2}{t}$ where v represents the speed of arrow while taking off (m/sec) and h represents the height of the arrow (m),(t) represents the time (sec) find the speed of the arrow while taking off if you knew that its height is 275 through 5 seconds.

Think

31 **Challange:** Simplify the algebraic expression:

$$\frac{-8z^4 - 10z^3 + z^2}{2z^2} + \frac{4z^3 + 5z^2 + 6z}{z}$$

32 **Correct the mistake:** Ahmad divided the algebraic term: $\frac{20x^3y^2 + 12x^2y^2 + 28xy}{4xy}$ so the result was $5x^2y + 3x^2y^2 - 7x$, show Ahmad's mistake and correct it.

Write

An example about dividing an algebraic expression of three terms by an algebraic term and find the result.

Lesson [3-5] Factoring an Algebraic Expressions

Idea of the lesson:
*Factoring an algebraic expression by common factor.
*Factoring an algebraic expression by using difference between two squares.
*Factoring an algebraic expression by using difference between two square expressions.

Vocabulary:
*Common factor
*Difference between two square.
*Difference between two square expressions.

Learn

The bigest pyramid is located in Egypt where the pyramid is quaternary and with total area is $m=x^2+2xy$ where x represents the length of the base y represents the height of one of the faces, how can I find the greatest common factor for the expression that represents the total area of the pyramid?

[3-5-1] Factoring an Algebraic Expression by using a Common Factor (G.C.F)

You have previously learned multiplying an algebraic term with an algebraic expression and multiplying an algebraic expression with an algebraic expression and in this lesson you will learn factoring the greatest common factor (G.C.F) and its the opposite of multiplication operation and the greatest common factor include of the coefficient and the variables with smallest index.

Example 1 **Find the greatest common factor (G.C.F) which represents the area:**

$x^2 + 2xy$ *I will find the greatest common factor for expression which is x*

$\frac{x^2}{x} + \frac{2xy}{x}$ *I divide each algebraic term by the greatest common factor*

$x+2y$ *Then the factoring is x(x+2y)*

$x(x+2y) = x^2 + 2xy$ check: Multiplying the result by greates common factor.

Example 2 **Factorize the algebraic expression by using the greatest common (G.C.F) factor and check your answer:**

i) $25c^2d + 45d - 5cd^3$

$= 5d(\frac{25c^2d}{5d} + \frac{45d}{5d} - \frac{5cd^3}{5d})$, G.C.F= 5d

$= 5d\,(5c^2 + 9 - cd^2)$ Check

$= 5d(5c^2 + 9 - cd^2) = 25c^2d + 45d - 5cd^3$

ii) $28h^2k - 8k + 12$ G.C.F= 4

$= 4(\frac{28h^2k}{4} - \frac{8k}{4} + \frac{12}{4})$

$= 4(7h^2k - 2k + 3)$

$4(7h^2k - 2k + 3) = 28h^2k - 8k + 12$ Check

iii) $\frac{1}{4}z^2w^2 + \frac{5}{16}zw + \frac{3}{8}zw^3$ G.C.F= $\frac{1}{4}zw$

$= \frac{1}{4}zw(\frac{\frac{1}{4}z^2w^2}{\frac{1}{4}zw} + \frac{\frac{5}{16}zw}{\frac{1}{4}zw} + \frac{\frac{3}{8}zw^3}{\frac{1}{4}zw})$

$= \frac{1}{4}zw(zw + \frac{5}{4} + \frac{3}{2}w^2)$

$= \frac{1}{4}zw(zw + \frac{5}{4} + \frac{3}{2}w^2)$ Check

$= \frac{1}{4}z^2w^2 + \frac{5}{16}zw + \frac{3}{8}zw^3$

[3-5-2] Factoring an Algebraic Expression Using Difference between Two Squares

You have previously learned factoring algebraic expression using greatest common factor and in this section you will learn factoring by using difference between two squares or the both methods together.
i) $a^2 - b^2=(a+b)(a-b)$
ii) $ka^2 - kb^2= k(a^2 - b^2)= k(a+b)(a-b)$

Example 3 **Factorize each expression using the difference between two squares:**

i) x^2-y^2
$= (x+ y)(x-y)$

ii) $z^2-36 = (z)^2 - (6)^2$
$= (z-6)(z+6)$

iii) $4h^2 - 81w^2$
$=(2h-9w)(2h+9w)$

iv) $y^2-7= (y)^2- (\sqrt{7})^2$
$= (y-\sqrt{7})(y+\sqrt{7})$

v) $225m^2n^2 - 625a^2b^2$ G.C.F= 25
$= 25(9m^2n^2-25a^2b^2)$
$= 25(3mn-5ab)(3mn+5ab)$

vi) $25h^4-5a^2$ G.C.F= 5
$= 5[(\sqrt{5}\,h^2)^2 -(a)^2]$
$= 5(\sqrt{5}\,h^2-a)(\sqrt{5}\,h^2+a)$

vii) $144z^2w^2-2 = {}^2(72z^2w^2-1)$
$= 2(\sqrt{72}\,z\,w -1)(\sqrt{72}\,z\,w +1)$

[3-5-3] Factoring an algebraic Expression Using Difference between Two Square Expressions

You have previously learned factoring using difference between two squares, and for factoring the more complex expressions you can use factoring of the difference between two square expressions for making it easier in some operations.

Example 4 **Factorize the expression by using the difference between two square expressions:**

i) $(x+y)^2 - (x+z)^2$
$= [(x+y)+(x+z)][(x+y)-(x+z)]$ *Using difference of two squares*
$= (x+y+x+z)(x+y-x-z)$ *Open brackets*
$= (2x+y+z)(y-z)$ *Simplify the expression*

ii) $(2m+3)^2 - (3m-4)^2$
$= [(2m+3)+(3m-4)][(2m+3)-(3m-4)]$ *Using difference of two squares*
$=(2m+3+ 3m -4)(2m+3 -3m +4)$ *Open brackets*
$=(5m-1)(-m+7)$ *Simplify the expression*

iii) $(3w+5)^2 - (w+4)^2$
$= [(3w+5)+(w+4)][(3w+5)-(w+4)]$ *Using difference of two squares*
$=(3w+5+w+4)(3w+5-w-4)$ *Open brackets*
$=(4w+9)(2w+1)$ *Simplify the expression*

Make sure of your understanding

Factorize the expression by using the greatest common factor (G.C.F):

1 $12x+9$ 2 $36y-18$ 3 $15m+21$

4 $28z^2 -7z +7$ 5 $16n^2m +12m-4$ 6 $\sqrt{3}\, h^2- 3h$

Questions 1-6 are similar to example 2

Factorize the expression by using the difference of two squares:

7 $h^2- 16$ 8 $4y^2- 9$ 9 $169x^2 -11$ 10 $81a^2- b^2$

11 $49 -y^2$ 12 $25h^2- 9k^2$ 13 $36 - 25r^2$ 14 $14z^2 -2$

Questions 7-14 are similar to example 3

Factozie the expression by using difference of two square expressions:

15 $(3m+1)^2 - (n+6)^2$ 16 $(2x+y)^2 - (4x+3y)^2$

17 $(x-2)^2 - (x+5)^2$ 18 $(3-z)^2 - (6-z)^2$

Questions 15-18 are similar to example 4

Solve the Exercises

Factorize the algebraic expression by using the greatest common factor (G.C.F) and check your answer:

19 $2x^5 -6x^2+10x^3$ 20 $-24y^6+8y^5-4y^4$

21 $64h^2k^2-16hk$ 22 $15m^4n^4+6mn^3+3m^2n^2$

23 $72x^3+18x^2+9$ 24 $36m^2n^2 +4mn+8$

Factorize each expression using the difference between two squares:

25 $4x^2-16$ 26 $81-25n^2$

27 $36h^4- 4$ 28 $169a^2-3$

29 $625b^2-2$ 30 k^2-5

Factorize the expression by using greates common factor (G.C.F) and then difference between two squares:

31 $5y^2 -20$ 32 $12x^2 -27$

33 $14w^2- 2$ 34 $18k^2 -32$

Factorize the expression by using the difference between two square expressions:

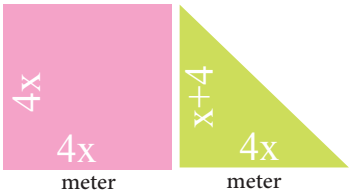
35 $(3x+5)^2-(x+4)^2$ 36 $(5y-3)^2-(32-y)^2$

37 $(4m+n)^2-(5m+2n)^2$ 38 $(6z+1)^2-(w+5)^2$

Solve the problems

39 If the total area for the near by figure in the rule was $x = r^2 + \frac{1}{8}\pi r^2$. Factorize the expression (x) by using the greates common factor (G.C.F) and find the value of x when r =4 .

40 If we had a square which the lengh of the side is 4x and a right-angled triangle which the length of its right sides 4x, x+4 . Find the area of the square and the triangle together and then factorize the expression.



41 Cylinderical shaped can which its total area is $(2\pi h + 2r^2\pi)$ square meter. Simplify the expression by using the greatest common factor (G.C.F) and then find the area when h=7, r=3 .

Think

42 Challenge: If the expression was $y = 5x^3-15x^2+6$, $z=5x^4-10x^3-6$ find the result of the addition of the two expressions and then factorize the result by using the greatest common factor and is it possible to factorize each z,y ?

43 Geometry: A square which length of its side is y cm and a rectangle which its length is more than the length of the square side by 3cm and its width is less than the length of the square side by 3cm. What is the area of the rectangle? and does the area represent difference of two squares?

44 Where is the mistake: Qasim and Hussam factorized the expression $36m^4-100n^2$ by using difference of two squares and wrote the result as:

Qasim Solution	**Hussam Solution**
$36m^4-100n^2$	$36m^4-100n^2$
$(6m^2+10n)(6m^2-10n)$	$(6m^2-10n)(6m^2-10n)$

Show which one wrote the correct result.

Write

Two different methods for factoring the following algebraic expression:

$(4x^2 - 4y^2)$

Chapter Test

Find the result of addition or subtraction of the following algebraic expressions:

1 $(\sqrt[3]{8}\,x^2y^2+ 4xy-2)+(\sqrt[3]{27}\,x^2y^2- 6xy+3)$

2 $(\sqrt{9}\,r^3v^2+ 12gh-6) + (\sqrt{100}\,r^3v^2- 2gh+1)$

3 $(\frac{1}{4}m^4n^4+ 7hk+8)+(\frac{1}{16}m^4n^4+9hk-12)$

4 $(\sqrt{5}\,h^2k^3- 20x+2)+(5\sqrt{5}\,h^2k^3+ 5x -3)$

5 $(\sqrt[3]{125}\,a^2b^2+4b+3)-(15a^2b^2+ 3b-6)$

6 $(\frac{3}{12}r^6v^3- 24h -6)-(\frac{3}{12}r^6v^3- 8h +1)$

Find the result of the multiplication for the following algebraic terms:

7 $(6x^2y)(12xy)$

8 $(\frac{1}{4}gh)(32gh)$

9 $(\sqrt{25}\,a^4b^2)(5a^2b^2)$

10 $(\sqrt[3]{1000}\,h^4k^3)(10hk)$

11 $\sqrt{7}\,rv(\sqrt{7}\,r^2v^2+rv+2\,)$

12 $\frac{1}{9}wz(81w^{-3}z^{-3}+zw+3)$

13 $-10z^2w^2(\sqrt{100}\,zw+10)$

Find the result of multiplying the two following algebraic expressions:

14 $(2x+y)(x+y)$

15 $(zw+4)(zw+5)$

16 $(\frac{3}{5}gh-3)(\frac{1}{5}gh-5)$

17 $(3x+4)(x^2+ 3x+1\,)$

18 $(9r-1)(2r^4- 3r+1\,)$

19 $(4m^2n^2-n)(4m^{-2}n^{-2}-n+2)$

Find the result of multiplying the two algebraic expressions using vertical method for the following:

20 $(\frac{4}{16}a^2b^2+ \frac{25}{5})(a^2b^2- \frac{1}{2})$

21 $(5y^2-y\,)(-3y^2+y+2)$

22 $(8z^{-2}w^{-2}+ 4)(2zw+2)$

Find the result of the dividing the following algebraic expressions:

23 $\frac{-35x^4}{5x^{-2}}$

24 $\frac{12y^3}{36y^2}$

25 $\frac{\sqrt{100}\,r^6}{\sqrt{25}\,r^{-8}}$

26 $\frac{30v^4}{6v^{-8}}$

27 $\frac{-35h^2}{5h^{-7}}$

28 $\frac{56m^2n^4-7m^2n^2+42}{7mn}$

29 $\frac{-25zw^2+10zw -5}{5zw}$

30 $\frac{81- 27ab -3a}{9b}$

Factorize the algebraic expressions by using (G.C.F):

31 $14y^2+ 2y-24$

32 $4z^4w^2- 16z^2w^2+48zw$

33 $100c^3d^3+50c^2d^2+ 25cd$

Factorize the algebraic expressions by using difference of two squares:

34 $4z^2 - 16$

35 $144x^4 - 64$

36 $25r^2 - 3$

37 $81g^2h^2 - 36$

Factorize the algebraic expressions by using (G.C.F) and difference of two squares:

38 $18v^2 - 32$

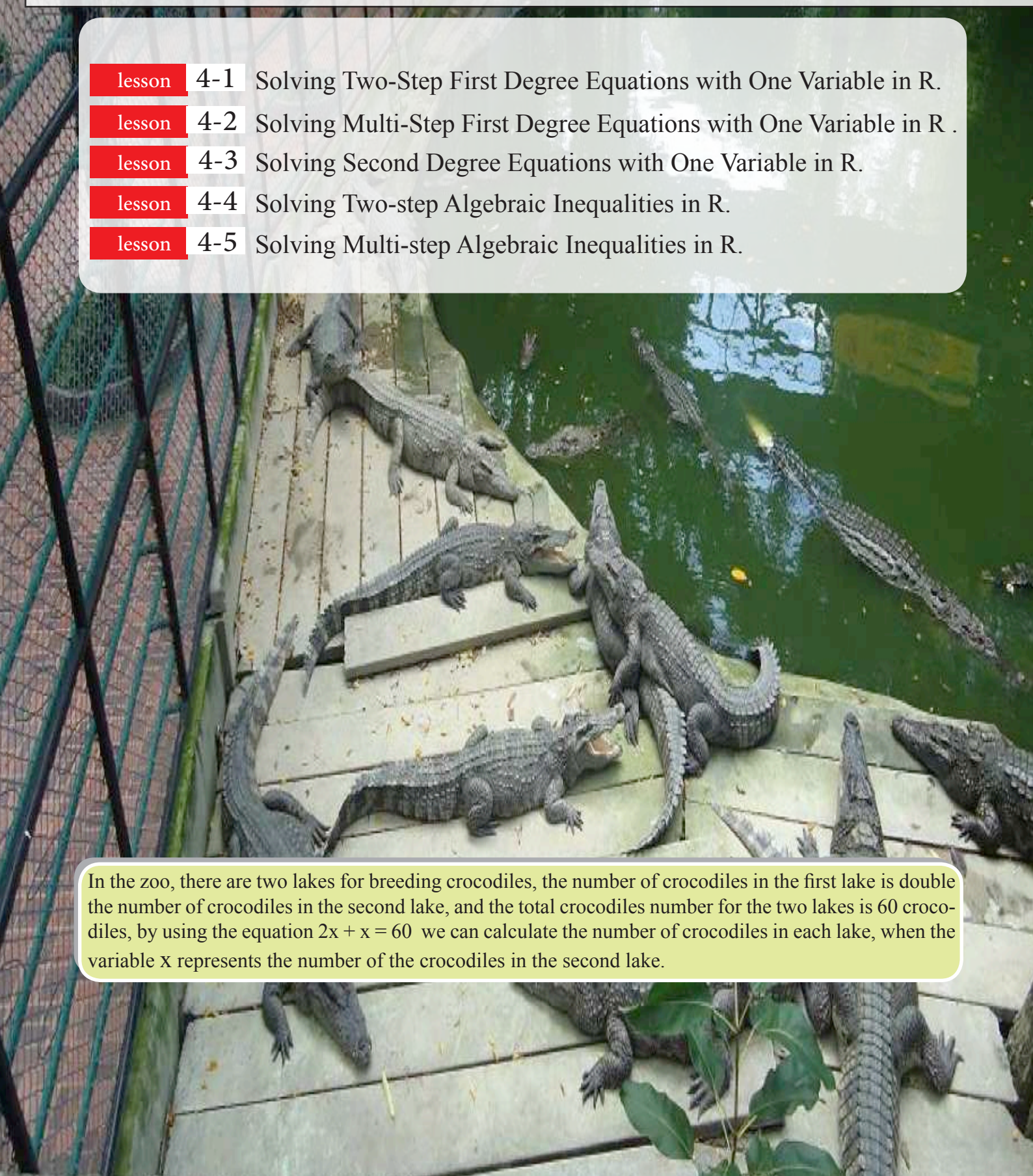
39 $50z^2 - 2$

40 $400x - 4xy^2$

41 $81w - 169w^3$

Chapter 4

Inequalities and Equations



In the zoo, there are two lakes for breeding crocodiles, the number of crocodiles in the first lake is double the number of crocodiles in the second lake, and the total crocodiles number for the two lakes is 60 crocodiles, by using the equation $2x + x = 60$ we can calculate the number of crocodiles in each lake, when the variable x represents the number of the crocodiles in the second lake.

Pretest

Find the value of algebraic statement for each of the following by using the given variable value:

1 $7x - 3^2 + 7$, $x = 4$

2 $3(y - 2) - 10$, $y = -5$

3 $2^3 (n - 6) - 15$, $n = -16$

4 $(36 \div d) - 4^2(1 - d)$, $d = 6$

5 $|-8| + y^3 - 24$, $y = 3$

6 $3v \div 5 - |-12| \div 2$, $v = -5$

Solve the addition and subtraction equations by using mental arithmetic:

7 $x + 21 = 21$

8 $y - 9 = 11$

9 $80 - z = 20$

10 $|-10| + x = 33$

11 $m - \sqrt{16} = 0$

12 $\sqrt{49} - n = 0$

Solve the addition and subtraction equations by using the relation between addition and subtraction.

13 $w + 132 = 61$

14 $m - 22 = -32$

15 $y + 14 = |-10|$

16 $63 - x = |-43|$

17 $\sqrt{64} - h = 8$

18 $d + \sqrt[3]{27} = 8$

Solve the multiplication and division equations by using the relation between multiplication and division:

19 $3k = 15$

20 $S \div 8 = -9$

21 $\sqrt{4}\ n = -24$

22 $|-7|\ m = 63$

23 $-88 \div y = |-11|$

24 $x \div \sqrt[3]{8} = 8$

Solve the following equations in Q:

25 $7y - 4 = 51$

26 $\sqrt{16} - 2x = 21$

27 $3x \div 9 = 5 + \frac{1}{2}$

28 $\sqrt[3]{27} \div m = 5^2 - 1$

29 $|-18|\ h = 72 \div (-9)$

30 $z \div |-11| = 3^3 \div 22$

Write one example for each property for the following properties:

31 For each $a, b, c \in Q$ if $a \leq b$ then $a + c \leq b + c$

32 For each $a, b, c \in Q$ if $a > b$ and $c < 0$ then $ac < bc$

33 For each $a, b, c \in Q$ if $a \geq b$ and $c > 0$ then $\frac{a}{c} \geq \frac{b}{c}$

Use Inequalities properties to solve each of the following inequalities:

34 $y - 10 < 12$

35 $x + 5^2 \geq 18$

36 $-9 + m \leq 0$

37 $\sqrt[3]{8} + h \leq 26$

38 $\frac{x}{11} > \frac{1}{3}$

39 $-7y \leq 19$

40 $\frac{y}{7} < -10$

41 $4x + 10 < -48$

42 $-3(n - 7) \geq 21$

Lesson [4-1] Solving First Degree Equations with One Variable Two-Step in R

Idea of the lesson:
* Solving Two-Step First Degree Equations with One Variable.

Vocabulary:
* One variable equation.
*First degree equation.
*Solving equation.
*Check.

Learn

Ahmad has 5 cages of cnary birds and Firas has 4 cages of birds. Ahmad took out 8 birds from his cages and added them to Firas's cage so each of them will have equal numbers of birds, if the birds are distributed equally in the cages, find the number of birds in each cage.

[4-1-1] Solving the Equations by using Addition and Subtraction

The first degree equation with one variable it is the equation that contains one variable it,s power is one
Solving equation means that finding the variable value in it, and for solving the equation you have to put the variables in a side and the numbers in the other side.

Example (1) **Find the number of the birds in each cage.**

Assume that the number of the birds in each cage is x
So the equation which represents the problem is $5x - 8 = 4x + 8$

First method: The vertical method

$5x - 8 = 4x + 8$ — *Write the equation*

$-4x \quad -4x$ — *Add to each side* $-4x$

$x - 8 = +8$

$+ 8 = +8$ — *Add* $+8$ *to each side*

$x = 16$

So the number of the birds in each cage is 16 birds.

Second method: The horizontal method

$5x - 8 = 4x + 8$ — *Write the equation*

$5x - 8 - 4x = 4x - 4x + 8$ — *Add to each side* $-4x$

$x - 8 = 8$

$x - 8 + 8 = 8 + 8$ — *Add* $+8$ *to each side*

$x = 16$

So the number of the birds in each cage is 16 birds.

Check: Replace the variable value ($x = 16$) that you have got it from the solving of the equation:

$5x - 8 \overset{?}{=} 4x + 8$

$5(16) - 8 \quad 4(16) + 8$

$72 = 72$ ✓

Example (2) **Solve the following equations by using addition and subtraction:**

i) $3y - 12 = 2y - |-30| \Rightarrow 3y-2y-12 = 2y-2y-|-30| \Rightarrow y - 12 = -30$

$\Rightarrow y-12+12 = -30+12 \Rightarrow y = -18$

ii) $20 + 2h = 3h - 3^2 \Rightarrow 20 + 2h - 2h = 3h - 9 - 2h \Rightarrow 20 = h - 9$

$\Rightarrow 20 + 9 = h - 9 + 9 \Rightarrow h = 29$

iii) $2x + 2\sqrt{3} = x - 3\sqrt{3} \Rightarrow 2x + 2\sqrt{3} - x = x - 3\sqrt{3} - x \Rightarrow x + 2\sqrt{3} = -3\sqrt{3}$

$\Rightarrow x + 2\sqrt{3} - 2\sqrt{3} = -2\sqrt{3} - 3\sqrt{3} \Rightarrow x = -5\sqrt{3}$

iv) $|-3|m = 10 - \sqrt[3]{-8}\ m \Rightarrow 3m = 10 + 2m \Rightarrow 3m - 2m = 10 + 2m - 2m \Rightarrow m = 10$

[4-1-2] Solving the Equations by using Multiplication and Division

Use the relation between the multiplication and division to find the Variable value in first degree equation with one variable.

Example (3) **Colouring pencils: Sura bought 5 cans of colouring pencils, she kept for herself 3 pencils from each can and gave the remaining to her four sisters equally, each of her sisters took 15 pencils. What is the number of the pencils in each can?**

Assume that the number of the pencils in each can is n

So the equation that represents the problem is $5(n-3) \div 4 = 15$

$5(n - 3) \div 4 = 15$

$\frac{5(n-3)}{4} = 15$

$\frac{5(n-3)}{4}(4) = 15(4)$ *Multiply each side by 4*

$5(n - 3) = 60$ *Multiply 5 with inside the bracket*

$5n - 15 = 60$

$5n - 15 + 15 = 60 + 15$ *Add +15 to each side*

$5n = 75$ *Divide each side by 5*

$n = 15$ So the number of pencils in each can is 15 pencils

Check: Replace the variable value (n=15) that you have got it from the solving of the equation:

$5(n - 3) \div 4 \overset{?}{=} 15$

$5(15 - 3) \div 4 \quad 15$

$15 = 15$ ✓

Example (4) **Solve the following equations by using multiplication and division:**

i) $x \div 12 = 4 \Rightarrow x = 4(12) \Rightarrow x = 48$

ii) $\sqrt{36}\, y \div 2 = |-5| \Rightarrow 6y \div 2 = 5 \Rightarrow 6y = 10 \Rightarrow \frac{6y}{6} = \frac{10}{6} \Rightarrow y = \frac{10}{6} \Rightarrow y = \frac{5}{3}$

Example (5) Solve the following equations:

i) $5y + 7 = 3y - 2^3 \Rightarrow 5y - 3y = -8 - 7 \Rightarrow 2y = -15 \Rightarrow y = \frac{-15}{2}$

ii) $\sqrt{16}\,x - 3\sqrt{7} = \sqrt{9}\,x \Rightarrow 4x - 3\sqrt{7} = 3x \Rightarrow 4x - 3\sqrt{7} - 3x = 3x - 3x$

$\Rightarrow x - 3\sqrt{7} = 0 \Rightarrow x - 3\sqrt{7} + 3\sqrt{7} = 0 + 3\sqrt{7} \Rightarrow x = 3\sqrt{7}$

iii) $3(6t + 5) = 3(3t + 12) \Rightarrow 18t + 15 = 9t + 36 \Rightarrow 18t - 9t = 36 - 15$

$\Rightarrow 9t = 21 \Rightarrow t = \frac{21}{9} \Rightarrow t = \frac{7}{3}$

iv) $\frac{2x}{5} = \frac{7}{20} \Rightarrow \frac{2x(5)}{5} = \frac{(5)7}{20} \Rightarrow 2x = \frac{7}{4} \Rightarrow \frac{2x}{2} = \frac{7}{4\,(2)} \Rightarrow x = \frac{7}{8}$

Make sure of your understanding

Solve the following equations by using addition and subtraction and check the solution:

1. $4x - 10 = 3x + 20$
2. $25 + m = 2m - 16$
3. $2y + 2^4 = y - 3$
4. $\sqrt{49} - d = 21 - 2d$
5. $|-13|\,x = \sqrt[3]{-27} + 12x$
6. $7y - 6^2 = 6y - 36$
7. $3h + 4\sqrt{5} = 2h + 7\sqrt{5}$
8. $\sqrt{9}\,x = |-17| + 2x$

Questions 1-8 are similar to examples 1,2

Solve the following equations by using multiplication and division and check the solution:

9. $2x \div 16 = \frac{1}{3}$
10. $\sqrt[3]{8}\,y \div |-6| = 3^2$
11. $3m - 9 = 5 - 2m$
12. $\sqrt{3}\,z \div 12 = \sqrt{3} \div 5$
13. $\frac{\sqrt{5}\,x}{2} = \frac{1}{3}$
14. $\frac{6y}{5\sqrt[3]{27}} = \frac{6}{5}$
15. $2(h + 5) = \sqrt{64}$
16. $|-14|\,n = 63 \div (-9)$

Questions 9-16 are similar to examples 3-5

Solve the Exercises

Solve the following equations by using addition and subtraction and check the solution:

17. $5y - 20 = 4y + 2$
18. $6x + 3^2 = 5x - 5$
19. $f = \sqrt[3]{-64} + 2f$
20. $3z - 2\sqrt{3} = 2z + 7\sqrt{3}$

Solve the following equations by using multiplication and division and check the solution:

21. $4y \div 24 = \frac{1}{5}$
22. $\sqrt[3]{27}\,z \div |-7| = 3^3$
23. $\sqrt{2}\,x \div 9 = \sqrt{2} \div 5$
24. $\frac{\sqrt{3}\,y}{2} = \frac{1}{3}$
25. $5(k + 6) = \sqrt[3]{-125}$
26. $|-8|\,n = 72 \div (-12)$

Solve the problems

27 **Buying:** Anwar bought a car with a price of 28 million Dinars. He paid 6 millions in advance. He is going to pay the rest amount as installments for 11 months. Write an equation which represents the problem and solve it in order to find the value of the monthly installment.

28 **Diving:** A research submarine descended under surface of the sea until it reached to $\frac{6}{10}$ from the depth of sea. What is the depth of the sea if the research diver stoped at the depth of 180m from the surface of the sea?

29 **Zoo:** Solve the equation $3n + 15 = 2n+32$ to find the value of n that represents the number of the monkeys in the zoo.

30 **Gardens:** A rectangle shaped area planted with flowers its length is three times its width. What are the dimentions of the area that planted with flower if its perimeter is 52m?

Think

Challenge: Solve the following equations:

31 $\sqrt{2}\ x - 3\sqrt{2} = 5\sqrt{2} - \sqrt{2}\ x$

32 $|-11|\ f = \sqrt[3]{-64} + 12f$

33 **Correct the mistake:** Eman solved the following equation : $\frac{\sqrt{5}}{5^3} = \frac{v}{5^2}$ and she wrote $v = \frac{1}{5}$. Determine Eman's mistake and correct it.

34 **Numerical sense:** Sameer's age is double Saad's age, after 6 years Sameer will be 22 years old. How old is Saad before 6 years?

Write

Problem that represents the following equation and find the solution:

$6n - 50 = 20$

Lesson [4-2] Solving First Degree Equations with One Variable Multi-Step in R

Idea of the lesson:
*Solving Multi-Step First Degree Equation with One Variable.

Vocabulary:
*Distributive Property
*Associative Property

Learn

There are 600 seals on a beach, its number increased about $\frac{1}{6}$ as a result of giving birth, and then they had attacked by the blue whiles, their numbers decreased to 550 seals. What is the number of the missing seals?

[4-2-1] Solving the Equations which involving variable in one side of Equation or both sides

To solve an equation that contains one variable in one side, isolate the part that contains the variable in one side of the equation then make its coefficint equals to one by using real number properties (distributive , associative ,...).

Example (1)

To find the number of the missing seals:
Assume that the number of the missing seals is n
The equation will represent the problem in this way

$600 + \frac{1}{6} \times 600 - n = 550$

$600 + 100 - n = 550$ *Find result of* $600 \times \frac{1}{6}$

$700 - n = 550$

$-n = 550 - 700$ *Isolate the variable in one side*

$-n = -150$ *Multiply both side by (-1)*

$n = 150$

So the number of the missing seals are 150 seal.

Check: replace the variable value with (n = 150) that you have got it by solving in the left side (LS) of the equation:

$600 + \frac{1}{6} \times 600 - n = 550$

$LS = 600 + \frac{1}{6} \times 600 - n = 600 + 100 - 150 = 550 = RS$ (right side) ✓

Example (2)

Solve the following equations by using real numbers properties:

i) $2(z - 8) + 16 = |-36| \Rightarrow 2z - 16 + 16 = 36 \Rightarrow 2z = 36 \Rightarrow z = 36 \div 2 \Rightarrow z = 18$

ii) $4(x - 5\sqrt{3}) = 3x - 2\sqrt{3} \Rightarrow 4x - 20\sqrt{3} = 3x - 2\sqrt{3}$

$\Rightarrow 4x - 3x = 20\sqrt{3} - 2\sqrt{3} \Rightarrow x = 18\sqrt{3}$

iii) $\frac{1}{5}(3y + 10) - 7 = \frac{2}{5}(y - 15) \Rightarrow \frac{3}{5}y + 2 - 7 = \frac{2}{5}y - 6$

$\Rightarrow \frac{3}{5}y - \frac{2}{5}y = 5 - 6 \Rightarrow \frac{1}{5}y = -1 \Rightarrow y = -5$

[4-2-2] Solving the Equations involving Absolute Value

Solving equation that contains absolute variable value for example $|x| = 3$ means finding the distance between x and the number 0 on a number line

So the solution of the equation $|x| = 3$ is either $x = 3$ or $x = -3$ and the solution set is $S = \{ -3 , 3 \}$

Example (3)

zoo: The equation $|x-27| = 2$ represents the temperature of the place that is specified for the snakes.
Find the greatest and the least temperature for the snake's place in the zoo.

First case: $x-27 = 2$

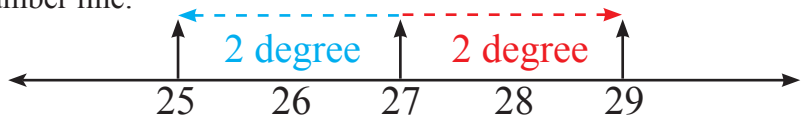
$x = 2+ 27 \Rightarrow x = 29$

So the greatest temperature is $29^{o}C$

Second case: $x-27 = -2$

$x = 27- 2 \Rightarrow x = 25$

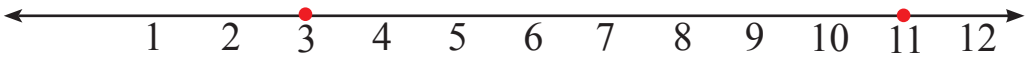
So the least temperature is $25^{o}C$
and it could be represented on a number line:



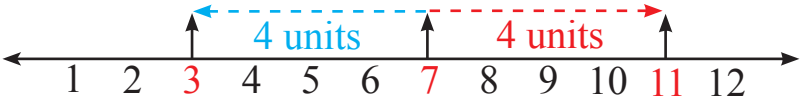
Set of the solution is $\{25, 29\}$

Example (4)

Write the absolute value equation that its graphic representation on a number line is:



Find a point that is the same distance between 11 and 3 , and this point is the middle distance between the two numbers so the number is 7,



So the requested equation is $|x-7| = 4$

Example (5)

Solve the following equations:

i) $|y + 9| = 5 \Rightarrow$ or $\begin{cases} y+9=5 \Rightarrow y= -4 \\ y+9=-5 \Rightarrow y= -14 \end{cases} \Rightarrow \{ -4,-14\}$ set of the solution

ii) $|2n - 7| = 6 \Rightarrow$ or $\begin{cases} 2n-7 = -6 \Rightarrow 2n=1 \Rightarrow n = \frac{1}{2} \\ 2n-7 = 6 \Rightarrow 2n=13 \Rightarrow n = \frac{13}{2} \end{cases} \Rightarrow \left\{ \frac{1}{2} , \frac{13}{2} \right\}$ set of the solution

iii) $|x - 5| = -3$

That means the distance between x and 5 equals to -3
Since the distance cannot be negative, so the set of the solution for this equation is an empty set ($\varnothing$).

Make sure of your understanding

Solve the following equations by using the real numbers properties:

1. $8y - 12 = 4y + 12$
2. $|-15| + z = 3z + 15$
3. $3(y + 5^2) = y + 70$
4. $3\sqrt{2} - x = x - 5\sqrt{2}$
5. $\sqrt{5}(n + 3) = 4\sqrt{5}$
6. $\sqrt{25}\,y = 3(y - 15) + 2$

Questions 1-6 are similar to example 2

Solve the following equations by using the properties and check the answer:

7. $2(x + 20) = 5(x - 10)$
8. $\sqrt[3]{64}\,y \div |-6| = 8\sqrt{2}$
9. $\frac{1}{5}(t - \sqrt{25}) + 3 = 2(4 - t)$
10. $\frac{2v}{1 + \sqrt[3]{27}} = \frac{3v}{8}$

Questions 7-10 are similar to examples 1,2

Solve the following equations:

11. $|x - 22| = 8$
12. $|4y + 30| = \sqrt{49}$
13. $|\frac{1}{5}m + 9| = \sqrt[3]{-27}$
14. $|3z - 9| = 2^3$

Questions 11-14 are similar to examples 3,5

Solve the Exercises

Solve the following equations by using the real numbers properties:

15. $4x + 8 = 12 - 2x$
16. $7(t + 1^2) = \frac{1}{2}t - 2$
17. $\sqrt{7}(v + 8) = 2\sqrt{7}$

Solve the following equations by using the properties and check the answer:

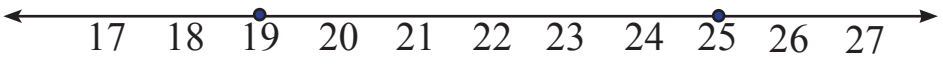
18. $4(y - 15) = 3(y + 15)$
19. $\frac{1}{2}(x - 5) + \frac{5}{2} = \frac{1}{4}(x - 4)$

Solve the following equations:

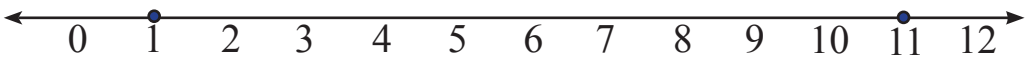
20. $|y - 13| = 9$
21. $|\frac{1}{2}m + 9| = \sqrt[3]{-125}$

Write the absolute value equation that its graphics representation on a number line is:

22.



23.



Solve the problems

24 **Weather:** The average temperature at the north of Iraq in February $2^{o}C$, it decreases or increases by $3^{o}C$. Write an equation that represents the least and the greatest temperature for February.

25 **Subway:** The average speed for a subway train is 60km/h and its speed decreases by the curves by amount 20km/h, and its speed increases when the way is straight by amount 20km/h. Write an equation to find the greatest and least speed for the train.

26 **Buying:** Waleed wants to buy a computer with price 650000 dinars, he has 200000 and every week he saves 50000 dinars. After how many weeks Waleed is going to collect the computer's money?

Think

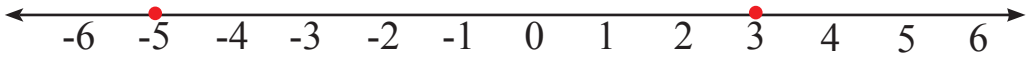
Challenge: Solve the following equations:

27 $\dfrac{\sqrt{3}\,x}{4+\sqrt[3]{-27}} = \dfrac{2\sqrt{3}\,x}{5}$

28 $|4y - 9| = |26 - 5\sqrt{64}\,|$

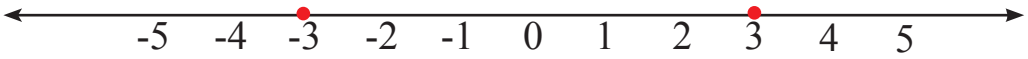
29 **Correct the mistake:** Hiba solved the following equation $\frac{1}{6}(z - 6) = \frac{1}{3}(z - 3)$, and she wrote that $z = 6$. Determine Hiba's Mistake and correct it.

30 **Numerical sense:** Write the absolute value equation that its graphic representation on a number line is:



Write

A life problem that represents absolute value equation which its graphic representation is:



Lesson [4-3] Solving Second Degree Equations with One Variable in R

Idea of the lesson:
*Solving Second Degree Equation with One Variable in R .

Vocabulary:
*Second Degree Equation
*Zero Product Property

Learn

Baghdad Tower is located at Yarmuk section in the west of Baghdad and it's built in 1991 and the height of the tower is 204m, and the area of the square base for the tower is $36m^2$, find the length of the side for the base of the tower.

[4-3-1] Solving the Equations by Using Square Root

Second degree equation with one variable is the equation that contains the biggest power for the variable which is the second power , ex:
$14x^2 - 2x = 0$, $x^2 = 25$, and its solution means finding two values for the variable x .

Example (1) **The area of the base is $36m^2$, find the length of the side of the base.**

The equation that represents the area of base is:

$x^2 = 36$

$x = \sqrt{36}$ or $x = -\sqrt{36}$ There are two square roots for the number 36

$x = 6$ or $x = -6$ -6 and 6 are the square roots for the number 36

So the length of the side of the base for the tower is 6m , and the value $x = -6$ is neglected because the length of the base cannot be negative.

Example (2) **Solve the following equations by using the square root:**

i) $y^2 = 32 \Rightarrow y = \sqrt{32}$ or $y = -\sqrt{32} \Rightarrow y = 4\sqrt{2}$ or $y = -4\sqrt{2}$

ii) $16z^2 = 4 \Rightarrow \frac{1}{16}(16z^2) = \frac{1}{16} \times 4 \Rightarrow z^2 = \frac{1}{4}$

$\Rightarrow z = \sqrt{\frac{1}{4}}$ or $z = -\sqrt{\frac{1}{4}} \Rightarrow z = \frac{1}{2}$ or $z = -\frac{1}{2}$

iii) $8x^2 = 1 \Rightarrow \frac{1}{8}(8x^2) = \frac{1}{8} \times 1 \Rightarrow x^2 = \frac{1}{8} \Rightarrow x = \sqrt{\frac{1}{8}}$ or $x = -\sqrt{\frac{1}{8}}$

$\Rightarrow x = \frac{1}{\sqrt{8}}$ or $x = -\frac{1}{\sqrt{8}} \Rightarrow x = \frac{1}{2\sqrt{2}}$ or $x = -\frac{1}{2\sqrt{2}}$

iv) $t^2 - 1 = 11 \Rightarrow t^2 = 12 \Rightarrow t = \sqrt{12}$ or $t = -\sqrt{12} \Rightarrow t = 2\sqrt{3}$ or $t = -2\sqrt{3}$

[4-3-2] Solving the Equations by Using Zero Product Property

Zero Product Property: If the result for the multiplication of two numbers is zero so one of the numbers must be zero, ex:
$5 \times 0 = 0$, $0 \times 8 = 0$, so if $ab = 0$ tend to $a = 0$ or $b = 0$

Example (3) Sport: **The following law $L = -5t^2 + 30t$ represents the height of the arrow in meters, that Mukhtar threw a raw in the air, where (t) represents the time in seconds. Calculate the time that it is needed to come back to the height that it started.**

$L = 0$ *The arrow will be at height, which started it when $L = 0$*

$-5t^2 + 30t = 0$ *So*

$5t(-t + 6) = 0$ *We will analyse it by extraction of the greatest common factor*

$5t = 0 \Rightarrow t = 0$ *Zero Product Property*

$-t + 6 = 0 \Rightarrow t = 6$ *Or*

$t = 0$ is the time in the second, of started the arrow

$t = 6$ is the time in the second, that it takes the arrow to come back to the height that it started.

Example (4) **Solve the following equations by using the zero product property:**

i) $(x - 3)(x + 5) = 0 \Rightarrow x - 3 = 0$ or $x + 5 = 0 \Rightarrow x = 3$ or $x = -5$

ii) $(t + 8)(t + 8) = 0 \Rightarrow t + 8 = 0$ or $t + 8 = 0 \Rightarrow t = -8$ or $t = -8$

iii) $(y - 12)(y - 9) = 0 \Rightarrow y - 12 = 0$ or $y - 9 = 0 \Rightarrow y = 12$ or $y = 9$

iv) $(2z - 7)(z + 3) = 0 \Rightarrow 2z - 7 = 0$ or $z + 3 = 0 \Rightarrow z = \frac{7}{2}$ or $z = -3$

v) $(n + \sqrt{3})(n - \sqrt{2}) = 0 \Rightarrow n + \sqrt{3} = 0$ or $n - \sqrt{2} = 0 \Rightarrow n = -\sqrt{3}$ or $n = \sqrt{2}$

vi) $x^2 - x = 0 \Rightarrow x(x - 1) = 0 \Rightarrow x = 0$ or $x - 1 = 0 \Rightarrow x = 1$

vii) $4y^2 - 16y = 0 \Rightarrow 4y(y - 4) = 0 \Rightarrow 4y = 0$ or $y - 4 = 0 \Rightarrow y = 0$ or $y = 4$

viii) $5z - 5z^2 = 0 \Rightarrow 5z(1 - z) = 0 \Rightarrow 5z = 0$ or $1 - z = 0 \Rightarrow z = 0$ or $z = 1$

ix) $\sqrt{12}\,h^2 + 2h = 0 \Rightarrow 2\sqrt{3}\,h^2 + 2h = 0 \Rightarrow 2h(\sqrt{3}\,h + 1) = 0$

$\Rightarrow 2h = 0$ or $\sqrt{3}\,h + 1 = 0 \Rightarrow h = 0$ or $h = -\frac{1}{\sqrt{3}}$

Make sure of your understanding

Solve the following equations by using the square root:

1. $x^2 = 25$
2. $4y^2 = 1$
3. $12z^2 = 4$
4. $n^2 - 3 = 13$
5. $7 + m^2 = 43$
6. $\frac{1}{2}x^2 = 9$

Questions 1-6 are similar to example 2

Solve the following equations by using zero product property:

7. $(y - 4)(y + 7) = 0$
8. $(x + 10)(x + 10) = 0$
9. $(13 - m)(6 - m) = 0$
10. $(h - 15)(h - 8) = 0$
11. $(3x - 11)(x + 9) = 0$
12. $(v + \sqrt{5})(v - \sqrt{7}) = 0$

Questions 7-12 are similar to example 4

13. $y^2 - y = 0$
14. $5z^2 + 25z = 0$
15. $3t - t^2 = 0$
16. $\sqrt{18}\,x^2 + 3x = 0$

Questions 13-16 are similar to example 4

Solve the Exercises

Solve the following equations by using the sqare root:

17. $y^2 = 36$
18. $7z^2 = 1$
19. $t^2 - 4 = 12$
20. $7 + n^2 = 56$
21. $z^2 = \frac{4}{9}$
22. $v^2 - \frac{1}{2} = \frac{1}{2}$

Solve the following equations by using zero product property:

23. $(x - 5)(x + 6) = 0$
24. $(15 - n)(7 - n) = 0$
25. $(5t - 13)(t + 8) = 0$
26. $(\sqrt{3} - v)(\sqrt{3} + v) = 0$
27. $z^2 - z = 0$
28. $12n - 2n^2 = 0$
29. $2\sqrt{5}\,v^2 + 2\sqrt{5}\,v = 0$

Solve the problems

30 **Carpets:** The length side of a squared shape room is x meter, it is furnished in the middle with a squared shape carpet which its area is $25m^2$, the area that is not covered with the carpet was $24m^2$.
What is the length of the room's side?

31 **Badminton ball:** Deena hit the badminton ball with the racket to the upward with speed 30m /sec, if the law $H = -5t^2 + 25t$ represents the height of the badminton ball in the air with meters with indicating the time with seconds.
Calculate the time that it takes the ball to come back to the surface of earth.

32 **Shooting:** The following Law $H = -5t^2 + 40t + 3$ represents the height of the arrow that Khalid threw in the air, where t represents the time in seconds.
Find the height of the arrow after two seconds from its throwing and what is the time needed to the arrow to come back to height 3m?

Think

Challenge: Solve the following equations:

33 $x^2 - \frac{2}{3} = \left|-\frac{1}{2}\right|$

34 $3y^2 - 48 = 0$

35 $(2z + 2\sqrt{5})(z - 2\sqrt{5}) = 0$

36 **Correct the mistake:** Jameela solved the following equation:
$\left(\frac{1}{3}x - \frac{1}{4}\right)\left(\frac{1}{3}x + \frac{1}{4}\right) = 0$,and she wrote $x = \frac{4}{3}$ or $x = -\frac{4}{3}$ determine Jameela's mistake and correct it.

37 **Numerical sense:** : Proof that $(y + 5)(y - 3) = y^2 + 2y - 15$, then find solution of the equation: $y^2 + 2y = 15$.

Write

A life problem that represents the following equation:

$$x(x + 4) = 60$$

Lesson [4-4] Solving Two-step Algebraic Inequalities in R

Idea of the lesson:
*Solving Two-step Algebraic inequalities by using the properties and represent it on a number line.

Vocabulary:
*Algebraic Inequality
*Solution Set

Learn

Yaseen is 14 years old, training in a football team, and thinking to join the national team. Write an inequality and the solve it to determine after how many year he can join the national team.
Junior team (ages 16 – 21), youth team (ages 22 – 26),
National team (ages 27 and over).

[4-4-1] Solving Two-step Algebraic Inequalities by using Addition and Subtraction

The inequality that contains one variable or more called Algebraic Inequality, and every number that makes the inequality true is the solution for the inequality , and the solution set for the inequality called solution set, and it could be represented on the real number line.
Properties of inequalities on the real number:

1) Addition property: $\forall$ a, b ,c $\in$ R if $a \geq b$ then $a + c \geq b + c$

2) Subtraction property: $\forall$ a, b ,c $\in$ R if $a \geq b$ then $a - c \geq b - c$

The relations (1) , (2) will stay correct in case of changing $\geq$ with the relations ($<$, $>$, $\leq$) .

Example (1) **Write an inequality that represents the problem and solve it to find the number of years that Yaseen waiting for in order to join the national team.**

$x + 14 \geq 27$ *The inequality that represents the problem*

$x + 14 - 14 \geq 27 - 14$ *Add -14 to the both side of the inequality*

$x \geq 13$

Yaseen can join to the national team at least after 13 years.

Example (2) **Solve the following inequalities in R by using the properties of addition and subtraction and represent it on a number line:**

i) $3x - 12 \leq 2x - 6 \Rightarrow 3x - 2x \leq 12 - 6 \Rightarrow x \leq 6$

ii) $2z - \frac{5}{7} > z - \frac{12}{7} \Rightarrow 2z - z > \frac{5}{7} - \frac{12}{7} \Rightarrow z > -1$

Example (3) **Solve the following inequalities in R by using the property of addition and subtraction:**

i) $3(y - \sqrt{2}) < 2y + \sqrt{2} \Rightarrow 3y - 3\sqrt{2} < 2y + \sqrt{2} \Rightarrow 3y - 2y < \sqrt{2} + 3\sqrt{2} \Rightarrow y < 4\sqrt{2}$

ii) $8(\frac{1}{8}h + \frac{3}{16}) < 0 \Rightarrow 8 \times \frac{1}{8}h + 8 \times \frac{3}{16} < 0 \Rightarrow h + \frac{3}{2} < 0 \Rightarrow h < -\frac{3}{2}$

iii) $11(m + 3) > 10(m - 2) \Rightarrow 11m + 33 > 10m - 20 \Rightarrow m > -53$

[4-4-2] Solving Two-step Algebraic Inequalities by Using Multiplication and Division

The algebraic inequalities can be solved by using the properties of multiplication and division on the real numbers:

3) multiplication property: i) $\forall$ a, b ,c $\in$ R , if $a \geq b$, $c > 0$ then $ac \geq bc$

ii) $\forall$ a, b ,c $\in$ R , if $a \geq b$, $c < 0$ then $ac \leq bc$

4) Division property: i) $\forall$ a ,b ,c $\in$ R , if $a \geq b$ and $c > 0$ then $\frac{a}{c} \geq \frac{b}{c}$

ii) $\forall$ a ,b ,c $\in$ R , if $a \geq b$ and $c < 0$ then $\frac{a}{c} \leq \frac{b}{c}$

Example (4) Birds: **Anwar has 18 birds and Atheer has 98 birds, Anwar wants to duplicate his bird's number to get more birds than Atheer's birds at least 10 more birds. How many times Anwar must duplicate the numbers of the birds that he has?**

$18x - 10 \geq 98$ *Write the inequality that represent the problem*

$18x - 10 + 10 \geq 98 + 10$ *Add 10 to the both sides of the inequality*

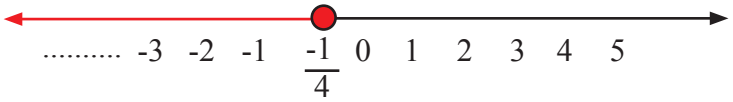
$18x \geq 108$ *Divide both sides of the inequality by 18*

$x \geq 6$ Anwar must duplicate the numbers of the birds at least 6 times

Example (5) **Solve the following inequalities in R by using the multiplication and division properties and represent it on a number line:**

i) $\frac{6y}{-2} < 3 \Rightarrow \frac{6y}{-2} \times \frac{-2}{6} > 3 \times \frac{-2}{6} \Rightarrow y > -1$

ii) $2x - 2 \leq \frac{-5}{2} \Rightarrow 2x \leq \frac{-5}{2} + 2 \Rightarrow 2x \leq \frac{-1}{2} \Rightarrow x \leq \frac{-1}{4}$



Example (6) **Solve the following inequalities in R by using the multiplication and division properties:**

i) $\frac{9x}{4} < \frac{3}{5} \Rightarrow \frac{9x}{4} \times \frac{1}{9} < \frac{3}{5} \times \frac{1}{9} \Rightarrow \frac{x}{4} \times 4 < \frac{3}{45} \times 4 \Rightarrow x < \frac{12}{45} \Rightarrow x < \frac{4}{15}$

ii) $\frac{-2t}{7} \geq \frac{5}{14} \Rightarrow \frac{-2t}{7} \times \frac{7}{2} \geq \frac{5}{14} \times \frac{7}{2} \Rightarrow -t \geq \frac{5}{4} \Rightarrow -t(-1) \leq \frac{5}{4}(-1) \Rightarrow t \leq \frac{-5}{4}$

Solve the following inequalities by using the properties of the inequalities on the real numbers:

iii) $6z > 3(z - 6) \Rightarrow 6z > 3z - 18 \Rightarrow 6z - 3z > -18 \Rightarrow 3z > -18 \Rightarrow z > -6$

iv) $\frac{m}{8} < \frac{1}{3} - 2 \Rightarrow \frac{m}{8} < \frac{-5}{3} \Rightarrow \frac{m}{8} \times 8 < \frac{-5}{3} \times 8 \Rightarrow m < \frac{-40}{3}$

v) $\frac{5}{-9} \leq \frac{k}{3} \Rightarrow \frac{5}{-9} \times 9 \leq \frac{k}{3} \times 9 \Rightarrow -5 \leq 3k \Rightarrow 3k \geq -5 \Rightarrow k \geq \frac{-5}{3}$

Make sure of your understanding

Solve the following inequalities in R by using the properties and represent it on a number line:

1. $2y - 8 \leq 3y - 8$
2. $2x - 6 < x - \sqrt{16}$
3. $4t + \frac{2}{3} \geq 3t - \frac{5}{3}$
4. $\frac{3}{5} > z - \frac{9}{10}$

Questions 1-4 are similar to examples 1,2

Solve the following inequalities in R by using the properties of addition and subtraction:

5. $7(x - \sqrt{3}) < 6x + \sqrt{3}$
6. $2y + \sqrt[3]{-27} \geq 3y - \sqrt[3]{8}$
7. $5(\frac{1}{5}m + \frac{3}{10}) < 0$
8. $9(z - 4) > 10(z + 3)$

Questions 5-8 are similar to example 3

Solve the following inequalities in R by using the multiplication and division properties:

9. $\frac{3t}{4} \geq \frac{5}{7}$
10. $\frac{-5x}{7} < \frac{7}{21}$
11. $\frac{y}{7} \leq \frac{5}{14}$

Questions 9-11 are similar to example 6

Solve the following inequalities by using the properties of the inequalities on the real numbers:

12. $5(v + \sqrt{7}) \geq 2v - \sqrt{7}$
13. $2z + \sqrt[3]{-125} < 6z - \sqrt[3]{27}$
14. $9(\frac{1}{3}x + \frac{7}{9}) > 0$
15. $\frac{5}{6}(t - 6) > 11(t + 2)$
16. $\frac{-h}{13} < \frac{1}{26} - 1$
17. $\frac{2x}{3} + 4 \leq \frac{8}{3} - 5$

Questions 12-17 are similar to examples 3,6

Solve the Exercises

Solve the following inequalities in R by using the property of addition and subtraction:

18. $9(z - \sqrt{5}) < 8z - \sqrt{5}$
19. $7(\frac{1}{7}m + \frac{5}{14}) < 0$

Solve the following inequalities in R by using the multiplication and division properties:

20. $\frac{2p}{3} \geq \frac{-6}{21}$
21. $\frac{-4x}{9} < \frac{-8}{27}$

Solve the following inequalities by using the properties of the inequalities on the real numbers:

22. $6(x - \sqrt{3}) \geq 4x - \sqrt{3}$
23. $8y + \sqrt[3]{-8} < 4y - \sqrt{121}$
24. $7(\frac{1}{4}x - \frac{3}{14}) \leq 0$

Solve the problems

25 **Beads:** Saba wants to give a necklace to her sister as a birth-day gift, the necklace consists of 100 beads and it needs 7 hours to make it, if there are 16 beads in the necklace what is the least number of the beads that she must use each hours to complete the necklace?

26 **Football:** In the first league of football tournaments, the air force team won 7 matches and lost 3 matches and there are 22 more matches that it can play.
What is the least number of the remaining matches that must play them and win them with the most half of number overall the games?

27 **Fun fair:** A ticket seller in a fun fair needs 1400000 dinars to cover the operating expenses per a day if he sold till the noon 650000 dinars with the price of ticket 750 dinars, how many more tickets has he to sell in order not to fall under the fiscal deficit ?

Think

Challenge: Solve the following inequalities:

28 $2y - \frac{3}{4} < |-\frac{3}{2}|$ 29 $7(\frac{1}{5}z - 1) > 1$ 30 $(\frac{1}{2}x - \frac{\sqrt{5}}{2})(4 + \sqrt[3]{-8}) \geq 0$

31 **Correct the mistake:** Osman solved the following inequality: $\frac{v}{5} - \frac{1}{4} \leq \frac{3}{4} - 1$, and he wrote the solution set is: $\{5,6,7,...\}$, determine Osman's mistake and correct it.

32 **Numerical sense:** Proof that the values of (h) are making the following inequality true and it is just a negative numbers:

$$3h + \frac{1}{3} < |-\frac{1}{6}| - \frac{1}{3}$$

Write

A life problem that represents the following inequality:

$$500\,x + 600\,000 \leq 1\,600\,000$$

Lesson [4-5] Solving Multi-step Algebraic Inequalities in R

Idea of the lesson:
*Solving multi-step algebraic inequalities by using the properties and representing it on a number line.

Vocabulary:
*Algebraic Inequality
*The variable

Learn

If 8 soldiers want to get on a helicopter and each of them have 20kg personal equipments. Write an inequality and solve it to find the allowed extra weight for each soldier so that will not exceed their total cargo which is 880kg.

[4-5-1] Solving Multi-step Algebraic Inequalities which have variable in one side

To solve an inequality that has one variable in one side, use the properties to isolate the extended that contain a variable in the inequality side, then make its coefficient equal to one by using the properties of multiplication or division then find the solution set for the inequality, and the solution set can be represented in the real number line

Example (1) **Write an inequality represented the problem and solve it to find the number of the extra kilograms that allowed to each solider**

$8(w + 20) \leq 880$ *Suppose that the variable (w) is representing the extra weight that allowed to each soldier*

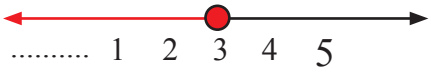
$8w + 160 + (-160) \leq 880 + (-160)$ *Add -160 to the both sides of the inequality*

$8w \leq 720$ *Divide the both sides of the inequality by 8*

$w \leq 90$ The maximum extra weight that each solider can take with him into the helicopter is 90kg.

Example (2) **Solve the following inequalities in R by using the properties and represent it on a number line:**

i) $3(y-2) \leq 6- \sqrt[3]{27} \Rightarrow 3y- 6 \leq 3 \Rightarrow 3y \leq 9 \Rightarrow y \leq 3$



ii) $\frac{1}{2}(x - \frac{4}{3}) + \frac{1}{2}x > -\frac{10}{6} \Rightarrow \frac{1}{2}x + \frac{1}{2}x > -\frac{10}{6} + \frac{4}{6} \Rightarrow x > -1$

Example (3) **Solve the following inequalities in R by using the properties of inequalities on the real numbers:**

i) $5(z -\sqrt{3}) \geq 10(2 -\sqrt{3}) \Rightarrow z -\sqrt{3} \geq 4 -2\sqrt{3} \Rightarrow z \geq 4 -2\sqrt{3} +\sqrt{3} \Rightarrow z \geq 4- \sqrt{3}$

ii) $\frac{1}{3}v + \sqrt[3]{-27} - \frac{4}{3}v < |-3| \Rightarrow \frac{1}{3}v - \frac{4}{3}v - 3 < 3 \Rightarrow -v < 6 \Rightarrow v > -6$

iii) $9 - \sqrt[3]{-8} > 5(x -1) \Rightarrow 9 +2 > 5x -5 \Rightarrow 11 > 5x -5 \Rightarrow 16 > 5x \Rightarrow x < \frac{16}{5}$

iv) $\frac{-4}{5}(\frac{7}{2}h + \frac{14}{8}) < 0 \Rightarrow \frac{-4}{7} \times \frac{7}{2}h + \frac{-4}{7} \times \frac{14}{8} < 0 \Rightarrow -2h -1 < 0 \Rightarrow -2h < 1 \Rightarrow h > \frac{-1}{2}$

[4-5-2] Solving Multi-step Algebraic Inequalities which have variable in both sides

To solve Multi-step Algebraic Inequalities Which have variable in both sides follow the following steps:

1- Use the distribution property to remove the brackets if exist.

2- Isolate the variable in one side of the inequality.

3- Use the operations arrangement to simplify the inequality.

4- Use the properties to find the solution set for the inequality.

Example (4) Animals: **The weight of the bear 600kg before hibernation its normal weight is not less than 440kg, during the hibernation it can lose 8kg in a week.**
So how many weeks can it be in hibernation to reach to its normal weight?

Suppose that the variable (x) is representing the number of the weeks

$600 - 8x \geq 440$ *Write the inequality that represent the problem*

$-8x \geq 440 - 600$ *Add - 600 to each side of the inequality*

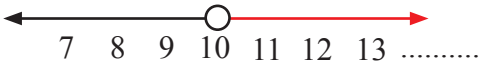
$-8x \geq -160$ *Divide both sides of the inequality by -8*

$x \leq 20$ The bear can be in hibernation for 20 weeks at most.

Example (5) **Solve the following inequalities in R by using the properties and represent it on a number line:**

i) $\frac{2x}{5} < 4(x-9) \Rightarrow \frac{2x}{5} < 4x-36 \Rightarrow 2x < 20x-180$

$\Rightarrow 180 < 18x \Rightarrow 10 < x \Rightarrow x > 10$



ii) $3(y-2) \leq 5(2-y) \Rightarrow 3y-6 \leq 10-5y$

$\Rightarrow 3y+5y \leq 10+6 \Rightarrow 8y \leq 16 \Rightarrow y \leq 2$

Example (6) **Solve the following inequalities in R by using the multiplication and division properties:**

i) $8(t-3) < 7(t-5) \Rightarrow 8t-24 < 7t-35 \Rightarrow 8t-7t < 24-35 \Rightarrow t < -9$

ii) $\frac{-2x}{5} \leq \frac{x}{7} \Rightarrow 7(-2x) \leq 5x \Rightarrow -14x \leq 5x \Rightarrow 0 \leq 5x+14x \Rightarrow 0 \leq 19x \Rightarrow x \geq 0$

iii) $\frac{1}{4}(z+4) > \frac{1}{2}(\sqrt{2}-z) \Rightarrow \frac{1}{4}z+1 > \frac{1}{\sqrt{2}} - \frac{1}{2}z \Rightarrow \frac{1}{4}z+\frac{1}{2}z > \frac{1}{\sqrt{2}} - 1$

$\Rightarrow \frac{3}{4}z > \frac{1-\sqrt{2}}{\sqrt{2}} \Rightarrow z > \frac{4-4\sqrt{2}}{3\sqrt{2}}$

iv) $\frac{h}{4}+\frac{1}{3} \geq \frac{h}{2}-\frac{1}{3} \Rightarrow \frac{1}{3}+\frac{1}{3} \geq \frac{h}{2}-\frac{h}{4} \Rightarrow \frac{2}{3} \geq \frac{h}{4} \Rightarrow 8 \geq 3h \Rightarrow \frac{8}{3} \geq h$

v) $\frac{7}{\sqrt[3]{-27}} - 2y \leq \frac{2}{3} - y \Rightarrow \frac{7}{-3} - 2y \leq \frac{2}{3} - y \Rightarrow \frac{7}{-3} - \frac{2}{3} \leq 2y - y \Rightarrow \frac{-9}{3} \leq y \Rightarrow -3 \leq y$

Make sure of your understanding

Solve the following inequalities in R by using the properties and represent it on a number line:

1. $5(x-1) \leq 8 - \sqrt[3]{-8}$
2. $\frac{1}{3}(z - \frac{7}{2}) + \frac{1}{3}z \geq -\frac{23}{6}$
3. $\frac{t}{2} > 2(\frac{1}{\sqrt{16}} - t)$
4. $7(n - 1) \leq 9(1 - n)$

Questions 1-4 are similar to examples 2,5

Solve the following inequalities by using the inequalities properties on the real numbers:

5. $2(x - \sqrt{5}) \leq 11(1 - \sqrt{5})$
6. $\frac{1}{2}y - \sqrt[3]{-8} - \frac{3}{2}y < |-7|$
7. $12 - \sqrt[3]{-125} \geq 6(z - 1)$
8. $\frac{-5}{7}(\frac{7}{3}m + \frac{14}{5}) > 0$

Questions 5-8 are similar to example 3

9. $6(4 - h) \leq 7(h - 5)$
10. $\frac{-3z}{7} \geq \frac{1}{7} + z$
11. $\frac{1}{9}(x - \sqrt{2}) < \frac{1}{3}(\sqrt{2} - x)$
12. $\frac{n}{6} + \frac{3}{2} > \frac{n}{3} - \frac{1}{3}$
13. $\frac{5}{\sqrt[3]{-8}} - 4k \leq \frac{5}{2} - k$
14. $\frac{1}{2}(y - 3) \geq \frac{1}{4}(\sqrt{2} + y)$

Questions 9-14 are similar to examples 3,6

Solve the Exercises

Solve the following inequalities by using the inequalities properties on the real numbers:

15. $7(y+3) < 9 - \sqrt[3]{-27}$
16. $\frac{1}{7}(t - \frac{1}{8}) + \frac{1}{7}t \geq -\frac{6}{7}$
17. $\frac{x}{5} \leq 4(\frac{1}{\sqrt{36}} - 2x)$
18. $8(h + 3) > 12(1 - h)$
19. $5(z - \sqrt{3}) < 7(1 - \sqrt{3})$
20. $\frac{1}{6}m - \sqrt[3]{-27} - \frac{2}{3}m \leq |-9|$
21. $14k - \sqrt[3]{-64} \geq 7(k - 1)$
22. $\frac{-8}{3}(\frac{6}{8}x + \frac{3}{4}) > 0$
23. $9(3 + y) < 6(y - 4)$
24. $\frac{-2n}{7} \geq \frac{3}{5} + n$
25. $\frac{1}{6}(z - \sqrt{3}) \leq \frac{1}{6}(\sqrt{3} - z)$
26. $\frac{h}{4} + \frac{2}{3} < \frac{h}{3} + \frac{4}{3}$
27. $\frac{8}{\sqrt[3]{-125}} - 2x > \frac{2}{5} - x$
28. $\frac{1}{9}(y + 6) \geq \frac{1}{6}(\sqrt{6} - y)$

Solve the problems

29 **Zoo:** A ticket seller in a zoo sold 450000 dinars with the price of a ticket 500 dinars till the noon, and the ticket seller needs 1850000 dinars to cover the daily operating coast. What is the least number of ticket that he must sell to cover the coast of the operations?

30 **Basketball:** In a basketball tournaments, the yellow team won 22 matches and lost 10 matches and there are 40 more matches that it can play.
What is the smallest number of the remaining matches that must play them and win them with the most half of number overall the games?

31 **kites:** Nadia wants to make a kite from a rectangles shape paper its height is 15cm longer than its width.
What is the least length for a kite if the perimeter of the paper is more than 130cm?

Think

Challenge: Solve the following inequalities:

32 $\frac{1}{2}x + \sqrt[3]{-125} - \frac{5}{2}x > |-6|$

33 $(\frac{3}{5}y - \frac{1}{3})(5 + \sqrt[3]{-125}) \geq 2y$

34 **Correct the mistake:** Warda solved the following inequality: $\frac{5}{3}(\frac{h}{5} + \frac{3}{4}) \leq \frac{5}{4} - 2$, and wrote the solution set is $\{-6, -5, -4, \ldots\}$. Determine Warda's mistake and correct it.

35 **Numerical sense:** Proof that the values of (z) are making the following inequality true and it is just a negative numbers.

$$7z - \frac{1}{7} < |-\frac{1}{7}| - \frac{4}{14}$$

Write

A life problem that represents the following inequality:

$$(x + 6) \leq 40$$

Chapter Test

Solve the following equations by using the properties of real numbers and check the answer:

1. $z = \sqrt[3]{-8} + 2z$
2. $4m - 5\sqrt{3} = 3m + 6\sqrt{3}$
3. $\sqrt{36}\,h = |-16| + 5h$
4. $3z \div 21 = \frac{1}{7}$
5. $\sqrt[3]{125}\,x \div |-9| = 5^2$
6. $\sqrt{3}\,x \div 9 = \sqrt{3} \div 3$
7. $\frac{\sqrt{5}\,h}{2} = \frac{1}{\sqrt{5}}$
8. $\frac{9y}{2\sqrt[3]{4}} = \frac{18}{4}$
9. $6z \div 13 = 5z \div 13$
10. $8(h - 1^2) = \frac{1}{2}h - 6$
11. $5\sqrt{3} - z = z - 7\sqrt{3}$
12. $\sqrt{64}\,y = 10(y-1) + 3^2$

Solve the following equations in R by using the properties:

13. $3(x-10) = 2(x+10)$
14. $\sqrt[3]{-8}\,y \div |-8| = 4\sqrt{2}$
15. $\frac{1}{3}(z-7) + \frac{7}{3} = \frac{1}{5}(z - 10)$
16. $\frac{t}{8+\sqrt[3]{-27}} = \frac{6t}{5}$
17. $|y - 12| = 7$
18. $|2v - 5| = \sqrt{36}$
19. $|\frac{1}{3}n + 8| = \sqrt[3]{-125}$
20. $|7x - 14| = |-18|$
21. $|z - \sqrt{3}| = 4\sqrt{3}$

Solve the following equations by using the square root:

22. $x^2 = 64$
23. $9y^2 = 1$
24. $12t^2 = 4$
25. $n^2 - 5 = 20$
26. $7 - z^2 = -42$
27. $\frac{1}{4}k^2 = 9$
28. $y^2 = \frac{4}{9}$
29. $h^2 - \frac{3}{4} = \frac{1}{4}$

Solve the following equations by using the zero product properties:

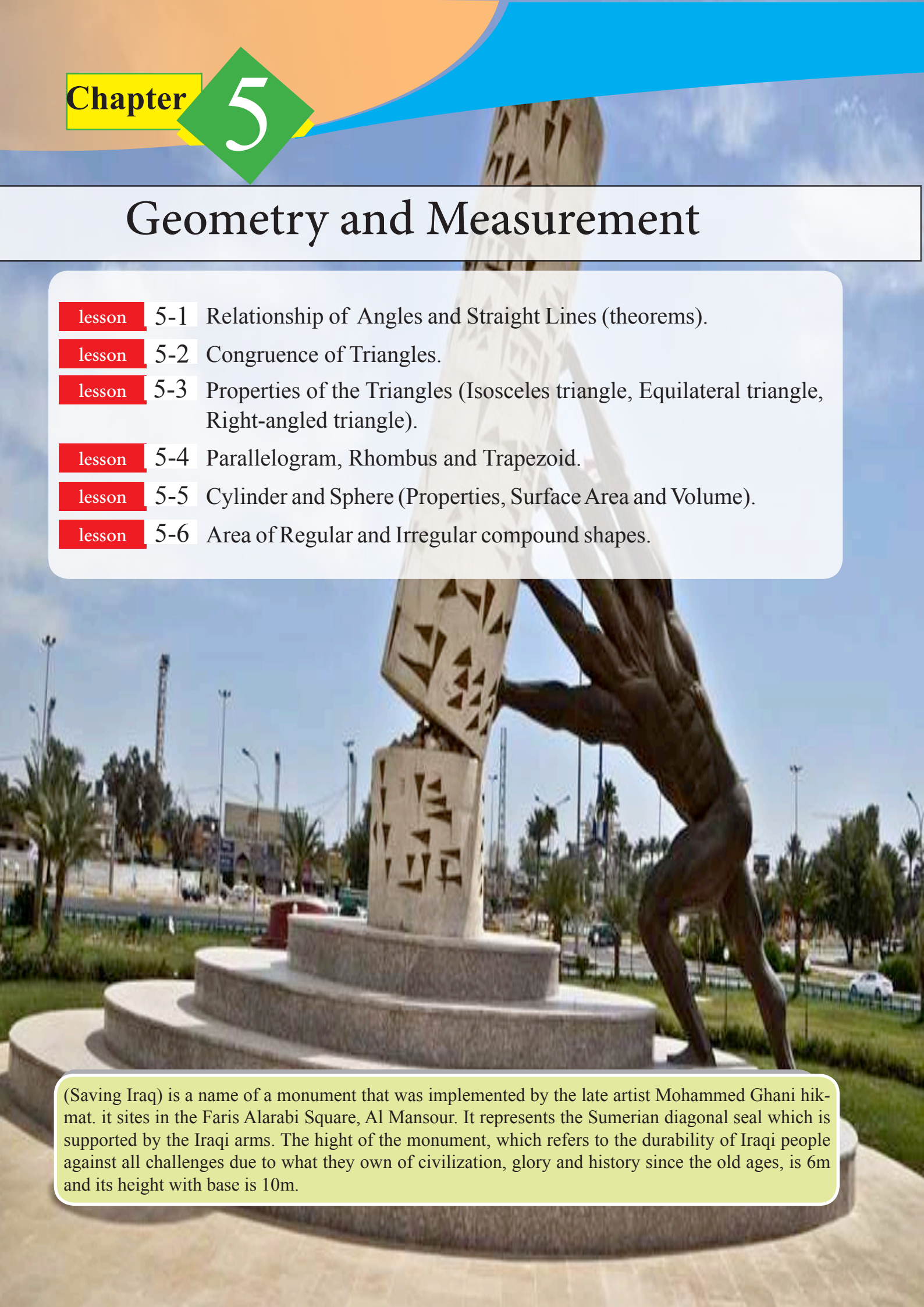
30. $(y - 4)(y + 4) = 0$
31. $(z-7)(z-7) = 0$
32. $(x + \sqrt{5})(x - \sqrt{3}) = 0$
33. $(\sqrt{2} - h)(\sqrt{2} + h) = 0$
34. $(4t+8)(3t-7) = 0$
35. $z^2 - z = 0$
36. $\sqrt{8}\,x^2 + 2x = 0$
37. $3\sqrt{7}\,n^2 - 3\sqrt{7}\,n = 0$
38. $\frac{1}{5}y^2 - \frac{1}{5}y = 0$

Solve the following inequalities in R by using inequality properties on the real number:

39. $2(x - \sqrt{2}) \geq x - \sqrt{2}$
40. $9z + \sqrt[3]{-27} < 10z - \sqrt[3]{125}$
41. $5(\frac{1}{2}y - \frac{3}{10}) \leq 0$
42. $\frac{t}{-7} - 1 > \frac{-1}{14}$
43. $\frac{2m}{9} - \frac{1}{9} \leq \frac{1}{9}$
44. $3(x+7) < 6 - \sqrt[3]{-64}$
45. $\frac{1}{6}(z - \frac{12}{5}) + \frac{5}{6}z \geq -\frac{3}{5}$
46. $\frac{y}{4} \leq 2(\frac{1}{\sqrt{16}} - \frac{1}{8}y)$
47. $5(x+1) > 2(1-x)$
48. $\frac{1}{3}(h - \sqrt{2}) \leq \frac{2}{3}(\sqrt{2} - h)$
49. $\frac{m}{6} + \frac{2}{5} < \frac{m}{3} - \frac{4}{5}$
50. $\frac{9}{\sqrt[3]{-27}} - 5y > \frac{1}{5} - y$

Chapter 5

Geometry and Measurement

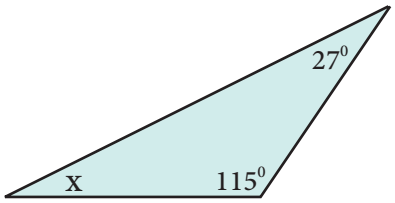


(Saving Iraq) is a name of a monument that was implemented by the late artist Mohammed Ghani hikmat. it sites in the Faris Alarabi Square, Al Mansour. It represents the Sumerian diagonal seal which is supported by the Iraqi arms. The hight of the monument, which refers to the durability of Iraqi people against all challenges due to what they own of civilization, glory and history since the old ages, is 6m and its height with base is 10m.

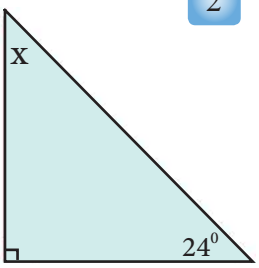
Pretest

Find the value of the unknown angle for each of the triangles shown in the following figures:

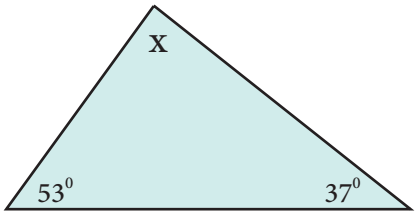
1



2

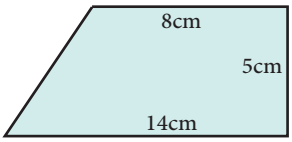


3

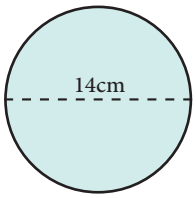


Calculate the area of the shaded region in the following plane shapes:

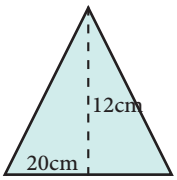
4



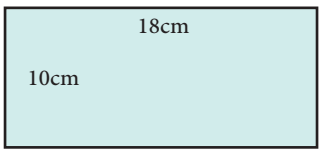
5



6



7



Write true or false for each of the following statements:

8 The two parallel lines do not meet.

9 The two perpendicular lines do not have intersection point.

10 The distance between the two parallel lines is not constant.

11 The two perpendicular lines form a right angle between them.

Complete the following blanks:

12 The sum of two supplementary angles is ..... .

13 The sum of two complementary angles is ..... .

14 The alternated angles are..... in measures.

15 The sum of triangle's angles is..... .

16 Ahmad wants to cover a squared area with tiles, the length of its side (square side) is 9m. The shape of tiles is square and the area of each tile is $0.25m^2$. Calculate the number of tiles required for paving.

Lesson [5-1] Relationship of Angles and Straight lines (theoroms)

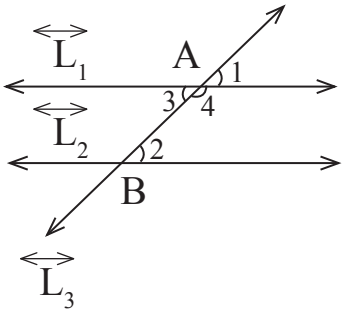
Idea of the lesson:
* Identify the relation of alternate, corresponding and internal angles and vice -versa.

Vocabulary:
* Corresponding angles
* Alternate angles
* Internal angles

Learn

In the near figure, $\overleftrightarrow{L_1} \parallel \overleftrightarrow{L_2}, \overleftrightarrow{L_3}$ intersect the two parallel lines in A,B.

* The two angles $\angle 1$ and $\angle 2$ are named corresponding angles and they are congruent (equal in measur).
* The two angles $\angle 2$ and $\angle 3$ are named alternate angels and they are congruent (equal in measur).
* The two angles $\angle 2$ and $\angle 4$ are named supplementary interior angles, and the sum of the two angles is 180^0.

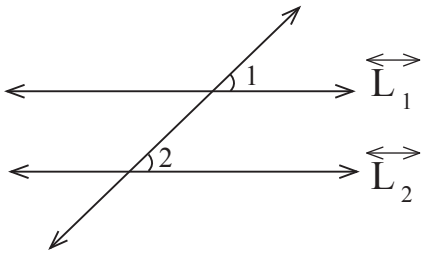


You have previously learnd if two parallel lines were intersected by a third one then the angles will be: vertically opposite, alternate, corresponding and equal in measure. In this lesson we will learn when would the two straight lines be parallel.

* Converse of corresponding angles theorem:

If a line intersected two lines in the same plane, if the two corresponding angles having the same measurement, then the two lines are parallel.

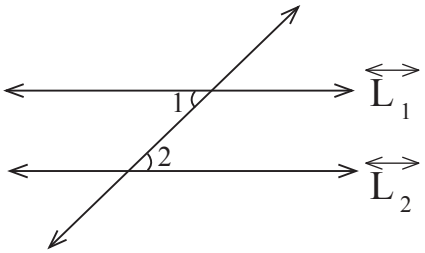
If: $m\angle 1 = m\angle 2$ corresponding, then $\overleftrightarrow{L_2} \parallel \overleftrightarrow{L_1}$



* Converse of alternate angles theorem:

If a line intersected two lines in the same plane, if the two alternate angles having the same measurement, then the two lines are parallel.

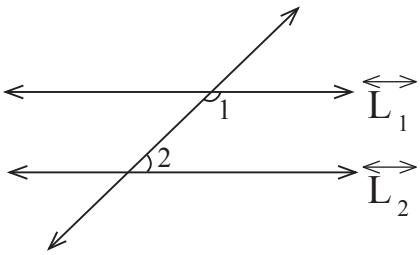
If: $m\angle 1 = m\angle 2$ alternate, then $\overleftrightarrow{L_2} \parallel \overleftrightarrow{L_1}$



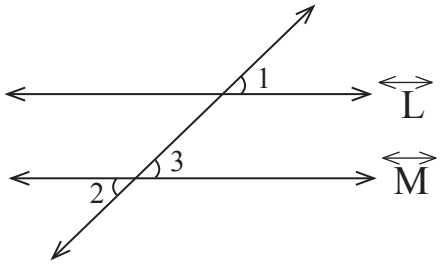
* Converse of internal angles theorem:

If a line intersected two lines in the same plane, if the two supplementary internal angles at the same-side of crossing, then the two lines are parallel.

If: $m\angle 1 + m\angle 2 = 180^\circ$ internal, then $\overleftrightarrow{L_2} \parallel \overleftrightarrow{L_1}$



Example 1 Use the given information in near by figure:



If $m\angle 1 = m\angle 2$ then show that $\overleftrightarrow{L} \parallel \overleftrightarrow{M}$

$m\angle 1 = m\angle 2$ *given*

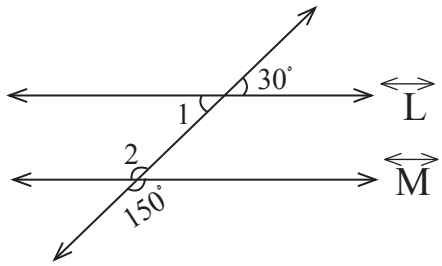
$m\angle 2 = m\angle 3$ *two vertically opposite angles*

$m\angle 1 = m\angle 3$ *(if two quantities equal one quantity, then both of them are equal)*

$\angle 1 , \angle 3$ *two corresponding angles*

Then $\overleftrightarrow{L} \parallel \overleftrightarrow{M}$ *converse of corresponding angels theoroem*

Example 2 Use the given information in near by figure and show $\overleftrightarrow{L} \parallel \overleftrightarrow{M}$.



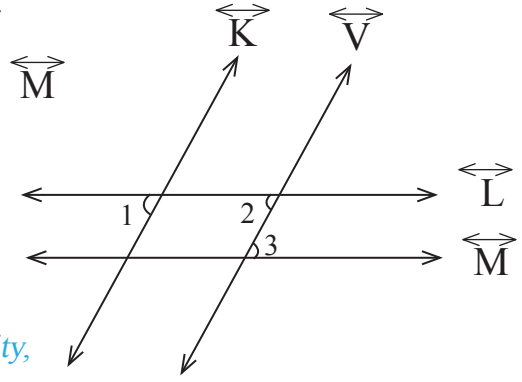
$m\angle 1 = 30^0$ *two vertically opposite angles*

$m\angle 2 = 150^0$ *two vertically opposite angles*

The two angles 1, 2 are internal and at the same-side of crossing and their sum is 180°

Then $\overleftrightarrow{L} \parallel \overleftrightarrow{M}$ *converse of internal angles theorem.*

Example 3 Use the given information in near by figure:



Since $m\angle 1 = m\angle 3$, $\overleftrightarrow{k} \parallel \overleftrightarrow{V}$ show that $\overleftrightarrow{L} \parallel \overleftrightarrow{M}$

$m\angle 1 = m\angle 3$ *given*

$m\angle 1 = m\angle 2$ *two corresponding angles*

$m\angle 2 = m\angle 3$ *(if two quantities are equal one quantity, then both of them are equal)*

$\angle 3 , \angle 2$ *two alternate angles*

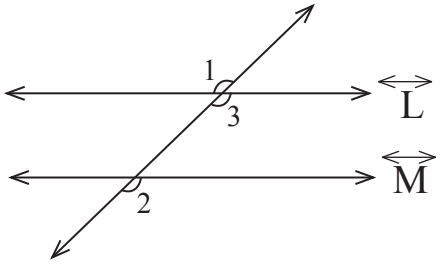
Then $\overleftrightarrow{L} \parallel \overleftrightarrow{M}$ *converse of alternate angles theorem.*

Make sure of your understanding

Use the given information and converse the theorems to show the following:

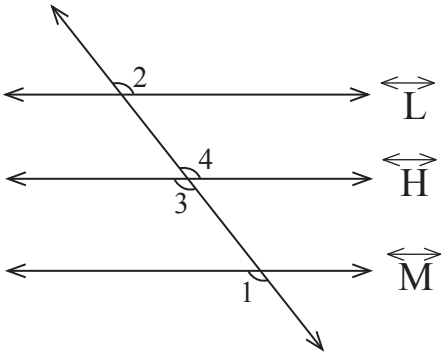
1 If $m\angle 1 = m\angle 2$

then: $\overleftrightarrow{M} \parallel \overleftrightarrow{L}$



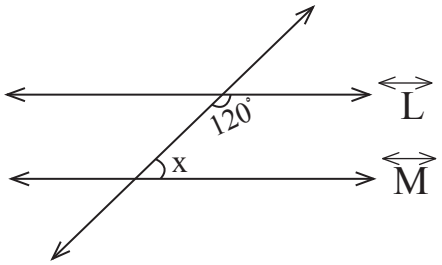
2 If $\overleftrightarrow{M} \parallel \overleftrightarrow{H} \parallel \overleftrightarrow{L}$

then: $m\angle 1 = m\angle 2$



3 If $X=15a$, $a = 4$

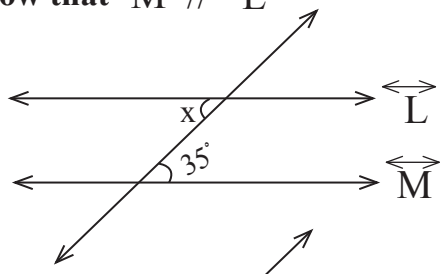
then: $\overleftrightarrow{M} \parallel \overleftrightarrow{L}$



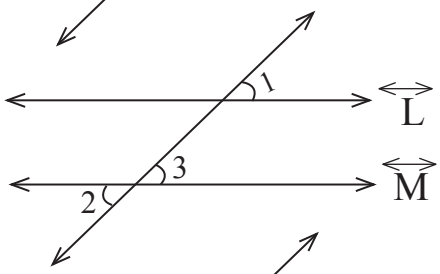
Solve the Exercises

Use the given information and converse the theorems to show that $\overleftrightarrow{M} \parallel \overleftrightarrow{L}$:

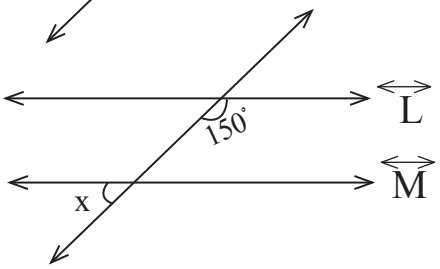
4 $X=7a$, $a = 5$



5 $m\angle 1 = m\angle 2$



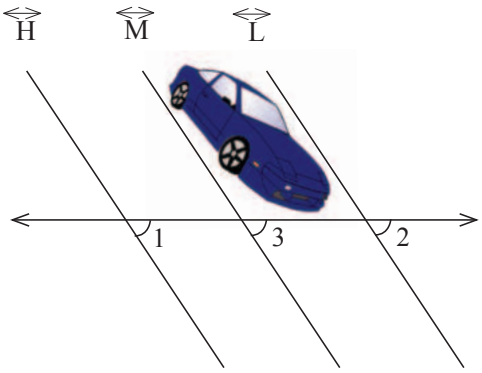
6 $m\angle x = 30^0$



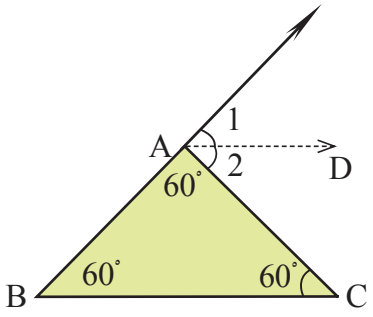
Solve the problems

7 **parking:** An engineer wants to design a car parking in a parallel shape. Use the given and converse the theorems to show that:

$\overleftrightarrow{M} \parallel \overleftrightarrow{L}$ since $m\angle 1 = m\angle 2$ and $\overleftrightarrow{M} \parallel \overleftrightarrow{H}$

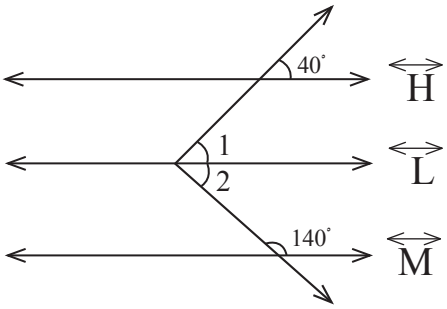


8 **Drawing:** Mohammed draws an equilateral triangle as shown in the near by figure since $m\angle 1 = m\angle 2$.
Help Mohammed to prove that $\overrightarrow{AD} \parallel \overline{BC}$

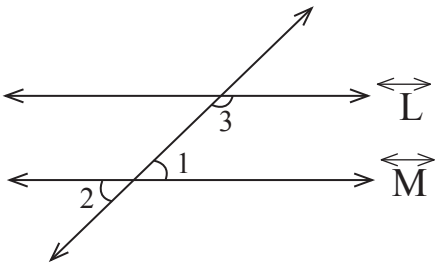


Think

9 **Challenge:** In near by figure $\overleftrightarrow{H} \parallel \overleftrightarrow{L}$, $m\angle 1 = m\angle 2$
Prove that $\overleftrightarrow{M} \parallel \overleftrightarrow{L}$.

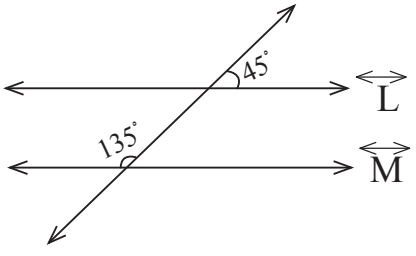


10 **Correct the mistake:** Mohannad draws the near by figure. He says, since $m\angle 1 + m\angle 2 = 180^0$, so $\overleftrightarrow{L} \parallel \overleftrightarrow{M}$.
Discover and correct the mistake.



Write

Does the information in the near by figure allow you to conclude that $\overleftrightarrow{L} \parallel \overleftrightarrow{M}$? Illustrate that.



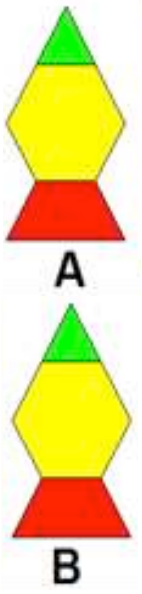
Lesson [5-2] Congruence of Triangles

Idea of the lesson:
* Identify the concept of congruence and triangles congruence cases.

Vocabulary:
*Congruence.
*Triangle's six elements.
* Two sides with one included angle.
*Two angles with one included side.
*Three sides.

Learn

The near by figure shows two congruent shapes. We mean that each of the two congruent shapes represent a real copy of the other.
If we say shape (A) is congruent on shape (B), that means the shape (A) is another copy of shape (B). The segments are congruent if they have the same measurement i.e. the same length. The angles are congruent if they have the same measurement.
The sides are also congruent if we could put one of them on the other in a way that makes the vertices of each polygon congruent on other.
We use symbol $\cong$ to denote congruence.



[5-2-1] Concept of Congruence of Two Triangles

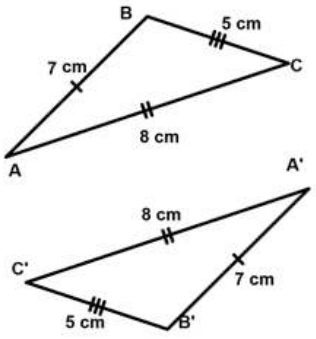
It is known that triangle has three sides and three angles (called triangul six elements). The two triangles will be congruent if each element of their six elements is congruent with each and another, and vice-versa. So: if two triangles were congruent, then each element of their six elements will be congruent and each element which is congruent to it in the other triangle called (corresponding), if two triangles are congruent, then the area of the two triangle will be the same.

[5-2-2] Cases of Congruence of Two Triangles

It is not necessary to prove the congruence of the six elements of one triangle with its corresponding from the second triangle. It is enough to know 3 elements (at least, one side must be among them) from one of the two triangles with its corresponding from other triangle. following are cases of congruence of two triangles:

First case: (Congruence of three sides) two triangles are congruent if the sides of first triangle has a congruent side to the other triangle (S S S).

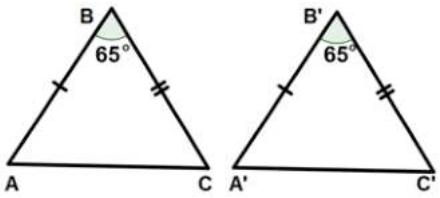
Example 1 In near by figure:



* Side $\overline{AB}$ is congruent on side $\overline{A'B'}$, ($\overline{AB} \cong \overline{A'B'}$)
* Side $\overline{AC}$ is congruent on side $\overline{A'C'}$, ($\overline{A'C'} \cong \overline{AC}$)
* Side $\overline{BC}$ is congruent on side $\overline{B'C'}$, ($\overline{BC} \cong \overline{B'C'}$)
* It means $\triangle ABC \cong \triangle A'B'C'$ (S S S)

Second case: (Congruence of two sides and their included angle). Two triangles are congruent if there are two congruent sides and included angle to the other triangle (S A S).

Example 2 In near by figure:

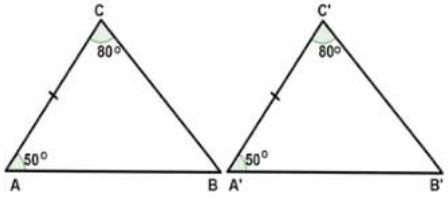


* Side $\overline{AB}$ is congruent on side $\overline{A'B'}$, ($\overline{AB} \cong \overline{A'B'}$)
* Side $\overline{BC}$ is congruent on side $\overline{B'C'}$, ($\overline{BC} \cong \overline{B'C'}$)
* Angle ABC is congruent on Angle $A'B'C'$, ($\angle ABC \cong \angle A'B'C'$)
* It means $\triangle ABC \cong \triangle A'B'C'$ (S A S)

Third case: (Congruence of two angles and included side). Two triangles are congruent if there are two congruent angles and included side to the other triangle (A S A).

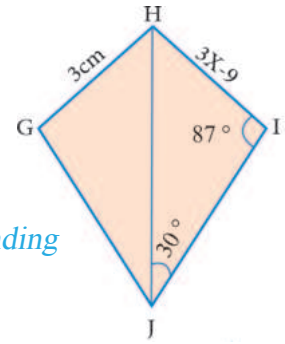
Example 3 In near by figure:

* Angle BAC is congruent on Angle $B'A'C'$, ($\angle BAC \cong \angle B'A'C'$)
* Angle ACB is congruent on Angle $A'C'B'$, ($\angle ACB \cong \angle A'C'B'$)
* Side $\overline{AC}$ is congruent on Side $\overline{A'C'}$, ($\overline{AC} \cong \overline{A'C'}$)
* It means $\triangle ABC \cong \triangle A'B'C'$ (A S A)



Example 4 In near by figure:

* Find value of X that makes $\Delta GHJ \cong \Delta IHJ$.
* If measure of $\angle HIJ$ equals 87°, what is the measure of $\angle HGJ$?
* If measure of $\angle IJH$ equals 30°, what is the measure of $\angle HJG$?



* $3X-9 = 3$ *one of the congruence properties (the corresponding sides are equaled)*

$3X = 3 + 9$ *relationship of addition by subtraction*

$3X = 12$ *simplifying*

$X = 4$ *result*

* $m\angle HGJ = m\angle HIJ$ *one of the congruence properties (the corresponding sides are equaled)*

Then: $m\angle HGJ = 87^0$ *by substitution*

* $m\angle HJI = m\angle HJG$ *properties of congruent (Equal of corresponding angles)*

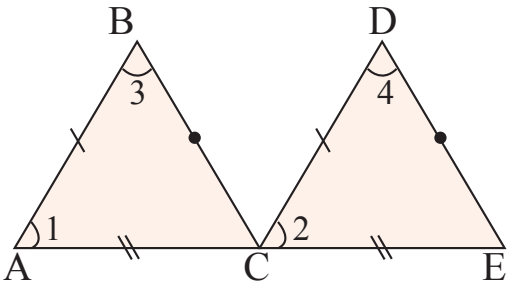
Then: $m\angle HJG = 30^0$ *by substitution*

Make sure of your understanding

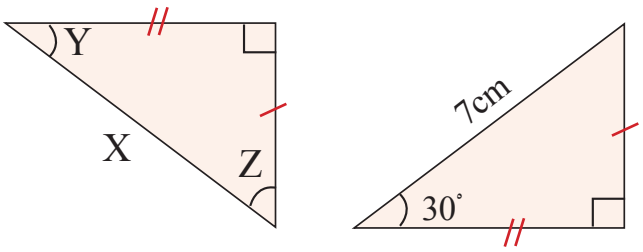
1 Notice the near by figure, if $\Delta ABC \cong \Delta CDE$, then complete the table:

Congruent elements	Congruence case
$m\angle 1 = m\angle 2$, $m\angle 3 = m\angle 4$ $AB = CD$	
$m\angle 1 = m\angle 2$, $AB = CD$ $AC = CE$	
$AB = CD$, $AC = CE$ $BC = DE$	

Questions 1-3 are similar to examples 1-3



2 Find value of X, Y, Z which indicated in near by figure if the two triangles are congruent:

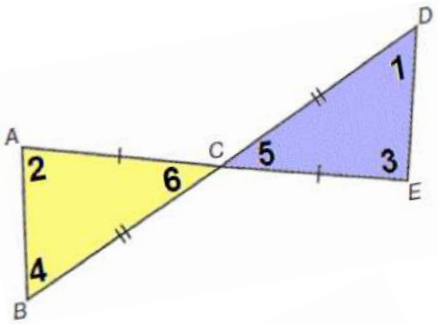


Question 4 is similar to example 4

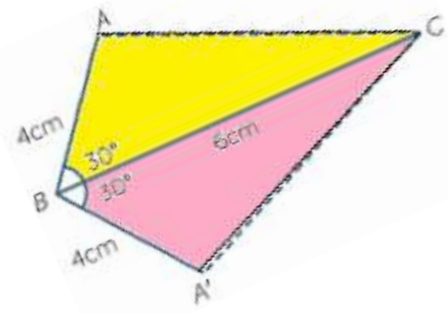
Solve the Exercises

3 Notice the near by figure $\overline{DE} \parallel \overline{AB}$, $\Delta ABC \cong \Delta CED$, complete the table:

Congruence case	Congruent elements
Two sides and included angle	
Two angles and included side	
Three sides	



4 Look at the two triangles ABC, A´B C in near by figure, then write using symbols the names of two congruent angles, and equaled sides, then express the congruentce by symbolic form and show the type of congruence.



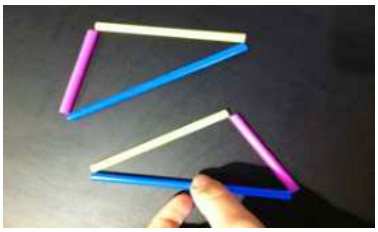
Solve the problems

Building: **Look at near by figure of two windows.**

5 How many congruent triangles can you count?

6 Which type of congruence of triangles is there?

Amusement: **Look at the two triangles in near by figure.**



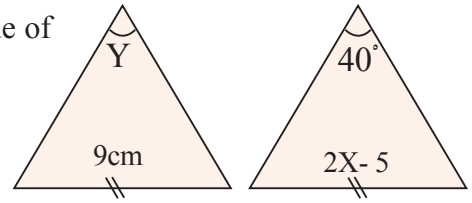
7 What type of congruence is between the two triangles?

8 Try (practically) by using the same manner to show the other cases of cogruence.

9 Garden: A garden of flowers was divided as shown in near by figure.

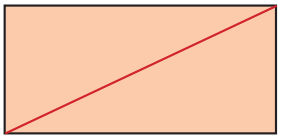
Prove that $\Delta ADC \cong \Delta BCD$

10 Geometry: From the two congruent triangles find the value of X,Y .



Think

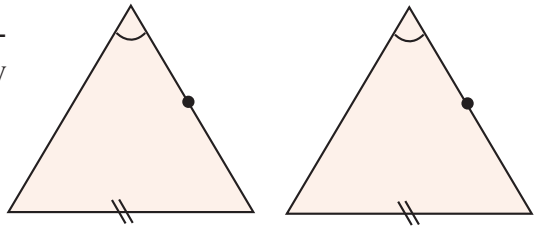
11 Challenge: Are two right-angled triangles congruent if they have equaled hypotenuses and both of them have equaled right sides.
Illustrate your answer.



12 Open problem: Clarify what is the difference between congruence of two triangles and similarity of two triangles?

13 Numerical sense: If we draw diameter of rectangular we get two triangles? Are the two triangles congruent? Why?

14 Correct the mistake: Tamara said that the two triangles are congruent as shown in below figure. Show Tamara's mistake and correct it.



Write

The cases that two triangles aren't congruent and give illustrative examples with drawing.

Lesson [5-3] Properties of the Triangles (Isosceles triangle, Equilateral triangle, Right-angled triangle)

Idea of the lesson:
* Identify properties of triangles (Isosceles triangle, Equilateral triangle, Right-angled triangle).

Vocabulary:
* Base of triangle
* Vertex angle
* Height of triangle

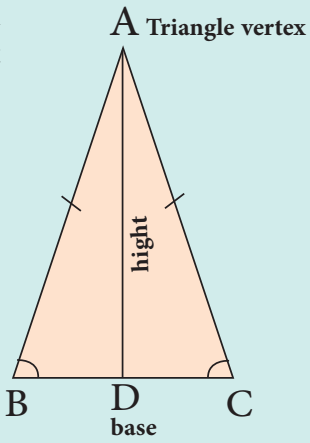
Learn

We can classify triangles according to its sides into:
1- Isosceles triangle
2- Equilateral triangle
3- Right-angled triangle
We will recognize properties of each by detail.

You have previously learnd the types of triangles according to its sides and angles measurement, in this lesson we will recognize properties of isosceles triangle, equilateral triangle and right-angled triangle.

[5-3-1] Properties of Isosceles Triangle

Isosceles triangle: Is a triangle that has two equal sides, the third side which is different in length from the two equal sides called triangle base, and the point which is opposite to triangle base is called triangle vertex.

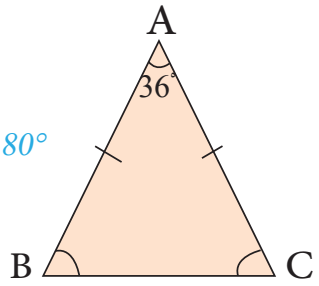


Properties of Isosceles triangle:

- The two base angles which are opposite to two equal sides, are equal.
  $m\angle B = m\angle C$
- Any triangle has two equal angles is an isosceles triangle.
- Bisector of angle of an isosceles triangle vertex is perpendicular on the base and divede it into two equaled parts.

  $BD = DC$, $\overline{AD} \perp \overline{BC}$

Example 1 In the near by figure, if measure of $\angle BAC=36°$, and triangle ABC is isosceles one. Find measure of angle ABC:



$m\angle ABC = m\angle ACB = X$, *AB=AC Isosceles triangle*

$m\angle A + m\angle B + m\angle C = 180°$, *Sum of angles of any triangle equals 180°*

$36°+X+X=180°$ *by substitution*

$36°+2X=180°$ *by simplifying*

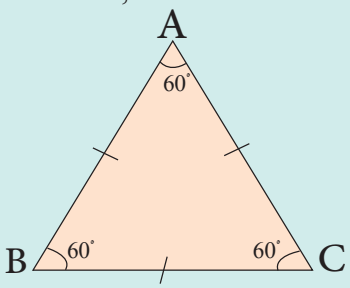
$2X = 180°-36°=144°$ *relationship between addition and subtraction*

$X = \frac{144}{2}$ *dividing the two sides of equation by 2*

$m\angle ABC = 72°$

[5-3-2] Properties of Equilateral Triangle

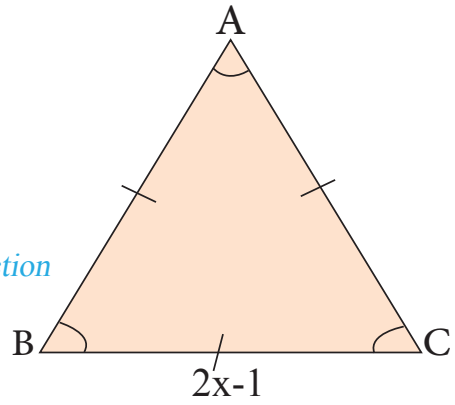
Equilateral triangle: Is a triangle which its three sides are equal in measurement, it could be called a regular triangle too.



Properties of equilateral triangle:

- All its angles are equal in measurement and each one of them is 60° .
- The triangle that its angles are equal in measurement, its all sides will be equal too.

Example 2 In near by figure an equilateral triangle. Its perimeter 57cm. Find length of each side then find value of x:



$$\text{Side length} = \frac{\text{perimeter of equilateral triangle}}{3} = \frac{57}{3} = 19$$

To find value of x:

$2x-1=19$ *write equation of question*

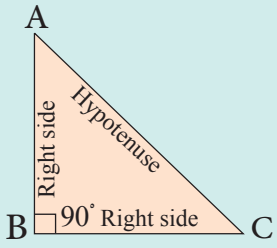
$12x = 19 + 1$ *relationship between addition and subtraction*

$2x = 20$ *simplifying*

$x = \frac{20}{2} = 10$ cm *dividing the two sides of equation by 2*

[5-3-3] Properties of Right-Angled Triangle

Right-angled triangle: Is a triangle which has two perpendicular sides, i.e. (angle between them is right and equals 90°). We called the opposite side to right angle, which is the longest side of triangle (hypotenuse), we also called the two other sides as two right sides.



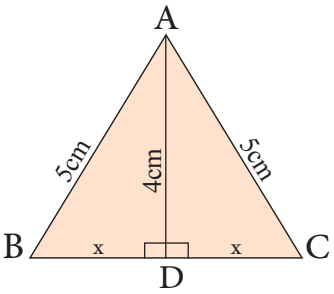
Properties of right-angled triangle (pythagorean theorem):

In any right-angled triangle the sum of two right sides square length equal to square of hypotenuse length.

We can express this theorem mathematically as follows:

$(AC)^2 = (AB)^2 + (BC)^2$

Example 3 Use the near by figure and find length of $\overline{BC}$:



Triangle ABC is isoscels

$\overline{AD} \perp \overline{BC}$ *given information*

$(AC)^2 = (AD)^2 + (DC)^2$ *(Pythagorean theorem)*

$5^2 = 4^2 + X^2$ *by substitution*

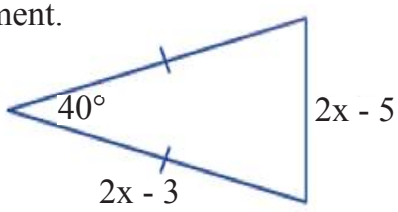
$25 = 16 + X^2$ *by simplifying*

$X^2 = 25-16 \Longrightarrow X^2 = 9 \Longrightarrow X = 3$cm *relationship between addition and subtraction, square root of two sides.*

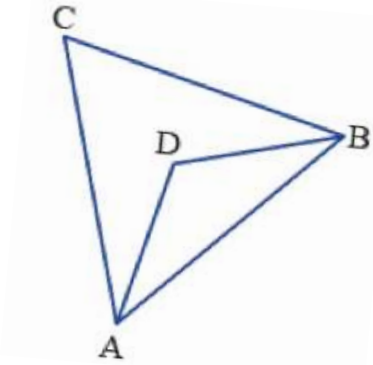
From the given information we found that length of side $\overline{BC}$ equals 6cm. (properties of isosceles triangle).

Make sure of your understanding

1 In near by isosceles triangle, if you know that the perimeter is 19cm, find value of x, length of each side and the two remaining angles measurement.



2 Triangle ABC is equilateral, $\overline{AD}$, $\overline{BD}$ are bisectors of two angles CAB and CBA. Find measurement of angle ADB.

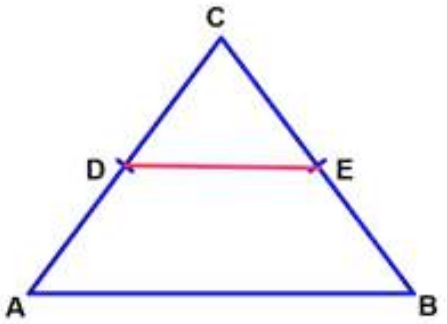


3 A triangle its three sides length are 6cm, 10cm, 8cm. Is the triangle a right-angled one?
Clarify that with drawing.

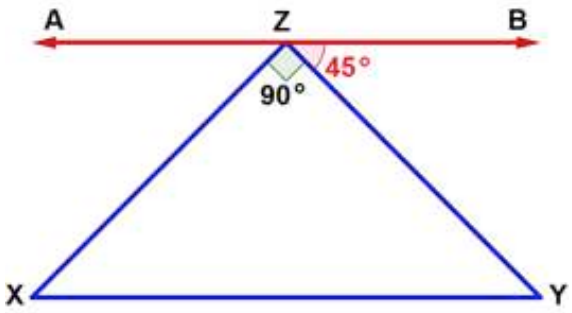
Questions 1-3 are similar to examples 1-3

Solve the Exercises

4 In near by figure m ∠CAB= m ∠CBA
$\overline{DE}$ parallel to $\overline{AB}$. Show why triangle CDE is an isosceles triangle?

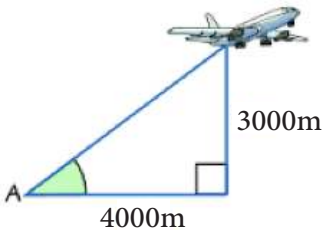


5 In near by figure triangle XYZ is right-angled triangle in Z, draw line AB crossing by vertex Z and parallel to base $\overline{XY}$, m ∠ BZY=45°.
Prove that triangle XYZ is isosceles.

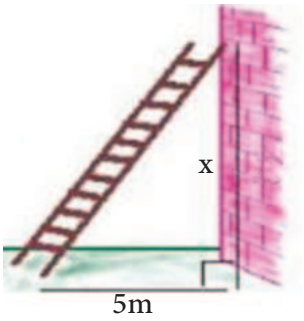


Solve the problems

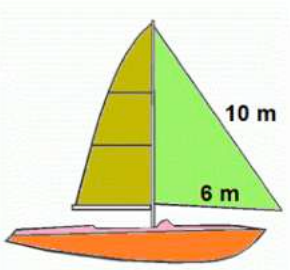
6 **Building:** In the near by illustrative figure, find the distance in metre, between the plane and point A.



7 Find value of x in near by figure if you know the length of ladder is 13m.

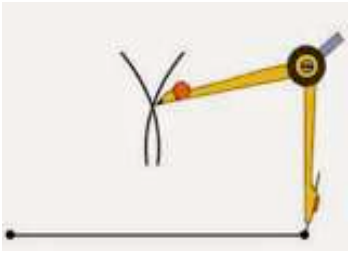


8 In the sailboat which is illustrated in near by figure, find height of the green part of sail then calculate its area.



Think

9 **Challenge:** By using a compass and ruler try to draw an equilateral triangle its side length 4cm (look at picture and conclude the method).



10 **Open problem:** What is the measure of each angle in a right-angled triangle and isosceles triangle? Clarify your answer by drawing.

11 **Numerical sense:** Is there any right-angled triangle and it can be equilateral triangle at the same time? Clarify by numerical examples.

12 **Correct the mistake:** Ahmed says that the triangle which its sides length 4cm, 3cm, 2cm, represents sides of right-angled triangle. Discover Ahmed's mistake and correct it.

Write

Three sets of positive integers which deals with the mathematical formula of pythagorean theorem.

Lesson [5-4] Parallelogram, Rhombus and Trapezoid

Idea of the lesson:
* Using properties of parallelogram,rectangle, rhombus, and trapezoid to solve geometrical problems.

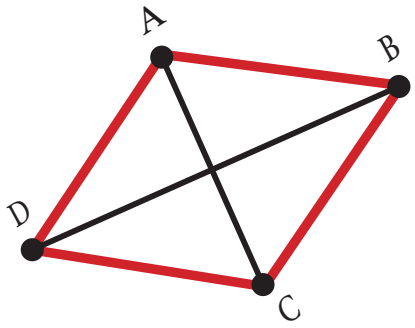
Vocabulary:
* Parallelogram
* Rhombus
* Trapezoid

Learn

The near by geometrical figure ABCD represents a parallelogram where:

1) $\overline{AB} \parallel \overline{CD}$, $\overline{AD} \parallel \overline{BC}$
2) AB = CD , AD = BC

The continued line between each two opposite vertices is called diagonals of parallelogram $\overline{BD}$, $\overline{AC}$

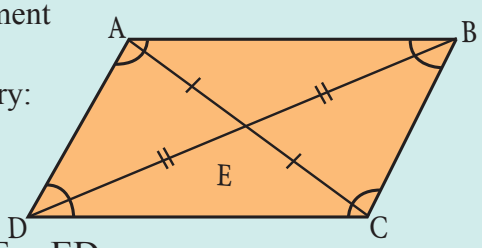


[5-4-1] Parallelogram

You have previously learnd that the parallelogram is a quadrangle in which each two opposite sides are parallel, now you will recognize the theorems and properties of parallelogram and the way of using it to solve geometrical problems.

Theorem of parallelogram properties:

* Each two opposite sides of parallelogram are congruent AD = BC , AB = DC.
* Each two opposite angles in parallelogram are equal in measurement
  $m\angle A = m\angle C$, $m\angle B = m\angle D$
* Each two consecutive angles in a parallelogram are supplementary:
  $m\angle A + m\angle D = 180^o$, $m\angle D + m\angle C = 180^o$
  $m\angle C + m\angle B = 180^o$, $m\angle B + m\angle A = 180^o$



* The two diagonals of parallelogram are bisector AE = EC , BE = ED
* Two triangles DAB , DCB are congruent, Two triangles ABC , ADC are congruent
* Two triangles EBA , ECD are congruent, Two triangles EBC , EAD are congruent

Example 1 Use properties of parallelogram to find measurements of angle C and angle D with degrees, then find the length of each side AB and DC in centimetres from near by figure:

$m\angle C + m\angle D = 180^o$ *two supplementary angles*

$2x + 5^o + 4x - 35^o = 180^o$ *by substitution with the angle value*

$6x - 30^o = 180^0 \Rightarrow 6x = 210^o \Rightarrow x = 35^o$

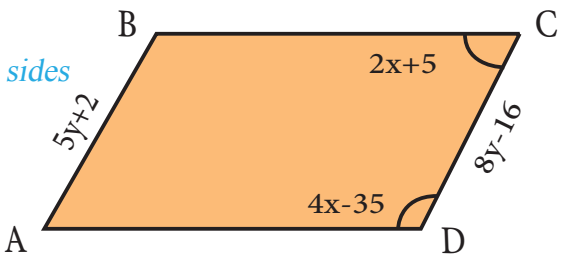
$m\angle C = 75^o$, $m\angle D = 180^o - 75^o = 105^o$

$8y - 16 = 5y + 2$ *two opposite congruent sides*

$8y - 5y = 2 + 16 \Rightarrow y = 6$ *by solving the equation*

AB = 5 ×6 +2 = 32cm

DC = 8× 6 -16 = 32cm



[5-4-2] Rhombus

Rhombus: Is a parallelogram which each two adjacent sides are equaled.

Theorem of rhombus properties:

* Rhombus's two diagonals are perpendicular.
* Each diagonals bisects the two angles at its two ends.

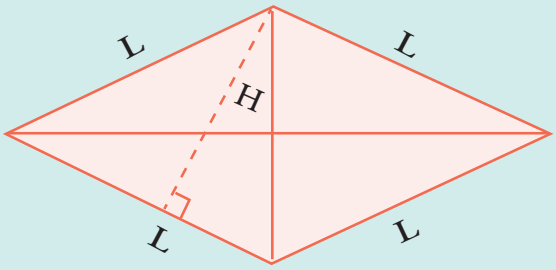
Rhombus area = length of side × height

That means A = L × H

Or (half result of multiplying its two diagonals length).

The perimeter = 4 × side length... i.e. :

P = 4 x L



Example 2 Use rhombus properties to find the length of side BC and rhombus perimeter:

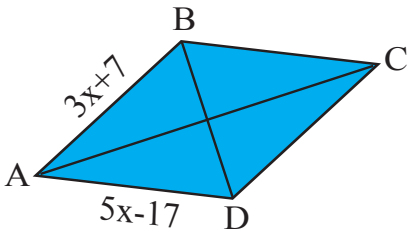
i) AD = AB

$5x - 17 = 3x + 7 \Rightarrow 5x - 3x = 17 + 7 \Rightarrow x = 12$

BC = AD $\Rightarrow 5 \times 12 - 17 = 43$ cm

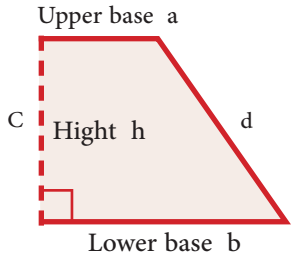
ii) P = 4 × L

P = 4 × 43 = 172cm

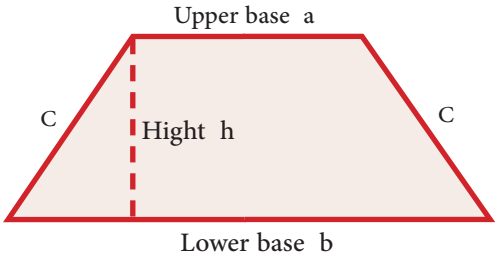


[5-4-3] Trapezoid

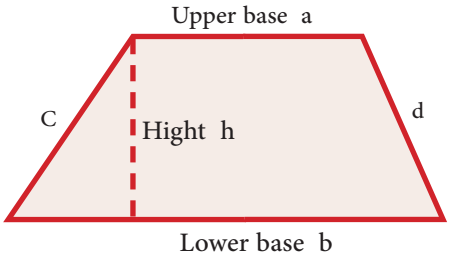
You have previously learnd the trapezoid which is quadrangle with different lengths. It has two parallel sides called two base of trapezoid and the other two sides which are the non parallel and they are known as the two legs of trapezoid. If the two non parallel sides are equal then trapezoid will be called isosceles trapezoid and if one of its angles is right then it is called right angle trapezoid.



Right-angled trapezoid



Isosceles trapezoid



Not isosceles Trapezoid

Area of trapezoid $A = \frac{1}{2}(a+b)\times h$, Perimeter of trapezoid $P = a+b+c+d$

Example 3 i) Find area of trapezoid which its two parallel sides lengths are 12cm, 8cm and its height 4cm.

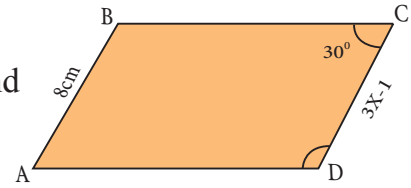
$A = \frac{1}{2}(a+b) \times h = \frac{1}{2}(8+12) \times 4 = 40 \text{ cm}^2$

ii) Find Perimeter of isosceles trapezoid, length of each of them is 8cm and length of its upper base is 5cm, and lower base is 10cm.

P = a+b+c+d =5+10+8+8=31cm

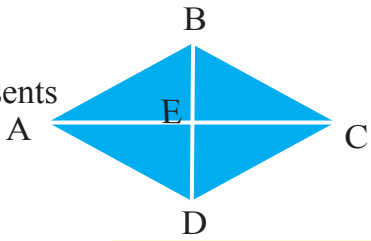
Make sure of your understanding

1 Use properties of parallelogram of near by figure to find measurement of each: $\overline{CD}$, $m\angle A$, $m\angle D$.



2 Find perimeter of a parallelogram if you know length of its one sides is 8cm and length of its near by side is three times of it.

3 Near by figure ABCD is a parallelogram which has $m\angle BCE + m\angle ADE = 90^o$, prove that this figure represents rhombus.



4 A rhombus has an area of $300cm^2$ and its height 15cm. What is its side length?

5 A rhombus has a perimeter of 36cm. What is its side length?

Questions 1-5 are similar to example 2

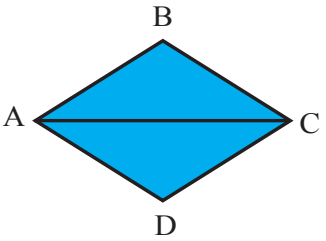
6 A trapezoid, the length of its two parallel bases upper and lower are respectively 7cm, 9cm, its height 3cm what is its area?

7 An isosceles trapezoid has an area of $90cm^2$ and its height 5cm. Find length of its two bases if you know that the length of its upper base is half of its lower base.

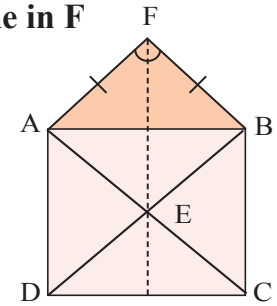
Questions 6-7 are similar to example 3

Solve the Exercises

8 Near by figure has AB = BC and $\overline{AC}$ is a bisector of two angles A, C. Prove that the figure ABCD represents parallelogram.



9 **Near by figure ABCD is a square, AFB is a right-angled triangle in F and isosceles. Prove that:**
i) AFBE represents square.
ii) $\overline{FE}$ is bisecting $\overline{DC}$
iii) AFED represents parallelogram.



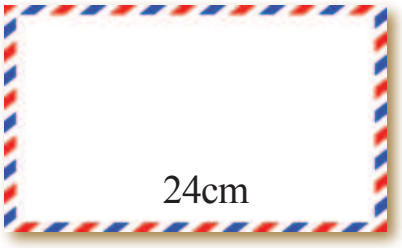
10 ABCD represents rhombus, the points E, F, G, H are midpoints of its sides. prove that figure EFGH represents rectangle.

11 A rhombus has two perpendicular diagonals, they are 8cm,10cm.What is its area?

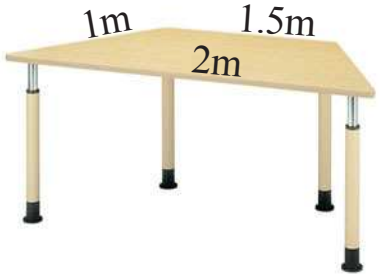
12 A trapezoid has two parallel, upper and lower, they are 16cm, 20cm, and its area is $180cm^2$. What is its height?

Solve the problems

13 **Postal envelope:** In near by figure of postal envelope if you know the width of the envelope is half of its length, what is its face area?



14 **Home furniture:** Surface of wood plank which is used in the table as an isosceles trapezoid. Calculate its perimeter.



15 **Garden:** A rectangular piece of agricultural land is, planted with trees to become a park. It needed to be surrounded by a fence, if its dimensions are 40m, 80m, what is the length of fence should be used to complete the surrounding?

Think

16 **Challenge:** A trapezoid, its minor base length is 3cm divided into three shapes, two triangles and a rectangle, the height of trapezoid is 4cm and length of right side of first triangle is 2cm, length of right side of second triangle is 1cm, calculate area of trapezoid. Using two methods.

17 **Open problem:** Could we consider each rectangle as a parallelogram, and on the contrary, we couldn't consider each parallelogram as a rectangle?

18 **Numerical sense:** What is the difference between rhombus and square? Draw and indicate the different parts.

Write

Properties of non isosceles trapezoid and isosceles trapezoid.

Lesson [5-5] Cylinder and Sphere (Properties, Surface Area and Volume)

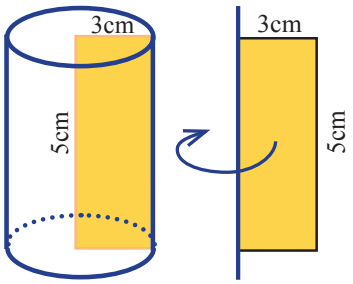
Idea of the lesson:
* Identify properties of each cylinder, sphere and how to find surface area and volume for each of them.

Vocabulary:
* Radius
* Height
* Lateral area
* Total area
* Volume

Learn

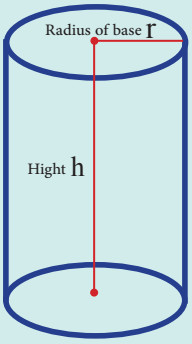
We have a rectangle its dimensions are 5cm, 3cm. We fix it on a vertical metallic cable then rotate as shown in near by figure. By using a motivator, we can note creating of a shape which we called right circular cylinder.

Note that radius of cylinder base which is generated and its height represent the value of dimensions of rectangle. Take cylindrical Ice-cream box then remove its two bases and then cut it vertically. After that open the metal piece you will note it represents a rectangle shape.



[5-5-1] Cylinder

It is a solid which has two congruent and parallel circular bases, Its walled by cylindrical lateral surface. The line which crosses the centre of two bases called cylinder axis, and the line which is tangent to the two base and parallel to cylinder axis is called cylinder generation. Each of cylinder generations is equal with each other in length. But the perpenicular segment which is fixed between the two bases of cylinder is called cylinder height and refered to it by Symbol **h**. If cylinder generation is perpendicular on its base, the cylinder will be named (right cylinder), and its height will be equal to its generation, but if cylinder generation is sloped on its base it will be named as (oblique cylinder).



Volume of right circular cylinder $V = \pi r^2 h$

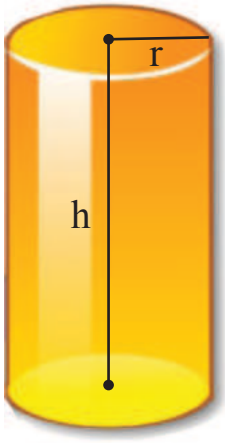
Lateral area $LA = 2\pi r h$

Total area equals sum of lateral area with area of the two bases, i.e. :

$$TA = 2\pi r h + 2\pi r^2$$

Example 1

A right circular cylinder, the length of its radius base 7cm, and its height 12cm. Calculate its lateral area then calculate its total area with its volume.



i) Lateral area $LA = 2\pi r h = 2 \times 7 \times \frac{22}{7} \times 12 = 528 \text{ cm}^2$

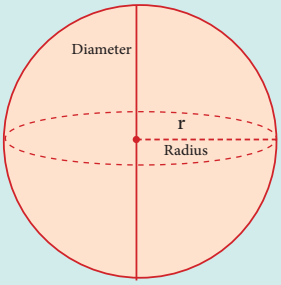
ii) Total area $TA = 2\pi r h + 2\pi r^2 = 528 + 2 \times \frac{22}{7} \times 7^2$

$= 528 + 308 = 836 \text{ cm}^2$

iii) Volume $V = \pi r^2 h = \frac{22}{7} \times 7^2 \times 12 = 1848 \text{ cm}^3$

[5-5-2] Sphere

Sphere: is a solid which represents the set of all points in the three dimensonal space. One of its properties that each point of its surface points is distant with equal distances from a certain fixed point inside it called centre of sphere. Each line which crosses through the sphere centre and reaches up to its surface called diameter of sphere, while the radius of sphere is the continued line between sphere centre and any point of its surface points.



Sphere volume: $V = \frac{4\pi}{3} r^3$

Surface area: $SA = 4\pi r^2$

Example 2 Find surface area and volume of sphere which its radius is 7cm.

i) Sphere area $SA = 4\pi r^2 = 4 \times \frac{22}{7} \times 7^2 = 616 \text{ cm}^2$

i i) Volume $V = \frac{4}{3} \pi r^3$

$V = \frac{4}{3} \times \frac{22}{7} \times 7^3 = \frac{4312}{3} \simeq 1437 \text{ cm}^3$

Example 3 Money box: A small box for saving money was made as a cylinder. Its base radius 10cm and its height 30cm, which is topped by hemisphere. Calculate the volume and surface area of it.

Volume of Money box = Volume of cylinder + Volume of hemisphere

$V = \pi r^2 h + \frac{1}{2}\left(\frac{4}{3} \pi r^3\right) = \pi (10)^2 (30) + \frac{1}{2}\left(\left(\frac{4}{3} \pi 10^3\right)\right)$

$V = 3000 \times 3.14 + \frac{2000}{3} \times 3.14 \simeq 9420 + 2093.3$

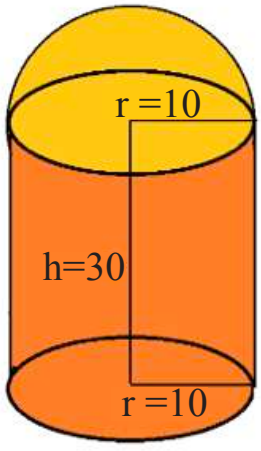
$V = 11513.3 \text{ cm}^3$

Total area = lateral area of cylinder + Area of one base + Half surface area of the sphere

$TA = 2\pi r h + \pi r^2 + \frac{1}{2}(4 \pi r^2)$

$TA = 2(3.14)(10)(30) + (3.14)(10)^2 + 2(3.14)(10)^2$

$TA = 1884 + 314 + 628 = 2826 \text{ cm}^2$



Make sure of your understanding

1 A right circular cylinder with hollow tube, its length is 21m with internal radius of 9m, what is its volume?

2 Find surface area and volume of sphere which has a radius of 10m.

3 A fuel tank was made as shape of cylinder. Its base radius 3m, and its height 9m topped by hemisphere, calculate the volume and surface area of the tank.

4 A spherical fuel dump has surface area of $576\pi m^2$.
Find its volume.

Questions 1-7 are similar to examples 1-3

5 A container, as shape of hemisphere, has surface area of $128\ \pi cm^2$. Find its volume.

6 A cylindrical beaker has volume of $128\ \pi cm^3$, and height of 8cm. Find its lateral area.

7 If the ratio of volume of sphere has radius r_1 to volume of second sphere has radius r_2 is equal to $\frac{8}{125}$, find the ratio of surface area of first sphere to surface area of second sphere.

Solve the Exercises

8 A lead-made-sphere has a radius of 6cm. It is melted and small spheres where made of it to be similar to baby games; Both radius are 1cm, calculate number of small spheres which generated from it.

9 A wood sphere its volume $2304\pi cm^3$. It was put in water, the area of floating part was the half of it. Find radius of circle which intersects surface of water with surface of sphere.

10 If you know that the surface area of sphere equals $1256cm^2$, what is the length of sphere radius?

11 The ratio of volume of two spheres is 27:8, find the ratio between their two surface areas.

12 Find radius of sphere which has surface area equals $100\pi cm^2$, then find its volume.

13 A piece of paper shaped as a rectangle has length of 33cm and width of 14cm, its base was folded in a way that the surface of paper was form as of right circular cylinder surface. Find the formed cylinder volume.

Solve the problems

14 **Liquid:** Find the capacity of cub near by if you know the base diameter is 7cm and its hight is 10cm.

15 **Industry:** Calculate the amount of capacity of four cylindrical cans which are equal in volume, if you know its base radius is 3.5cm, and hight 10cm, then find the required area of metallic plate which is used for manufacturing all of them.

16 **Geometry:** One of the famous buildings in world is the building which is known as Montreal dome in Canada. It is a circular sphere of transparent glass, its diametre 76cm.
Calculate the surface area and volume.

Think

17 **Challenge:** A plastic sphere has a radius of 14.7cm. it was painted. The thickness of painting is 0.3cm, calculate volume of sphere after painting.

18 **Open problem:** What is the height of a right circular cylinder which its lateral area is enough to make a sphere which has radius of the same cylinder?

19 **Numerical sense:** A cup is designed as a hemisphere with radius r and a cylindrical container whith radius r and height of r. Which one of them will contain more water than the other?

20 **Correct the mistake:** Sara wrote a sphere volume rule as $\left(\frac{3}{4}\pi r^3\right)$.
Discover Sara's mistake and correct it.

Write

An example of two right cylinders equal in volume but different by total area.

Lesson [5-6] Area of Regular and Irregular compound shapes

Idea of the lesson:
* To find area of regular and irregular compound plane shapes.

Vocabulary:
* Simple plane shape
* Compound plane shape

Learn

We want to calculate the path area which surrounded the pool as shown in near by picture so we will follow these steps:
We determine the two simple plane shapes which are composed of this compound shape, and they are rectangles:
• We find the external area of rectangle.
• We find the internal area of rectangle.
• We find area of the path by subtracting the internal area of rectangle from the external area of rectangle.

[5-6-1] Area of Regular compound Plane shapes

Regular compound plane: A regular compound plane shape is composed of two simple plane shapes or more shapes. To find its area we will divide regular compound plane shape into simple plane shapes.

Example 1

Try to find the path area in compound shape in paragraph (learn).

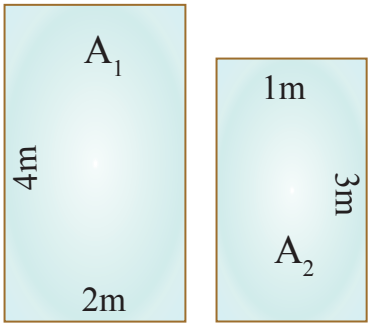
The two simple plane shapes which the compound shape is composed from, are rectangles.

We find external rectangle area: $A_1 = L \cdot W = 4 \times 2 = 8\ m^2$

We find internal rectangle area: $A_2 = L \cdot W = 3 \times 1 = 3\ m^2$

Area of bath equals the result of subtraction of internal rectangle area from external rectangle area, that means:

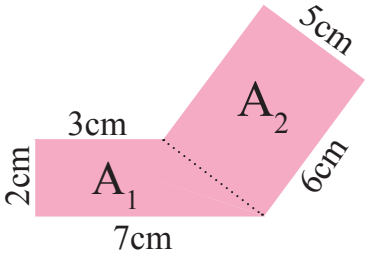
$$A = A_1 - A_2 = 8 - 3 = 5\ m^2$$

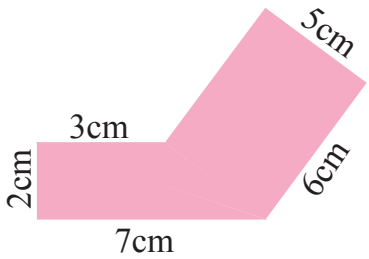


Example 2

To calculate the area of below regular compound shape:

The regular compound shape contains two simple plane shapes,which they are trapezoid and rectangle:





$A_1 = \frac{1}{2}(a+b) \times h = \frac{1}{2}(3+7) \times 2 = 10 cm^2$ *Area of trabezoid*

$A_2 = L \cdot W = 6 \times 5 = 30\ cm^2$ *Area of rectangle*

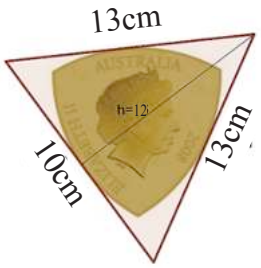
$A = A_1 + A_2 = 10 + 30 = 40\ cm^2$ *Area of compound shape*

[5-6-2] Area of Irregular compound Plane shapes

Irregular compound plane: An irregular compound plane shape is composed of two simple plane shapes or more irregular.
To find its area we divide the irregular compound plane shape into simple regular plane shapes which near to its origion shape. We calculate area of simple plane shapes then adding its results, then it will be as a proximate value of irregular compound plane shape.

Example 3

In near by figure a model of coin of one of the countries, it is designed as an irregular plane shape to calculate approximate value of coin surface area, we draw an isosceles triangle which its sides tangent to the coin curve. We fixed the sides measurement and triangle height as shown in figure:

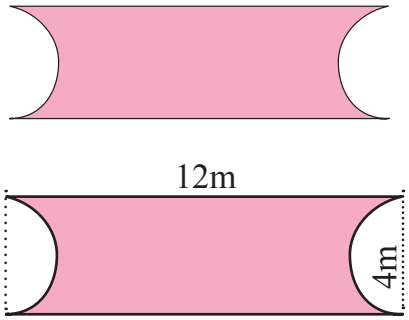


We calculate triangle area: $A = \frac{1}{2} \times b \times h = \frac{1}{2} \times 10 \times 12 = 60cm^2$

And because surface area of coin is less than surface area of triangle, we make an approximate result and say:
Surface area of the coin is approximately less than $60cm^2$.

Example 4

To calculate area of a near by shaded shape:



We divide the shape into two simple plane shapes which the compound shape composed from, and they are a rectangle and two halves of similar circle which are equal, (We can consider them as one circle)

$A_1 = L \cdot W = 12 \times 4 = 48\ m^2$ *rectangle area*

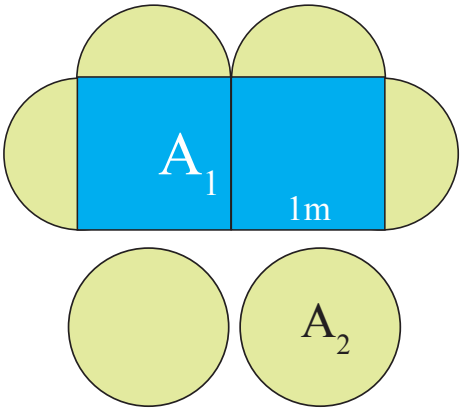
$A_2 = \pi \times r^2 = 3.14 \times 2^2 = 12.56\ m^2$ *area of two halves of circle = circle area*

Area of shaded shape equal to subtraction of circle area from rectangle's area.

$A = A_1 - A_2 = 48 - 12.56 = 35.44m^2$

Example 5

Find area of surface of compound plane shape which is shown below.
The compound shape is composed of two similar squares and two similar circles (four halves of similar circle)



$A_1 = L^2 = 1^2 = 1\ m^2$ *square area*

$A_2 = \pi \times r^2 = 3.14 \times (0.5)^2 = 0.785\ m^2$ *circle area*

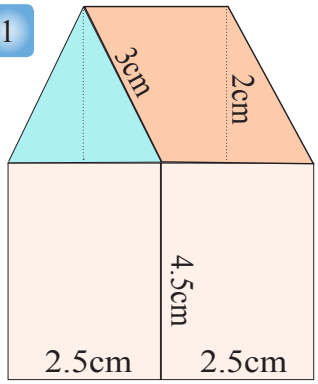
Area of shaded shape equal to

$A = 2A_1 + 2A_2 = 2 \times 1 + 2 \times 0.785 = 3.57m^2$

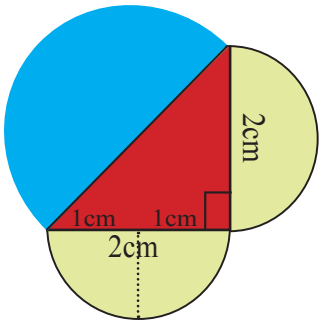
Make sure of your understanding

Find the area of the shaded surface of the following compound plane shapes:

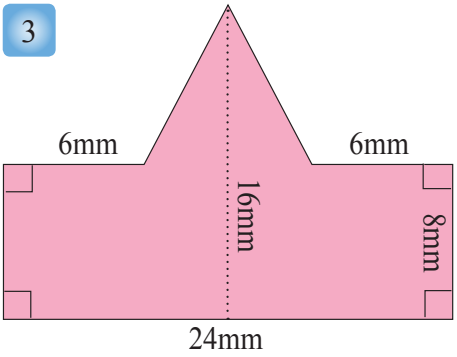
1



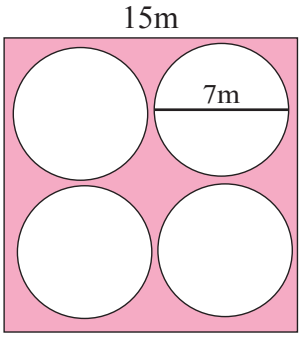
2



3

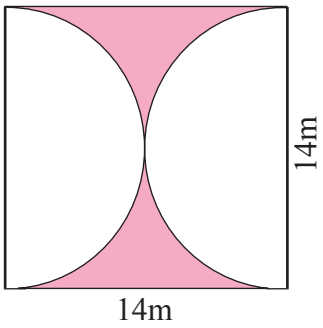


4



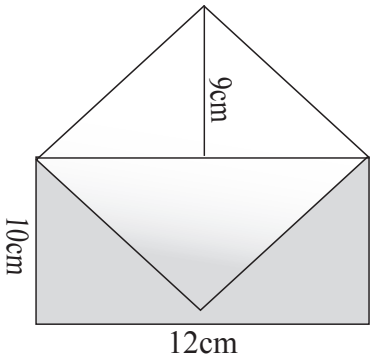
Questions 1-5 are similar to examples 1-5

5

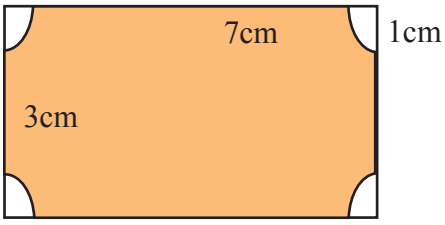


Solve the Exercises

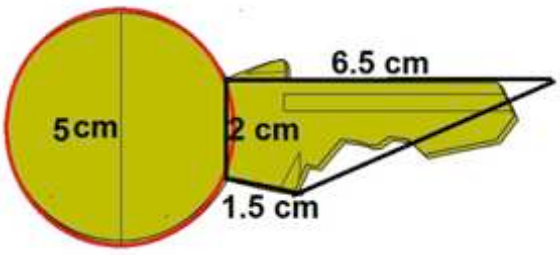
6 Find the surface area of the open envelope shown in the near by figure from one face:



7 Find the area of the shaded surface:

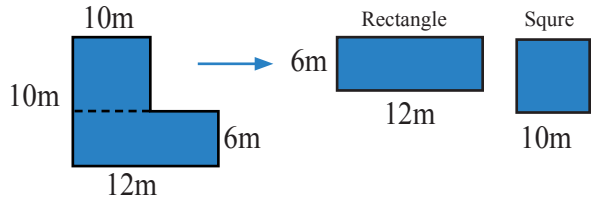


8 Find a suitable approximate for the area of the key surface in the near by figure:

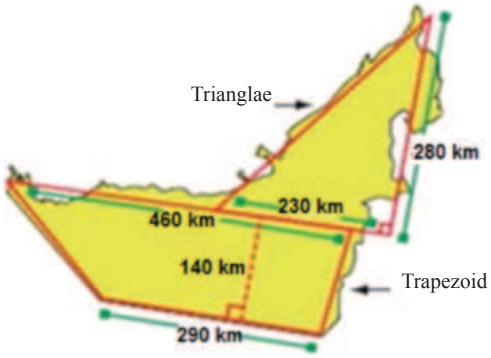


Solve the problems

9 Below is a picture of swimming pool with a diagram of its compound plane surface. Calculate the area of the swimming pool.

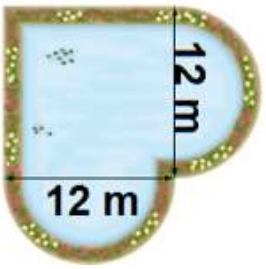


10 In the near by figure, a map of a country, which there is no doubt, represents an irregular compound plane. For the purpose of estimating the area on the map, two simple plane shapes were drawn which are trapezoidal and triangular, to surround the boundaries of the map using the fixed dimentions on the shape. Find suitable approximate to the area on the map.

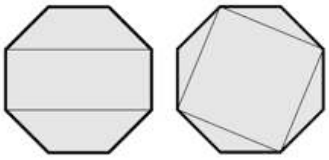


Think

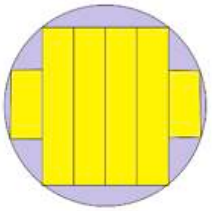
11 Challenge: In near by figure a pool surrounded by a path of tiles its width is 2m. Calculate the area of the path.



12 Open problem: The figure shows two different methods to find a regular polygon area with eight sides. Explain the content of the two methods and look for a third method.



13 Numerical sense: Can you calculate an approximate circle area by drawing adjacent rectangles inside it? What happens when we make the number of drawn rectangles too large?



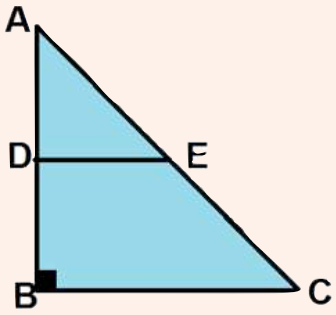
Write

A mathematical formula to calculate the area a compound plane shape composed of K of adjacent drawn rhombuses with length of each diametre n , m of units.

Chapter Test

1 ABC is an equilateral triangle, we draw line CE on an extension of side BC. Find m $\angle$ ACE.

2 In near by figure, AB=BC, $\overline{BC}$ // $\overline{DE}$, prove that triangle ADE is isosceles.



3 Give an example with drawing showing that two triangles can not be congruent if they have equal corresponding angles.

Complete the following blanks, to get true statement:

4 The two angles will be congruent if..... .

5 Triangle's six elements are..... and..... .

6 Polygons are congruent if we could put one of them on other so that..... .

7 Cases of two triangles congruence are..... and..... and..... .

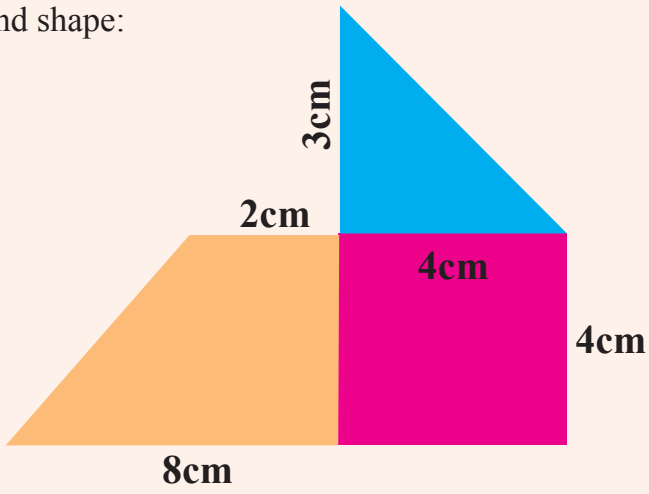
8 Isosceles trapezoid its area 180cm^2 and its height 10cm. Find the length of each of its base, if you knew that the length of its upper base is four times of its lower base.

9 Find perimeter and area of rhombus, its side length 4cm and its height 6cm.

10 A rhombus has two orthogonal diametres length of 2.5cm and 4cm. What is its area?

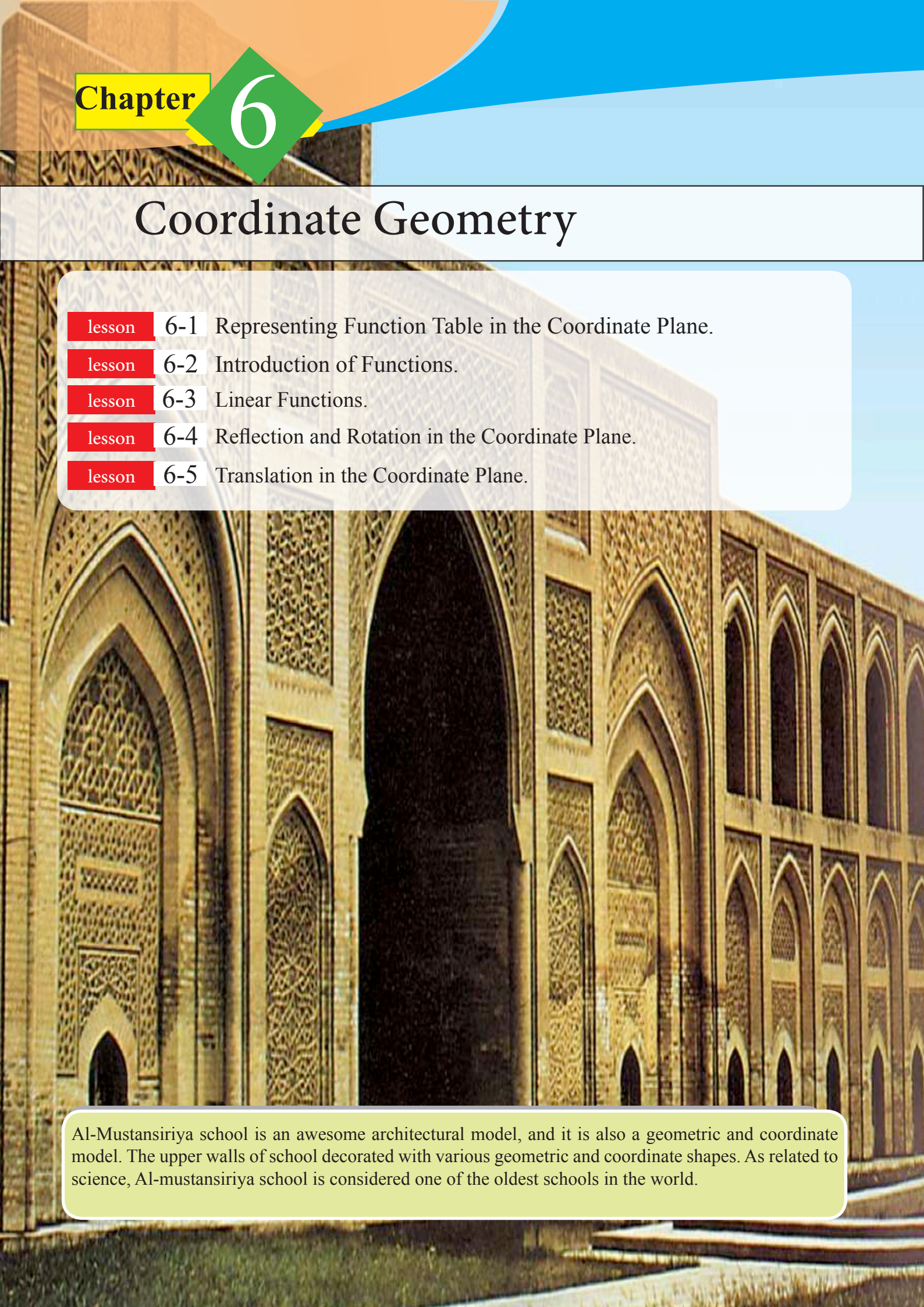
11 Table lamp as shape of cylinder, its radius is 8cm and its height is 12cm topped by hemisphere. Calculate its volume and surface area.

12 Find the area of the following compound shape:



Chapter 6

Coordinate Geometry



Al-Mustansiriya school is an awesome architectural model, and it is also a geometric and coordinate model. The upper walls of school decorated with various geometric and coordinate shapes. As related to science, Al-mustansiriya school is considered one of the oldest schools in the world.

Pretest

Write algebraic statement which represents:

1. Less than y by 15
2. Greater than N by 13
3. T-3 divided by T+3
4. 7^2 multiplied by L-9
5. Double 9-w multiplied by 5
6. Half w+9 multiplied by y
7. Quarter of T-5 divided by T
8. Cubic root for L-3T multiplied by 2+w
9. $2w-w^2$ is the function rule and {-1,0,1} are inputs, write the outputs for the function.
10. Write the function rule for the following inputs and outputs:

Intput	Function rule	Output
1		1
2		4
3		9
-1		1
-2		4

Intput	Function rule	Output
-2		0
0		2
2		4

11. $|y|+2y$ is the function rule, make a table and show the outputs where the intputs are {-1, 0, 1}
12. Construct a table and show in the intputs where the outputs are {3, 2, 1} and the function rule is X-2.

Represent the ordered pairs in the coordinate plane then draw segments to connect the points in each quarter, name the shape you will get:

13. A (0,2), B(0,-2), C(3,-2), D(3,2)
14. A(-1,3) , B(-1,-3) , C(-3,0)
15. A(-1,1) , B(0,2) , C(3,2) , D(3,1)
16. Show how the location of the point (-3,3) is different from the location of the point (3,-3).

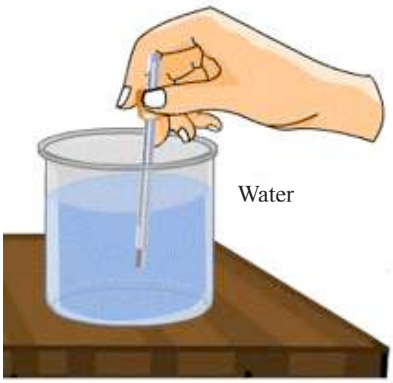
Lesson [6-1] Representing Function Table in the Coordinate Plane

Idea of the lesson:
* Representing function table in the coordinate plane.

Vocabulary:
* Input
* Output
* Function table
* Coordinate Plane
* The four quadrants

Learn

Ali wanted to measure the temperature of water in a flask. In the first hour he found it 3°C, in the second hour he found it 3°C. Find the temperature of water after 7 hours.



[6-1-1] Representation a Table In The Coordinate Plane

* The relation which has variable input and constant output, it represents a function of a line parallal to X-axis.
* The relation which has constant input and variable output, it represents a function of a line parallal to Y-axis.
* The relation which has variable input and output, it represents a function of a line non parallal to any axis.

Example 1 Find the temperature of water afetr 7 hours.

Step (1): We assume that number of hours is (X) and temperature of water in each hour is (Y).

Then we get the following table:

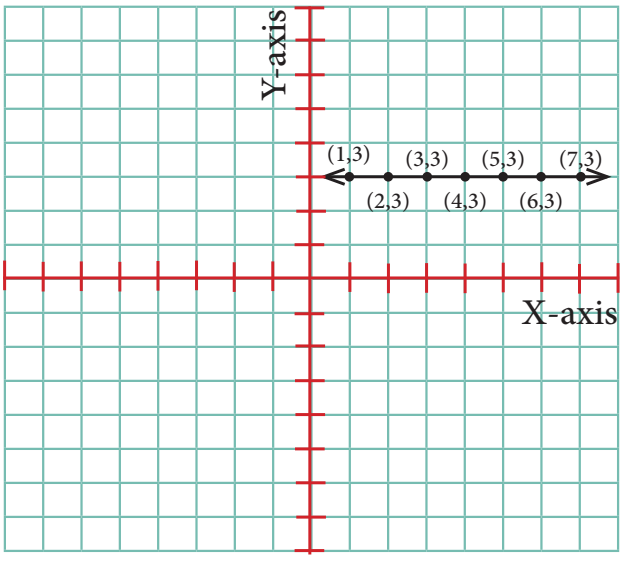
X	1	2	3	4	5	6	7
Y	3	3	3	3	3	3	3

Step (2): Use the table to find ordered pairs

(1,3), (2,3), (3,3), (4,3), (5,3), (6,3), (7,3).

Step (3): We determine the points in the coordinate plane, then we connect the points, we get a line parallel to X-axis.

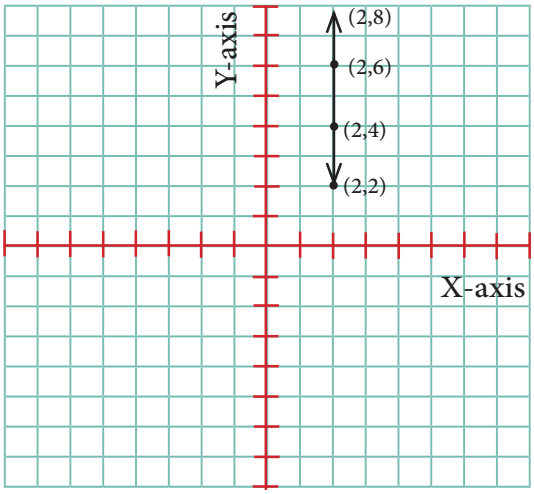
(x values are variable , y values are constant).



Example 2 Represent the following table in the coordinate plane.

X	2	2	2	2
Y	2	4	6	8

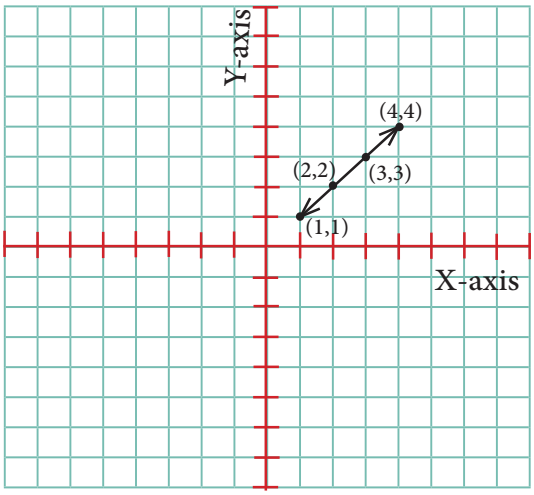
From the table we notice that x values are constant and we can get ordered pairs (2, 2) , (2,4) , (2,6) , (2,8).
We represent the points in the coordinate plane and connect them.
We notice the line is parallel to Y-axis,
(x values are constant , y values are variable)



Example 3 The following table shows the magnitude of oil produced by Rumaila oil field in four days, represent the table as set of ordered pairs.

Number of days	X	1	2	3	4
Magnitude of oil	Y	1	2	3	4

Show the table by ordered pairs {(1,1), (2,2), (3,3), (4,4)} and then we determine each point in the coordinate plane then connect them.
We notice that the line is not parallel to any axis,
(x values are variable , y values are variable).

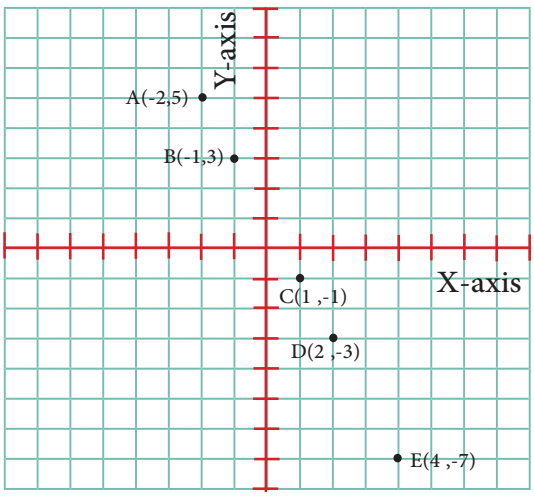


[6-1-2] Writing Table of Specific Points in The Coordinate Plane

Example 4 Write function table using specific points in the coordinate plane.

From near by figure we determine the ordered pairs
A(-2,5), B(-1,3),C(1,-1), D(2,-3), E(4,-7).
Then we form the function table:

Points	A	B	C	D	E
X	-2	-1	1	2	4
Y	5	3	-1	-3	-7



Make sure of your understanding

Represent the following tables in the coordinate plane then connect the points, what do you notice? and what is the shape?

1

X	2	1	1	1
Y	1	2	3	4

2

X	4	4	4	4
Y	1	2	3	4

Questions 1- 4 are similar to examples 1-3

3

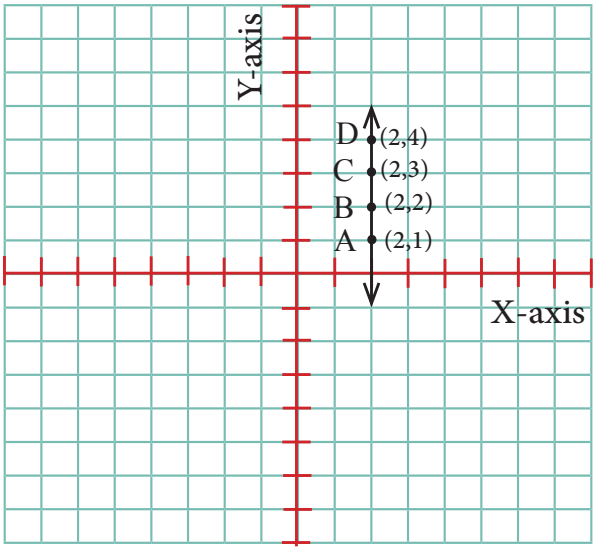
X	1	2	3	4
Y	-2	-2	-2	-2

4

X	-1	-2	-3	-4
Y	2	4	6	8

5 Complete the table from the specific points in the coordinate plane.

Points	A	B	C	D
X				
Y				



Question 5 is similar to example 4

Solve the Exercises

Represent the following tables in the coordinate plane then connect the points, What is the relation of line with X-axis?

6

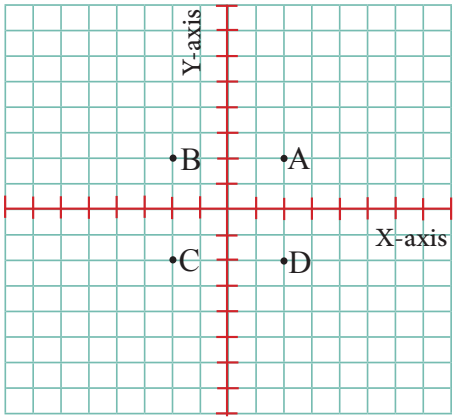
X	-5	-4	3	2
Y	2	1	0	-1

7

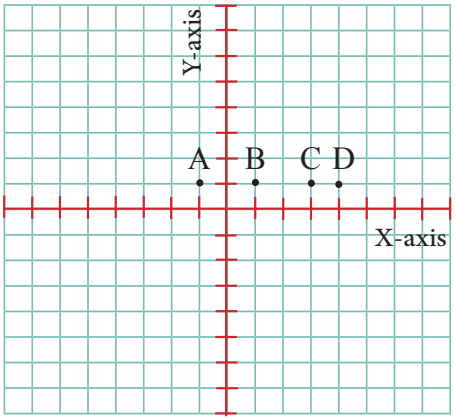
X	-1	0	1	2
Y	3	3	3	3

Write function table from the specific points in the coordinate plane, then show the type of shape.

8



9



Solve the problems

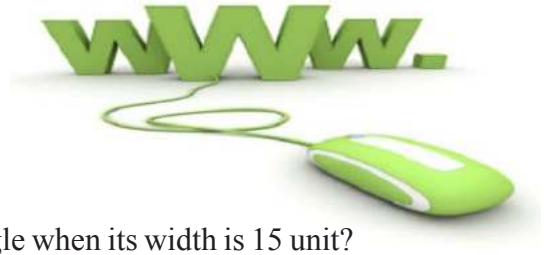
10 **Geology:** A scientist recorded four different temperatures in the south pole, thermometre reading for each 4 hours was:

time	9 morning	1 noon	5 afternoon	9 evening
temperature	-3	-7	-11	-15

Write the function which represents the table above, and write function table.

11 **Electronic design:** Aula designed a website, she drew in it similar rectangles. In the following function table given information was given about length and width of each rectangle aula drew, (x represents length , y represents width)

X	2	4	6	8
Y	3	6	9	12



i) Represent the function table in the coordinate plane.
ii) How you can use this table to find the length of rectangle when its width is 15 unit?

Think

12 **Table:** Write a function table that represents the distance which bike rider covers during four hours, knowing that he covers 15 kilometres in one hour.

13 **Open problem:** Write a set of data which represents a vertical line.

14 **Challenge:** How can I get a line parallel to X-axis by given a certain function table?

15 **Numerical sense:** Ali types 50 words in one hour, how many hours he needs to print a page containing 400 words?

Write

A problem innovate with it a function table with specific points represent the number of days a worker spends in drilling a Well during 5 days.

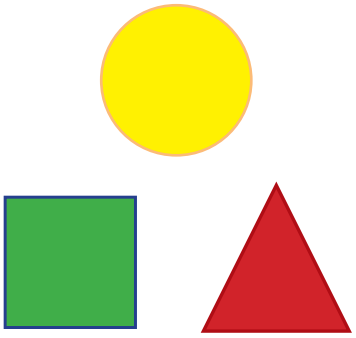
Lesson [6-2] Introduction of Functions

Idea of the lesson:
* Representing a function with a number of points in the coordinate plane.

Vocabulary:
* Function
* Function table
* Function rule
* Element
* Image

Learn

In a school atelier, Ayman, Wa·el and Thamir drew geometric figures.Ayman drew square and triangle,Wa·el drew a triangle and Thamir drew a circle.Find the relation between each student and the geometric figure which drew.



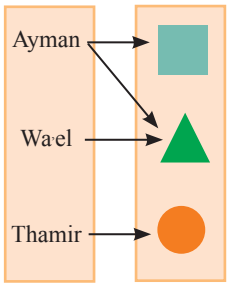
[6-2-1] Relation and Function

* **Function:** it is a relation dtermines only one output value for each input value.
* **Function rule:** it is the formula which is used for substituting each input value to obtain output value.

Example 1

Draw a graph of relation between each student and the geometric figure he drew.

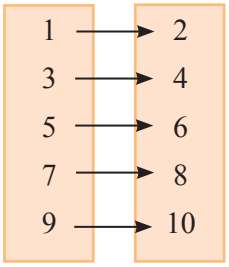
We notice that Ayman drew two geometric figures (square, triangle), it means he has two outputs for one input, so it doesn·t represent a function.



Example 2

Determine whether the following relations represent a function or not ? Illustrate that.
{(1,2), (3,4), (5,6), (7,8), (9,10)}

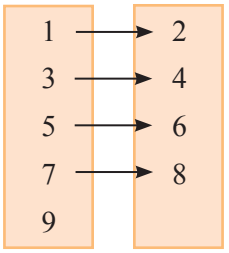
Represent the relation on near by graph:
We notice it represents a function since every input has one output only, so the relation is a function.



Example 3

Notice the near by graph and if it represents a function or not? Illustrate that.

We notice it doesn·t represent a function since one of input doesn·t have output. The number 9 did not associate with any output values.



Example 4

Determine whether the following relations represent a function or not? Illustrate that.

i) {(4,18), (3,15), (2,18), (1,9)}, we notice that the relation is a function since there is one output only for each input.
ii) {(2,8), (-1,6), (0,6), (-1,5)}, we notice that the relation is not a function since there are two outputs 5,6 for one input -1 .

[6-2-2] Complete Table of the Function

Table of function: The table that organize value of input and output.

Example 5

Complete the function table for the function Y= X-3, such that X = -1, 0 , 1 , 2

We complete the function table by substituting X values in their function rule to find Y corresponding values, as it is noticed in the near by table.

Element (input)	Function rule	Image (output)
X	X-3	Y
-1	- 1 - 3	-4
0	0-3	-3
1	1-3	-2
2	2-3	-1

[6-2-3] Representing the Function in Number of Points in the Coordinate Plane

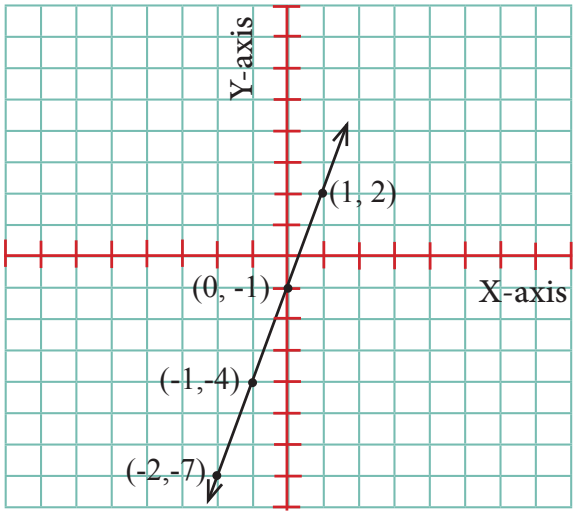
Example 6 Represent the function Y= 3X-1,in the coordinate plane such that X= -1, -2, 0, 1

Step (1): We construct the function table below.

Step (2): Locate the points in the coordinate plane.

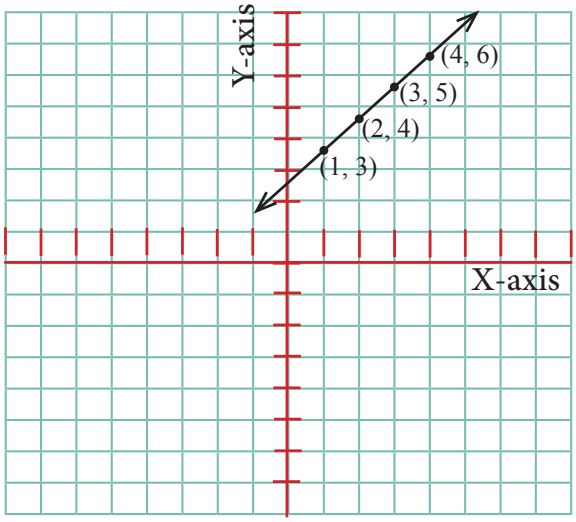
Step (3): We connect the points to get the line as shown in the figure below:

Element (input)	Function rule	Image (output)	Ordered pair
X	3X-1	Y	(X,Y)
-1	3(-1)-1	-4	(-1,-4)
-2	3(-2)-1	-7	(-2,-7)
0	3(0)-1	-1	(0, -1)
1	3(1)-1	2	(1, 2)



Example 7 Complete the table and represent it in the coordinate plane.

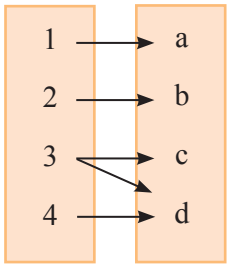
Element (input)	Function rule	Image (output)	Ordered pair
X	X+2	Y	(X,Y)
1	(1)+ 2	3	(1,3)
2	(2)+ 2	4	(2,4)
3	(3)+ 2	5	(3,5)
4	(4)+ 2	6	(4, 6)



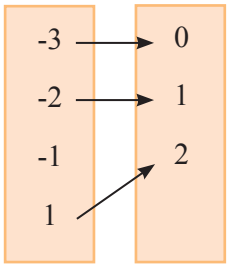
Make sure of your understanding

Determine whether the relation represent a function or not ? And state the reason.

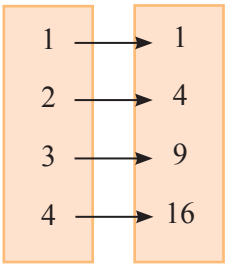
1



2



3



Questions 1-3 are similar to examples 1-3

If the set of inputs are { 3 , 5 , 6 , 7 } and outputs are: $\{\sqrt{3}, \sqrt{5}, \sqrt{7}, \sqrt{6}\}$, determine whether the following relations are a function or not? and state the reason.

4 $\{(3,\sqrt{3}), (5,\sqrt{5}), (6,\sqrt{6}), (7,\sqrt{7})\}$

5 $\{(3,\sqrt{3}), (5,\sqrt{7}), (3,\sqrt{6}), (6,\sqrt{6}), (7,\sqrt{6})\}$

Questions 4-5 are similar to example 4

6 A bat eats 600 insect per hour, how many insects it will eat in (2,3,4) hour? Construct the relation between the number of hours and number of insects. Then represent the table in the coordinate plane.

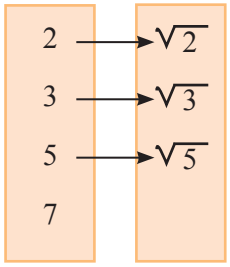
Question 6 is similar to examples 6-7

Element (input)	Function rule	Image (outcput)	Ordered pair
X		Y	(X,Y)
1		600	(1,600)
2			
3			
4			

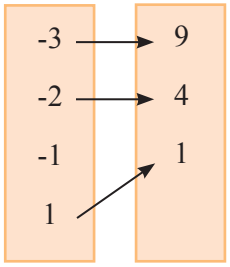
Solve the Exercises

Determine whether the relation represents a function or not ? And state the reason.

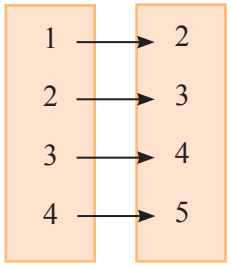
7



8



9



If the set of inputs are { 3 , 5 , 6 , 7 } and the set of outputs are: $\{\sqrt{3}, \sqrt{5}, \sqrt{7}, \sqrt{6}\}$, determine whether the following relations are a function or not? And state the reason.

10 $\{(3,\sqrt{3}), (5,\sqrt{5}), (6,\sqrt{5})\}$

11 $\{(3,\sqrt{3}), (5,\sqrt{7}), (5,\sqrt{6}), (7,\sqrt{5})\}$

12 Write the function from the following table:

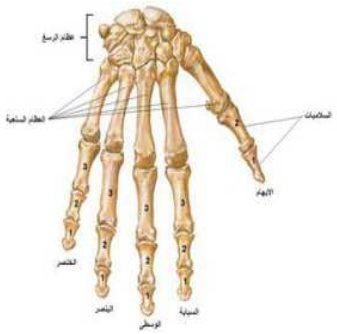
Age	X	10	20	30	40
Expenditure	Y	15	25	35	45

Solve the problems

13 **Sport:** Muhammad swims 9.5 km/h, construct a function table representing the total number of kilometres he can swim in {2, 4, 6} hours.

14 **Bacteria:** If the number of bacteria increases at a rate of double for each 20 minutes, how much it will increase in 2 hours? Construct a function table.

15 **Fuel:** If an ambulance is consuming 5 liter for each 25 km, make a function rule and show the number of km the car cover for {6,7,8,9,10}liter. Construct a function table and represent it.



16 **Biology:** Each hand of human body contains 27 bones, the number of the metacarpus bones is less than the number of phalanges in 9 bones, while the number of wrist bones is more than the bones of the metacarpus in 3. knowing that the number of phalanges is 14. Represent the above mentioned data in the function table.

Think

17 **Challenge:** Find the set of elements for the function $y = 2x - 1$ if the set of image is {41,49,57}.

18 **Correct the mistake:** Suha and Maha found a function rule one of its element is less than image by 7? Which one is correct? Illustrate your answer.

Maha Y=X+7 Y=X-7 Suha

19 **Numerical sense:** A function its rule is 42x-8, determine the image of the element 2 in the function.

Write

A daily life problem representing of a function and then construct the function table and represent it in the coordinate plane.

Lesson [6-3] Linear Functions

Idea of the lesson:
* Writing the general formula for the linear function from the function table.

Vocabulary:
* Linear function
* Equation of straight line
* Graphical diagram
* The four quadrants

Learn

Saeed has apiary for honey producing, he noticed that the Bee flies 24 k/h. What is the relation between the time and the distance in km?

[6-3-1] Representing Linear Function (Equation of Straight Line) in the Coordinate Plane)

* **Linear function:** It is a function in which all its points lies on a straight line not perpendicular and it's called linear graph function.
* **Equation of striaght line:** It is an equation represents the linear function y= mx+t such that t and m are constants.

Example 1 What is the distance covering by a bee in km in time unit in hours?

Step (1): Construct function:

We assume the time (number of hours) a bee flies by: X, and the distance that covered by Y, it representing by the function: Y= 24X

Step (2): Construct the function table:

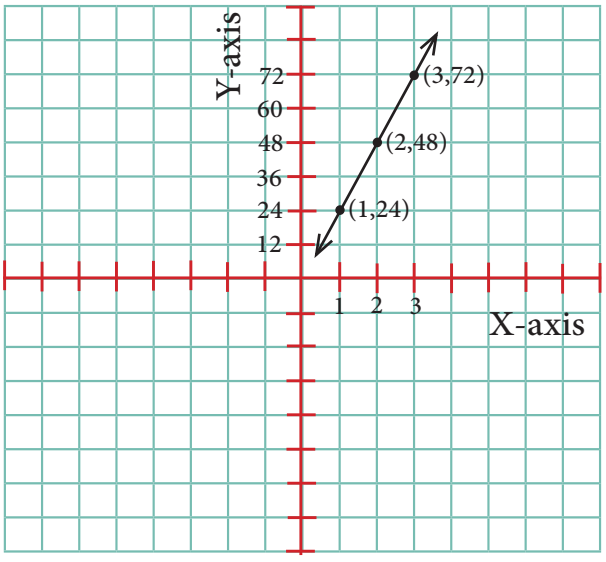
Element (input)	Function rule	Image (output)	Ordered pair
X	Y=24X	Y	(X,Y)
1	24(1)	24	(1,24)
2	24(2)	48	(2,48)
3	24(3)	72	(3,72)

Step (3): Represent a function in the coordinate plane:

To graph a linear function you need to locate at least two points and then connect them.

We notice that when the bee flies 2 hours, the distance will be 48 km, when it flies 3 hours the distance will be 72 km.

The equation represents a striaght line.

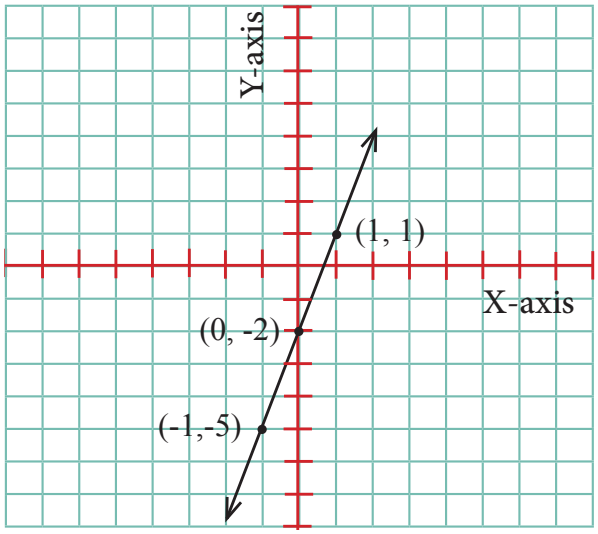


Example 2 Represent the linear function for y = 3x-2 in the coordinate plane.

Step(1): construct the linear function table.

Element (input)	Function rule	Image (output)	Ordered pair
X	y = 3x-2	Y	(X,Y)
-1	y = 3(-1)-2	-5	(-1,-5)
0	y = 3(0)-2	-2	(0, -2)
1	y = 3(1)-2	1	(1, 1)

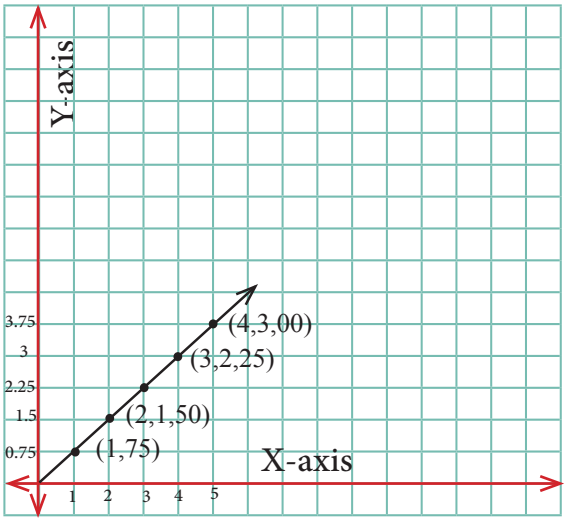
Step(2): Represent the function in the coordinate plane.



[6-3-2] Writing Linear Function (Equation of Straight Line) from the Graphical Diagram

Example 3 In 2011 NASA succeeded to land a lab on Mars, the lab was landing with a speed of 0.75m in one second, the graph shows a function represents the distance which the landed took at the time, construct linear function table and write the linear equation and represent it.

From the graph we can construct the function table by assuming the time is X and the distance by Y.



Linear function table

Element (input)	Image (output)
X	Y
1	0.75
2	1.50
3	2.25
4	3.00

From the table we conclude the function is:

y = 0.75x

Make sure of your understanding

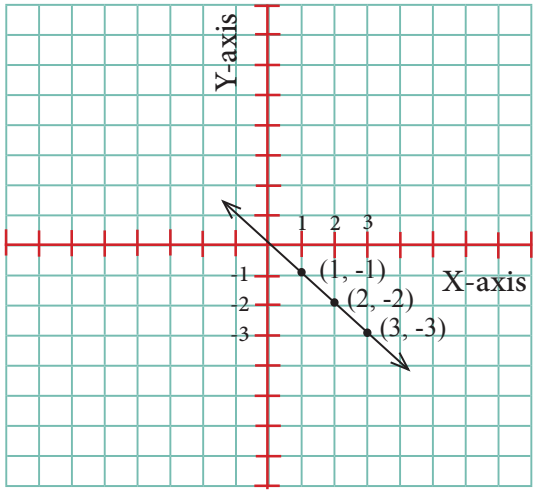
Questions 1-5 are similar to examples 1-2

Represent the following linear functions in the coordinate plane:

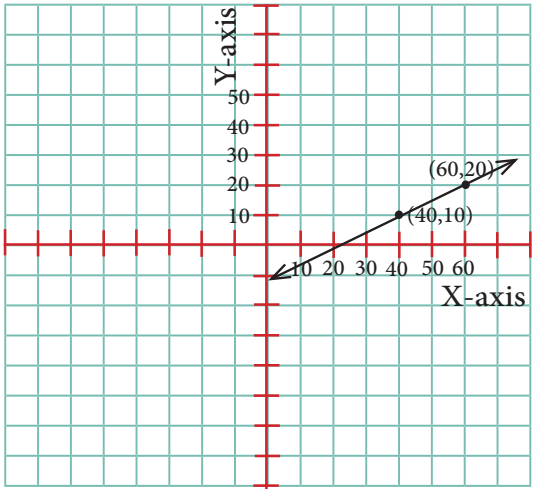
1 Y=X 2 Y=X-9 3 Y=X+2 4 $Y=\frac{3}{2}X$ 5 Y=12-X

Use the graphical diagram to construct the function table and write the linear function:

6



7



Questions 6-7 are similar to example 3

Represent the linear function in the coordinate plane:

8

X	-2	-1	0	1	2
Y	-1	0	1	2	3

9

X	2	1	3	4
Y	4	3	5	6

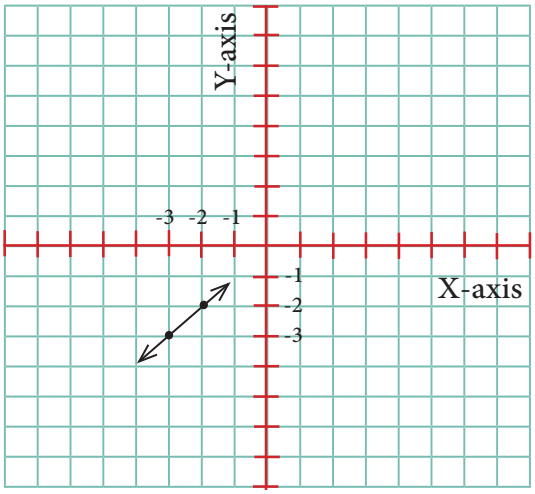
Solve the Exercises

Represent the following linear functions in the coordinate plane:

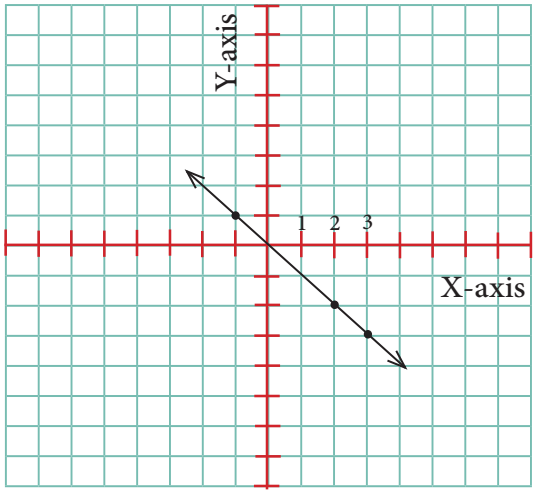
10 Y=7X 11 Y=3X-4 12 Y=X+4 13 Y=9X-0.5 14 $Y=\frac{X}{2}$

Use the graphical diagram to construct the function table and write the linear function:

15



16



Represent the linear functions table in the coordinate plane:

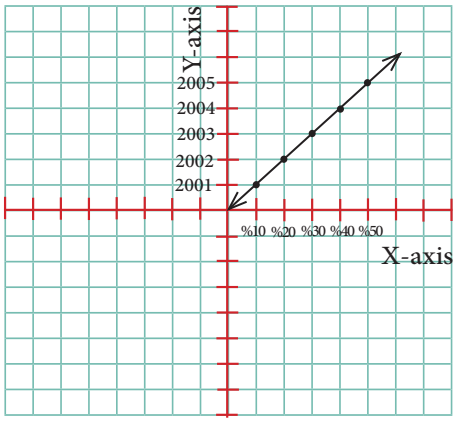
17

X	-2	-1	0	1	2
Y	2	1	0	-1	-2

18

X	2	1	3	4
Y	5	3	7	9

Solve the problems

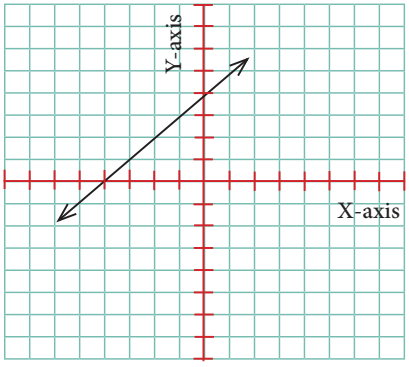


19 **Statistics:** A soap factory manager wanted to make statistics of earnings for 5years, such that the profit reached 50%, construct the linear function table from the graphical diagram and write the general linear equation for the profit with respect to number of years.

20 **Sports:** Bushra scored some points in basketball such that the number of points was less than the previous match by 5 points from the current game, write the general linear equation for the function, then Construct the linear function table and represent it in the coordinate plane .

21 **Money:** Hisham wants to buy a car for 12 million Iraqi dinars correntlly he has 4 millions and he has a plan to save 2 million each year? Write the function of saving money yearly. Then draw the linear function to determine the number of years needed it to buy the car after determining the linear function table. And represent information above in the function table.

Think



22 **Challenge:** Locate the point in the coordinate plane, check the linear function in near by graphical diagram, then write function rule.

23 **Numerical sense:** A natural number multiplied by 3 then 5 was subtracted from it after multiplication, the total result was 70.
What is the general linear equation for the total result with respect to the natural number?

Write

A daily life problem concerning general linear function (line equation). $y = 5x - 3$

Lesson [6-4] Reflection and Rotation in the Coordinate Plane

Idea of the lesson:
* Representing reflection and rotation in the coordinate plane.

Vocabulary:
* Geometric transformation
* Reflection
* Reflection line
* Rotation
* Coordinate plane

Learn

In the figure a bird's reflection appears on water, if Sahar determined three points A, B, C on the origional picture for the bird, then we find the points will appear as A´,B´, C´, B´, in the reflection on water.

[6-4-1] Reflection in the Coordinate Plane

* **Geometric transformation:** Is one of the geometry subjects studies definitions of geometric shapes and transforms every point in the coordinate plane to another in the same plane.
* **Reflection:** Is a geometric transformation from a figure to its mirror (reflectd) image. (The reflection keeps the figure structure).
* **Reflection line:** Is a horizontal or vertical line.

Example 1 i) Find the reflection of A, B, C which Sahar determined it.

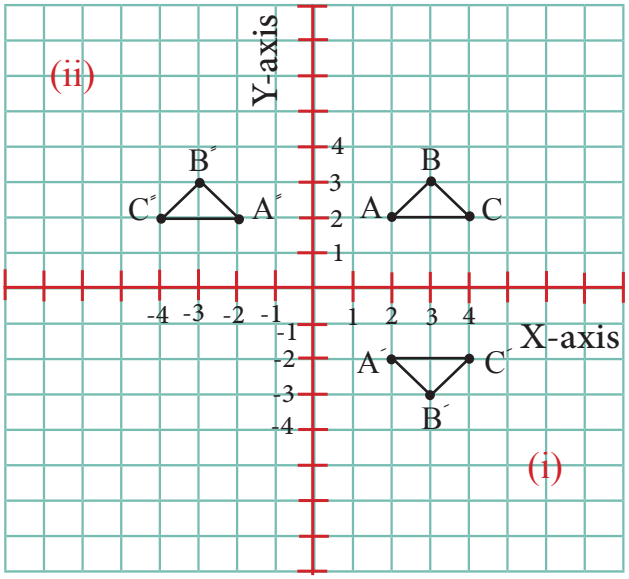
Step (1): Determine the ordered pairs of A, B, C such that

A(2,2), B(3,3), C(4,2)

Step (2): Determine the reflection line, let It be X-axis then we determine the number of units between each vertex and reflection line.

Step (3): Deteremine each point on the other side of X-axis with same distance is such that

A´(2,-2), B´(3,-3), C´(4,-2), generally the reflection of any point on X-axis: $R_x[(x,y)] = (x, -y)$



ii) Find the reflection of A(2,2), B(3,3), C(4,2) on Y-axis.

The points on Y-axis reflection are A´(-2,2), B´(-3,3), C´(-4,2).

Generally the reflection of any point on Y-axis is:

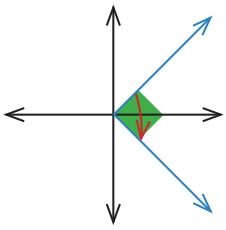
$R_y[(x,y)] = (-x,y)$

[6-4-2] Rotation in the Coordinate Plane

Murad has a clock on his room wall, he wanted to rotate the clock 90° about the origin on clockwise.

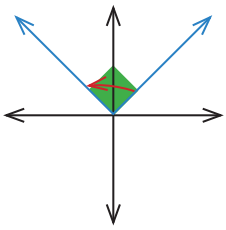
Rotation: Is a geometric transformation, transforms point (0, 0) to itself and transforms any point such that point A to point A´ according to the rotation angle measure and its direction.

Example 2 What is the image of point (1,2) under rotation effect?



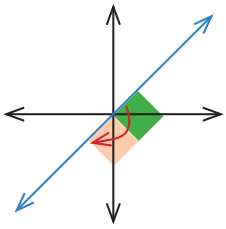
A- Rotation by 90° clockwise about the origin:

We apply the rule: $R_{90°}[(x,y)]=(y,-x)$ such that $R_{90°}[(1,2)]=(2,-1)$



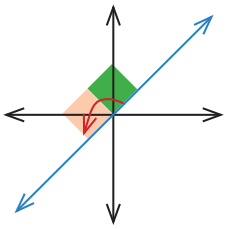
B- Rotation by 90° anti-clockwise about the origin:

We apply the rule: $R_{90°}[(x,y)]=(-y,x)$ such that $R_{90°}[(1,2)]=(-2,1)$



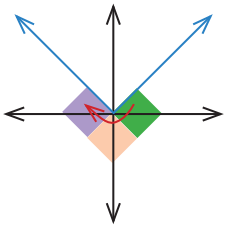
C- Rotation by 180° clockwise about the origin:

We apply the rule: $R_{180°}[(x,y)]=(-x,-y)$ such that $R_{180°}[(1,2)]=(-1,-2)$



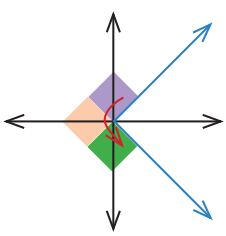
D- Rotation by 180° anti-clockwise about the origin:

We apply the rule: $R_{180°}[(x,y)]=(-x,-y)$ such that $R_{180°}[(1,2)]=(-1,-2)$



E- Rotation by 270° clockwise about the origin:

We apply the rule: $R_{270°}[(x,y)]=(-y,x)$ such that $R_{270°}[(1,2)]=(-2,1)$



F- Rotation by 270° anti-clockwise about the origin:

We apply the rule: $R_{270°}[(x,y)]=(y,-x)$ such that $R_{270°}[(1,2)]=(2,-1)$

Make sure of your understanding

Copy the figures in the coordinate plane, then draw their image about reflection line, if the points are:

1. A(2,2), B(4,4), C(4,2) Reflection line is X-axis
2. A(-5,2), B(-2,4), C(-4,6) Reflection line is Y-axis
3. A(1,1), B(2,3), C(5,3), D(3,1) Reflection line is X-axis

Questions 1-3 are similar to example 1

Find the image of the point (2,-1) in the following cases:

4. Rotation by 90° anti-clockwise about origin.
5. Rotation by 180° clockwise about origin.
6. Rotation by 270° clockwise about origin on.
7. A(1,-2), B(3, -2), C(1,-4) are vertices of a triangle, find rotation by 90° clockwise about origin.
8. A square with points A (-4,4), B(-2, 4), C(-4,2), D(-2,2) find the image of the square if rotated by 90° clockwise and anti-clockwise about origin, then find the area of square, what do you notice?

Questions 4-8 are similar to example 2

Solve the Exercises

Copy the figures in the coordinate plane, then draw their images about reflection line, if the points are:

9. A(1,1), B(6,1), C(1,5) Reflection line is X-axis
10. A(-3,3), B(-1,3), C(-2,1) Reflection line is Y-axis
11. A(-3,2), B(-2,4), C(-1,4), D(-1,2) Reflection line is Y-axis

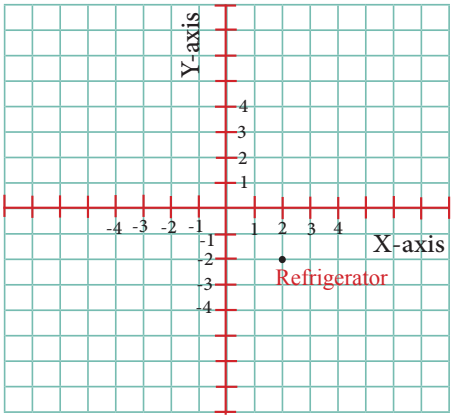
Find the image of the point (2,-1) in the following cases:

12. Rotation by 270° anti-clockwise about origin.
13. Rotation by 90° clockwise about origin.
14. Rotation by 180° anti-clockwise about origin.
15. (1,3), (-1, 4), (3,-1) are vertices of a triangle, find the image of the triangle if rotated by 180° anti-clockwise about origin once and then clockwise.

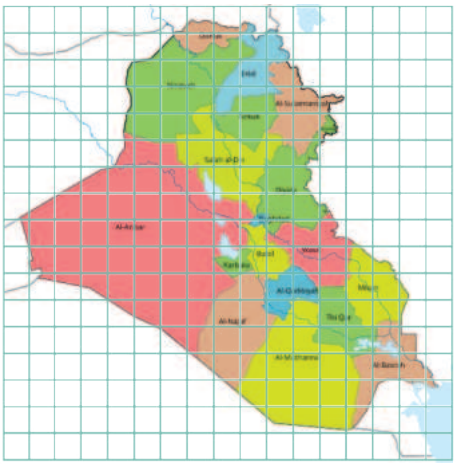
Solve the problems

16 **Raising Flag:** Every Thursday students stand to raise the flag, Kareem who stands on point (-3,3) decided to turn with angle 270° clockwise about origin, what is the point he stops after turning?

17 The figure shows the position of a refrigerator on point (2, -2). Jamil wanted to move it with 180° anti-clockwise, find its position after moving.



18 Determine the governorate which represents the reflection of Muthanna governorate by 90° anti-clockwise and with clockwise.



Think

19 **Challenge:** Let the point (-3, 4) is an image of the point (3, 4) in a reflection, determine in which axis the reflection done.

20 **Numerical sense:** On which reflection angle the image of a point is itself, clockwise or anti-clockwise?

21 **Correct the mistake:** Muhannad says the reflection of the point (-3, 2) on X-axis is (-2, 3). Correct Muhannad mistake.

Write

Steps of finding the coordinates for the image of point (-3,3) by reflection on Y-axis.

Lesson [6-5] Translation in the Coordinate Plane

Idea of the lesson:
* Translation in the coordinate plane.

Vocabulary:
* Translation
* Coordinate plane

Learn

Mahdi moved his bookshelf from one side to another side in his room, this is an example about concept of translation.

Translation: Is moving a figure from one place to another without rotation, and there is not changing in its measures.

[6-5-1] Translation to Right or Left Translation to Up or Down

Example 1 Find the coordinates of point A(3,4) by translation:

i) 2 units right. ii) 5 units left.

Translating point A(3,4), 2 units right, you will get

$A'(3+2, 4) = A'(5, 4)$

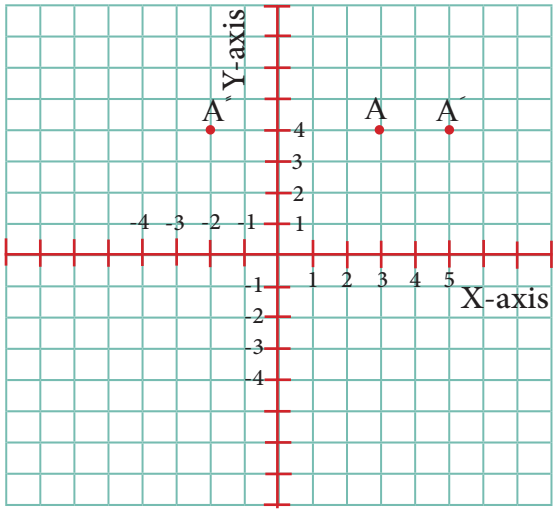
Translating point A(3,4), 5 units left, you will get

$A'(3-5, 4) = A'(-2, 4)$

Generally: Translating (x,y) parallel to X-axis, a- unite is done by this formula: $T_x[(x,y)] = (x + a, y)$

If the translation is done to the right then $a > 0$

If the translation is done to the left then $a < 0$.



Example 2 Find the coordinates of point A(3,4) by translation:

i) 2 units up. ii) 5 units down.

Translating point A(3,4), 2 units up, you will get

$A'(3, 4+2) = A'(3, 6)$

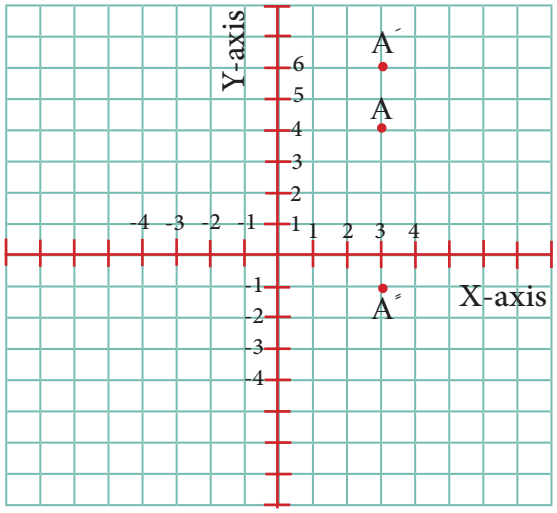
Translating point A(3,4), 5 units down,you will get

$A'(3, 4-5) = A'(3, -1)$

Generally: Translating (x,y) parallel to Y-axis,b - units is done by this formula: $T_y[(x,y)] = (x, y + b)$

If the translation is done up then $b > 0$,

If the translation is done down then $b < 0$.



[6-5-2] Oblique Translation

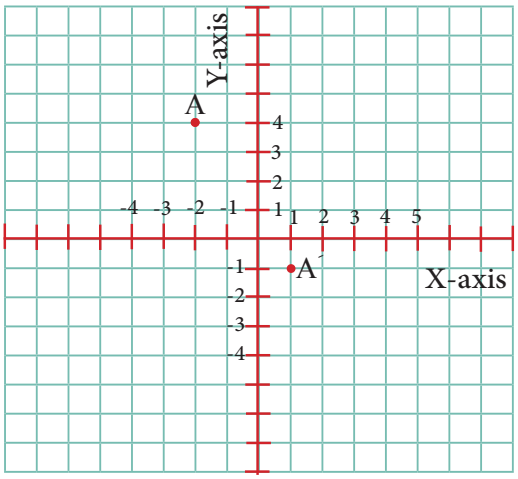
Example 3 Find coordinates of point A(-2,4) after translation 3 units right, and 5 units down.

Translating point A(-2,4), 3 units right and 5 units down

by $A'(-2+3 , 4-5) = A'(1,-1)$

Generally: Translating (x,y) by translation Italic:

$$T_{xy}[(x,y)] = (x + a , b + y)$$

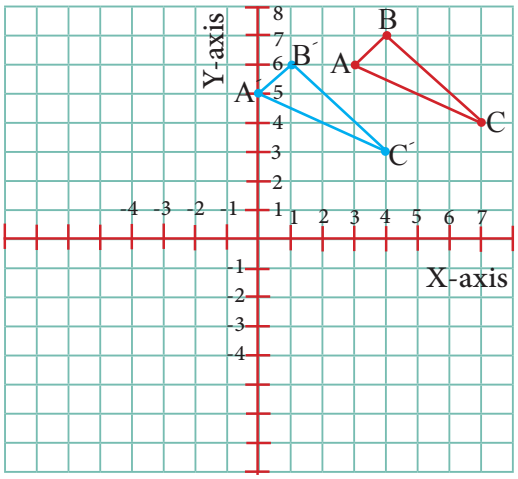


Example 4 Translate triangle ABC whose vertices are A(3,6), B(4,7), C(7,4), 3 Units left and 1 unit down:

$T_{xy}[(3,6)]=(3 - 3,6 - 1) = A'(0,5)$

$T_{xy}[(4,7)]=(4 - 3,7 - 1) = B'(1,6)$

$T_{xy}[(7,4)]=(7 - 3,4 - 1) = C'(4,3)$

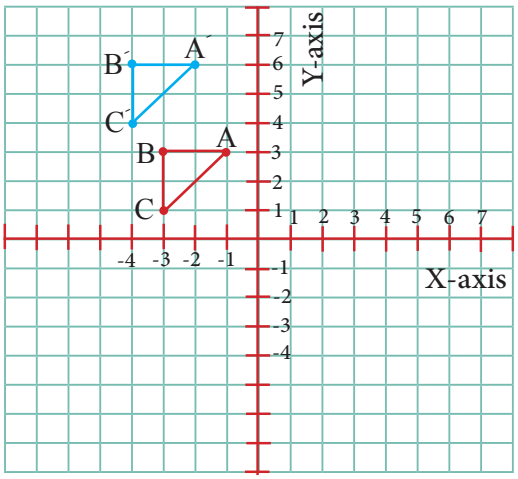


Example 5 Translate triangle ABC whose vertices are A(-1,3), B(-3,3), C(-3,1), 1 Unit left and 3 units up:

$T_{xy}[(-1,3)]=(-1 - 1 , 3 + 3) = A' (-2,6)$

$T_{xy}[(-3,3)]=(-3 - 1 , 3 + 3) = B'(-4,6)$

$T_{xy}[(-3,1)]=(-3 - 1 , 1 + 3) = C'(-4,4)$



Make sure of your understanding

Represent the following points and its images in the coordinate plane:

Questions 1-3 are similar to examples 1-3

1 Translate A(3,5), 4 units left.

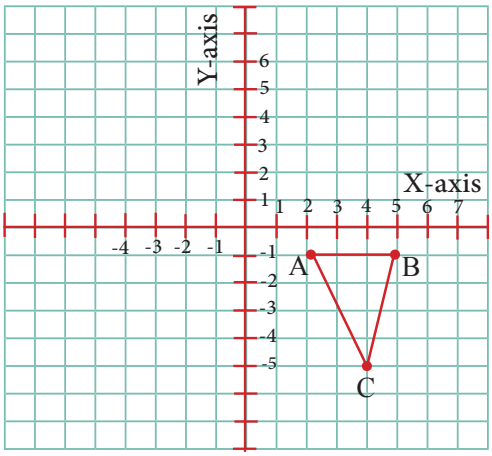
2 Translate B(-2,4), 2 units up.

3 Translate C(-2,4), 1 unit right and 2 units down.

4 Find translate of triangle ABC whose vertices are A(2,3), B(-1,4), C(0,2), 3 units down and represent it and its image in the coordinate plane.

Questions 4-5 are similar to examples 4,5

5 Copy the triangle ABC then determine ordered pairs in the coordinate plane and find its translation 3 units right, 2 units down.



Solve the Exercises

Represent the following points and its images in the coordinate plane:

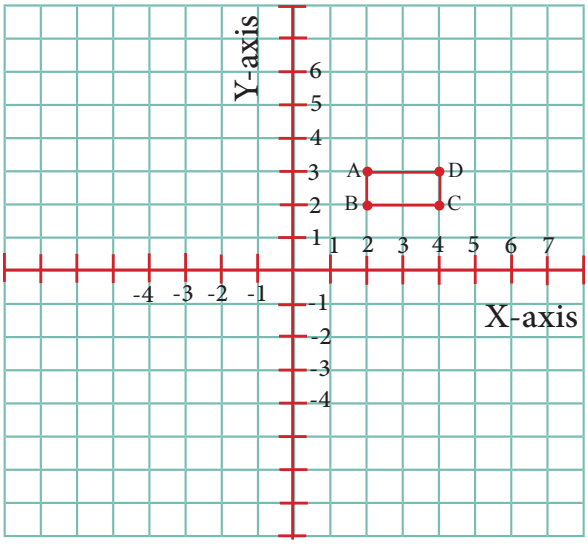
6 Translate A(-1,-2), 3 units right.

7 Translate B(-2,4), 2 units down.

8 Translate C(-1,-2), 1 unit left and 2 units up.

9 Find translation of the square ABCD whose vertices are A(2,3), B(-1,3), C(-1,0), D(2,0), 2 units right and represent it and its image in the coordinate plane.

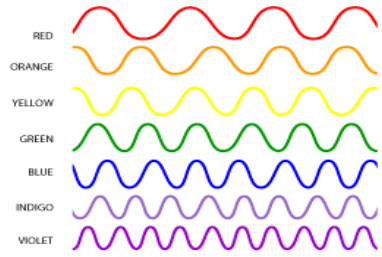
10 Copy the rectangle ABCD then determine ordered pairs and find the image of its translation, 2 units right and 3 units down.



Solve the problems

11 **Biology:** The figure is showing human Heart, find the repeating in the diagram, where is the translation locate it? And calculate the number of translations.

12 **Light beams:** There are light beams in the near by figure. Is there any translation in wave ABC and how many units on the right?



13 **Waves:** There are waves occuring in the sea have a point (-2,5) on top of it, if the waves moves to have the new position (8,2), calculate the magnitude of translation and the dierections of translation.

14 **Art:** Raghad drew four mountains. She finished the drawing of the first one, then she wanted the second mountain as a translation of first mountain by 2 units to the right and one unit up, find the translation of the mountain if the points of first mountain are A(3,3), B(3,0) and C(0,0).

Think

15 **Challenge:** What is the coordinates of point (X,Y) if translated by m units right and n units up.

16 **Conclution:** A translation was done lead to (-4,6) in a way and another, then another translation was done to the point image to become (4,-6) without using drawing, what is the final image after two translation? Illustrate that.

17 **Geometry:** After a translating the rhombus ABCD whose vertices are A(2,-1), B(3,-3), C(2,-4), D(1,-3) found that coordinates of A after the translation is A´(4,-3), describe B´,C´,D´, after the translation.

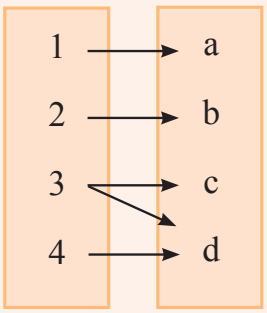
Write

A daily life problem about translating a figure, then solve it.

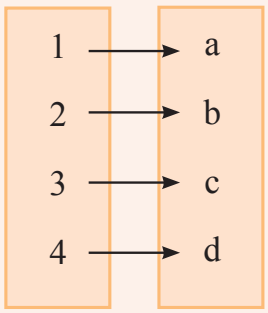
Chapter Test

Determine whether the following relations represent a function or not? Mention the reason:

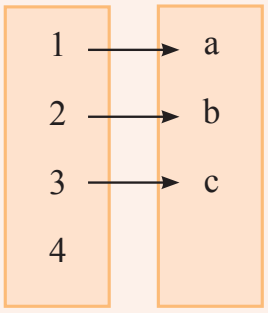
1



2



3



Represent the function table in the coordinate plane then connect the points:

4

X	1	2	3	4
Y	20	40	60	80

5

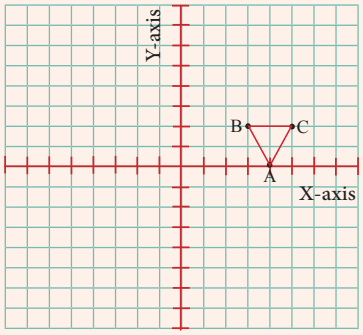
X	-1	0	1	2
Y	4	5	6	7

6 From the Input and output find the function ruly and represent the linear function:

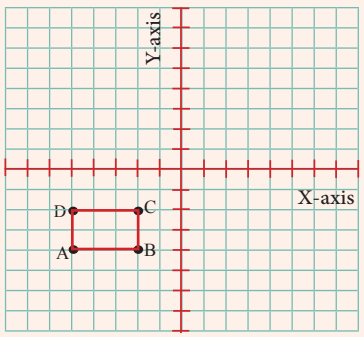
X	-2	-1	0	1
Y	-3	-2	-1	0

7 When the point (-3، 2) rotates 90° anti-clockwise, what is the image you will get?

8 Copy then find translation image for the figure one unit down and 4 units left.



9 Copy then find rotation image for the figure by 270° clockwise:



Chapter 7

Statistics and Probabilities

Statisticions focus on studying the statistical data by representing and illustrating them in various ways in a way that would help them to recognize and use the best experiments.

Pretest

Choose the appropriate word in the adjacent list so that the sentence is correct:

Mode
Median
Range
Arithmetic mean

1 ..... is the difference between greatest and least value in the giving set.

2 ..... is the most repeated value in the given set.

3 .....is the value which comes in the middle of data set after arranging data in an ascending or descending way.

4 ..... is the value which equals the sum of all given values divided by their number.

Arrange the following number in an ascending way: (from smallest to largest)

5 9 , 6 , 8 , 8 , 9 , 6 , 7 , 9

6 97 , 90 , 90 , 99 , 100 , 97 , 97 , 99

Choose the correct answer from brackets for the following:

7 Baghdad is the capital of Iraq (Possible , Certain).

8 The probability of drawing a red ball from a bag contains only white balls (50% , 0%).

9 Cards with numbers from 1-9 the percentage of even-numbered cards is (0% , less than 50%).

10 If the number 3 refers to the stem and 4 refers to the leaf then the number is (34 , 43)

11 Represent the given data in the near by table by stem and leaf methods:

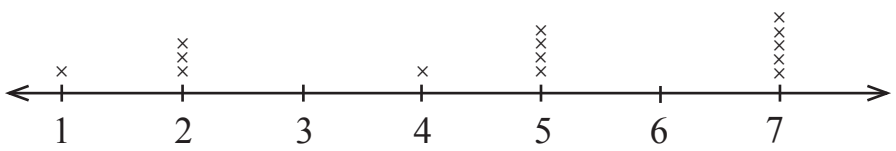
65	70	68	76	65
72	69	74	71	69
76	65	71	72	68

Find the median, mode, and range for each of the following:

12 4 ، 5 ، 0 ، 2 ، 3 ، 8 ، 1، 6 ، 2

13 87 ، 30 ، 55 ، 15 ، 12 ، 71 ، 77

14 From near by represented points:



Lesson [7-1] Measures of Central Tendency and Range

Idea of the lesson:
* Finding the measure of central tendency and range using stem and leaf display.

Vocabulary:
* Mode
* Median
* Mean
* Range
* Stem
* Leaf

Learn

The table near by shows the marks of the second intermediate students in math subject find:

1) Range 2) Median
3) Mode 4) Mean

Marks of students				
95	90	85	90	98
88	81	90	79	79
72	90	99	94	75

You have previously learnd how to represent data by using stem and leaf displays for one set, In this lesson you will learn how to represent two sets by using stem and leaf display and then comparing them with each other. You can find the measure of central tendency by using the same way.

Example 1 **Use the stem and leaf display to answer learn paragraph:**

Step (1): Use stem and leaf display to show the data after arranging them in ascending way.

Stem (Tens)	Leaf (Units)
7	2 5 9 9
8	1 5 8
9	0 0 0 0 4 5 8 9

Step (2): Use stem and leaf display to answer:

1 Largest value = 99 Smallest Value = 72

Range = Largest value - Smallest value

Range = 99 - 72
= 27

2 Median = 90 (The value which becomes in the middle of data in stem and leaf table)

3 Mode = 90 (The most repeated value)

4 Arithmatic mean = $\frac{72 + 75 + \ldots\ldots + 99}{15}$

= 87

You can compare two sets of data by using stem and leaf displays.

Example 2 **The table near by shows representing by step and leaf display the degrees of some students in physics and math:**

Leaf (Physics)	Stem	Leaf (Math)
8 5 1	7	3
9 8 4 4 2	8	0 2 3 3 7 8
9 6 5 3 0	9	0 0 1 5 6 7

i) Which subject has largest range?

Range of math set is 97 - 73 = 24

Range of Physics is 99 - 71 = 28

So range of physics set is larger than math set.

ii) Median of math set is 88, and median of physics set is 88.

iii) What is the highest degree in the math set? 97

IV) What is the smallest degree in the physics set? 71

Example 3 **A comparison was made on a distance covered by 9 cars in kilometres inside city and highway, the table below formed:**

Distance in kilometres									
Inside city	28	23	41	31	20	19	23	31	34
High way	28	38	32	41	38	28	32	30	27

i) Construct back-to-back stem and leaf method from data above.

Inside city	Stem	High way
9	1	
8 3 3 0	2	7 8 8
4 1 1	3	0 2 2 8 8
1	4	1

ii) Find the median, mode, and range for both.

Inside city	High way	
28	32	Median
23 , 31	28,32 , 38	Mode
41-19=22	41-27=14	Range

iii) Which set has the largest range?

The range of cars set inside city is = 22

The range of cars set on highway is = 14

So, the range of cars set inside city is a larger.

Make sure of your understanding

The table near by shows temperatures for somedays:

1 Use stem and leaf method to represent the data.

2 Find: Range, mode and median.

3 Find: Arithmatic mean for the data.

Temperatures in celsius				
24	30	36	32	38
2	31	35	13	15
38	32	38	38	13

Questions 1-3 are similar to example 1

The stem and leaf representation table shows the number of points that two sets of school basketball team gained in the matches.

4 Find: Range, median and mode for the first set data.

5 Which set has the smallest range?

6 Compare the medians of two sets.

Questions 4-6 are similar to example 2

7 Construct back-to-back representation by stem and leaf then find mode, median and range for both types data.

8 Which set has the largest range?

Set (2)	Stem	Set (1)
0	4	7 8 8
6 4 3 1	5	0 0 2 3 7
9 8 6	6	1 6
2 1	7	
3	8	4

Lengths of plants in cm.							
First type	49	64	72	63	45	52	43
Second type	54	58	54	64	58	52	61

Questions 7-8 are similar to example 3

Solve the Exercises

Use stem and leaf method near by the following and find:

9 Range, median and mode for data.

10 Arithmetic mean for data.

Stem	Leaf
0	8 9
1	0 2 4 6 8
2	7
3	4

The table bellow shows stem and leaf representation for the number of the Baghdadian museum visitors for two weeks:

The number of the Baghdadian museum visitors							
First week	58	61	70	63	36	50	44
Second week	50	59	70	66	30	52	40

11 Use the back-to-back stem and leaf method to represent the data.

12 Find: Range, median and mode for the first week data.

13 Find: The arithmetic mean for each week, what do you notice? Illustrate your answer.

Solve the problems

Speed: **The near by table shows speed of some cars**

Speed of cars km/h				
69	65	71	76	65
59	74	68	74	72
70	65	69	71	68

14 Use stem and leaf method to represent the data

15 Find range, median, and the mode for the data

16 Find the arithmatic mean for the data

Food: **Use the near by stem and leaf method and find the following:**

Quantity of fat in chicken and meat sandwiches ٪		
Chicken	Stem	Meet
1	0	8 9
9 5	1	0 2 4 6 7
7 6 5	2	7
4 3	3	
1	4	0

17 What is the highest quantity of fat in each sandwich kind?

18 Which one of sandwich kinds contains less quantity of fat generally?

19 Find the median for each kind of sandwiches and compare between them.

Think

20 Give an example for a set of data has the same median and mode.

21 Discover the mistake: The near by table shows the weight of some materials in Kilograms, Muhammad and Muhannad are trying to analyse the data represented in stem and leaf method, Muhammad says that half of weights are between 30 and 40 kg and Muhannad says that there is no weight above 70 kg, find who is right? and clarify your answer.

Weight in kilograms	
Stem	Leaf
3	4 5 6
4	
5	6
7	2 8

Marks: **The table below shows the degrees of two classes in math exam.**

22 Use the representation by back-to-back stem and leaf method for both of classes. And answer:

23 What is the median in a class A?

24 What is the mode in a class B?

25 Which class has the smallest range?

Students grades of both classes							
Class A	66	81	73	52	77	80	90
Class B	80	60	99	60	84	66	90

Write

A problem you can solve by back-to-back stem and leaf method, clarify how you can use it to find range, median and mode.

Lesson [7-2] Representation of Data by Box-Whisker

Idea of the lesson:
* Representing data by Box-Whisker.
* Comparison of Box-Whisker.

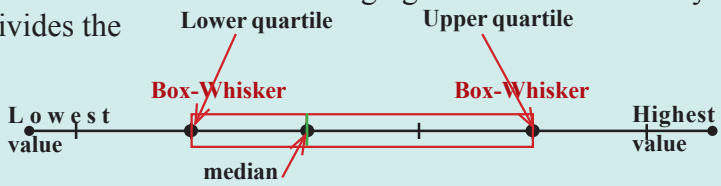
Vocabulary:
* Box-Whisker.
* Upper quartile
* Lower quartile
* Interquartile range

Learn

A basket ball player scored in each game the following points:
35, 37, 40, 35, 38, 46, 46, 42 ,37, 40 .
How do I represent data by Box-Whisker?

[7-2-1] Box and Whisker Graph

To construct Box and Whisker graph, you have to divide the data after arranging into four divisions، by using the quartiles, median or middle quartile divides the data into lower half and upper half.
The median in lower half: is the lower quartile
The median in upper half: is the upper quartile
these divisions is distribute on straight line.



Example 1 **Construct Box and Whisker graph for the following data:**
40 ، 37 , 42 , 46 , 46 , 38 , 35 , 40 , 37 , 35

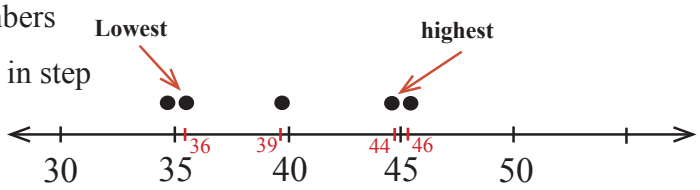
Step (1): Arrange the data ascendingly, determine the lowest value, highest value, median, lower quartile and upper quartile:

35 ، 37 | 37 | 38 ، 40 | 40 | 42 ، 46

Lowest value: 35 — Highest value: 46

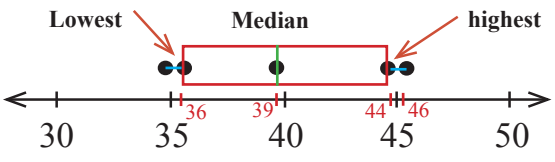
Lower quartile: $\frac{35+37}{2} = 36$ — Median: $\frac{40+38}{2} = 39$ — Upper quartile: $\frac{42+46}{2} = 44$

Step (2): Draw a number line and put a numbers at the top of a line for each part of data found in step (1).



Step (3): Draw a rectangle starting from lower quartile and ending in upper quartile, inside the rectangle draw a line showing median, then draw the Box-Whisker from the lowest and highest value to the rectangle.

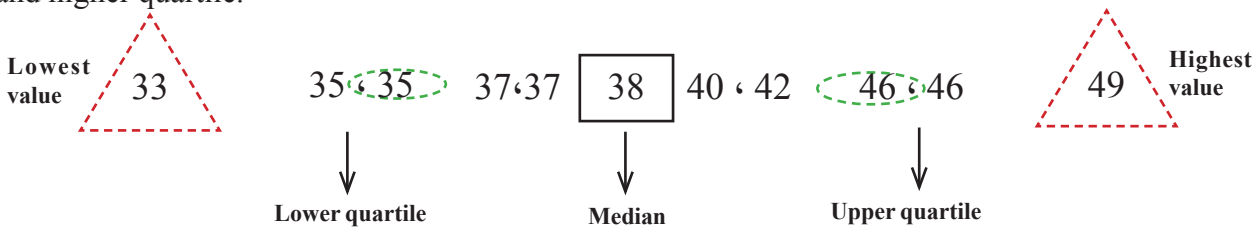
Interquartile range = Upper quartile - Lower quartile
Interquartile range = 44 - 36 = 8 (The length of rectangle)



Example 2 **Construct Box and Whisker graph for the following data:**

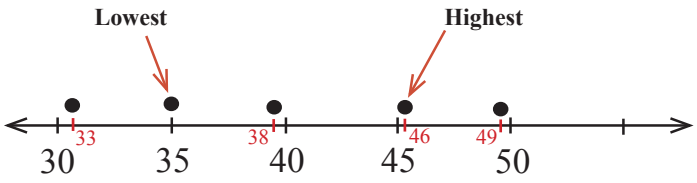
37 , 46 , 35 , 40 , 42 , 46 , 38 , 49 , 37 , 35 , 33

Arrange the data ascendingly, dtermine the lowest value, highest value, then the median, lower quartile and higher quartile.



35 | 38 | 46

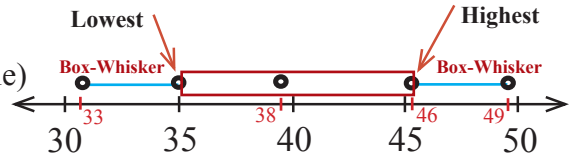
Draw a number line and set a point for each part of data.



Draw a rectangle starting from lower quartile and ending in upper quartile، then draw a line inside rectangle showing the median, then draw the Box-Whisker from the lowest and highest value to the rectangle.

Interquartile range = Upper quartile - Lower quartile

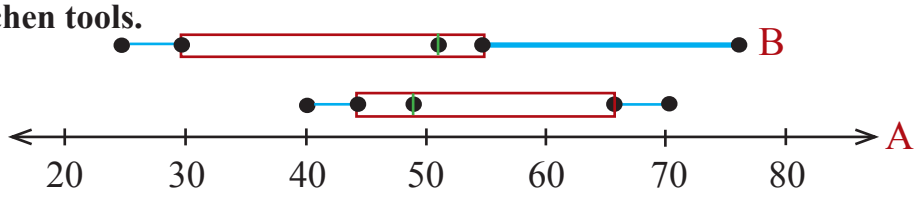
Interquartile range = 46 - 35 = 11 (The length of rectangle)



[7-2-2] Comparing by Using Box-Whisker

We can use the Box and Whisker to compare two sets by aligning both sets together.

Example 3 **The Box and Whisker graph shows information of companies A , B to produce a special kind of kitchen tools.**



Using the near by graph, answer the following:

- Which company has the largest median? The company B median is largest.
- Which company has the largest interquartile range? Company B has largest median, since the length of rectangle shows the interquartile range.
- Which company will produce more tools?

The Range and the interquartile range for the company A are smallest from the range and the interquartile range for the company B this means the change in the data of company A is smaller than the data of company B, so the company A might produce more.

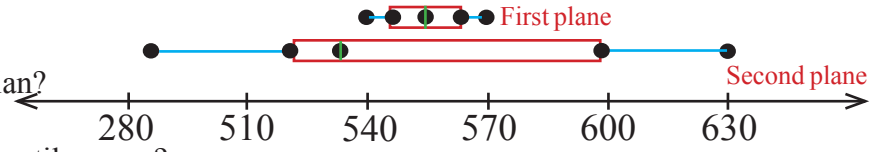
Make sure of your understanding

Construct Box-Whisker graph for the following data:

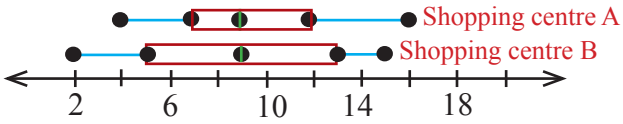
1. 20 , 28 , 19 , 21 , 17 , 29 , 26 , 23 , 17
2. 85 , 79 , 75 , 72 , 72 , 73 , 76 , 81
3. 16 , 26 , 17 , 23 , 21 , 14 , 17 , 25 , 10

Questions 1-3 are similar to examples 1,2

Muhammad and Muhannad have two kites, the Box-Whisker graph represents the altitude each kite has flied.



4. Which kite has smallest median?
5. Which kite has largest interquartile range?
6. Which kite seems to fly less distance?



The Box and Whisker graph shows the number of visitors of shopping centre A and shopping centre B.

Questions 4-8 are similar to example 3

7. Compare between the medians and the ranges.
8. Compare between the interquartile range of the numbers of shopping Centre A and B.

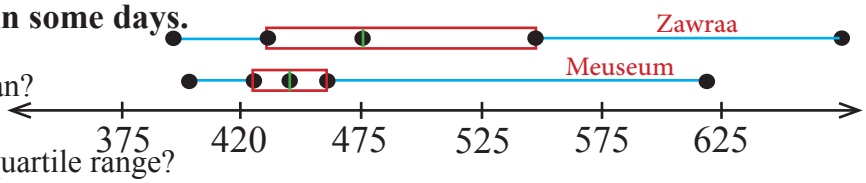
Solve the Exercises

Construct Box-Whisker graph for the following data:

9. 12 , 13 , 10 , 7 , 23, 15 , 8 , 20 , 15, 18 , 39
10. 23 , 29 , 18 , 24 , 24, 25 , 18 , 17 , 18 , 22
11.

Stem	7	6		5		4		
Leaf	2	3	4	2	7	3	5	9

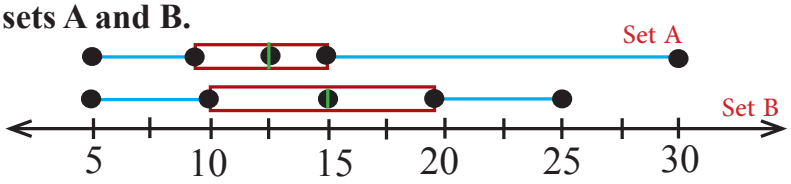
The Box and Whisker graphs show the number of Zawraa garden visitors and the Baghdadian museum visitors in some days.



12. Which place has largest median?
13. Which place has largest interquartile range?
14. Which place seems to have more visitors?

The Box-Whisker graphs show the sets A and B.

15. Find the median of each set and compare between them.



16. Find the interquartile range of each set and compare between them.

Solve the problems

17 **Clothes:** The frequency table shows the size of 12 suits.

Size	65	54	52	50	48
Frequency	1	2	2	4	3

Represent the table above in Box and Whisker graph.

18 **Agriculture:** The lengths of 8 plants are given in centimetres by stem and leaf representation, represent it in Box and Whisker graph.

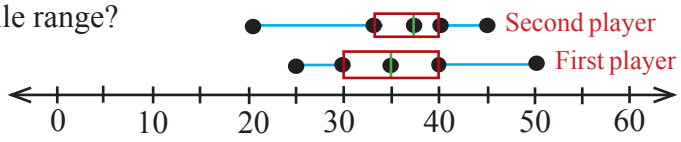
Stem	5		4			3		
Leaf	2	4	3	5	8	1	1	2

Sport: The Box and Whisker graph shows number of matches played by two football players during the first fourteen years from their professionalism.

19 Which player seems to have more matches?

20 Which player has largest median?

21 Which player has largest interquartile range?



Think

22 **Discover the mistake:** Mahmoud used the following data:
8 , 2 , 9 ,14 ,5 ,13 , 7 , 5 , 8
And he found that lower quartile (11). Find Mahamoud's mistake and correct it.

23 **Challenge:** If the interquartile range for a set of data is 9, and the upper quartile is 27. What is the value of lower quartile?

24 **Open problem:** Write a set of data in which the rectangle becomes long, and whiskers are short. Compare the values inside the rectangle with the values of whiskers

Write

Similarity and difference cases between stem and leaf display and Box-Whisker representation.

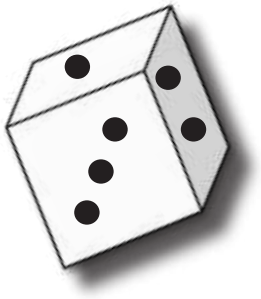
Lesson [7-3] Random Experiment

Idea of the lesson:
* Identify the random experiment.
* Writing the result of random experiment by tree diagram.
* Writing the results by using basic counting principle.

Vocabulary:
* Random action
* Random experiment
* Basic counting principle
* Event
* Outcome

Learn

Mohammed rolled a dice once, and asked Tamarra to record the results he got.
* What do we call the possible results he got?
* What do we call the prime numbers set?



[7-3-1] Random Experiment

Random experiment: Every activity its outcomes is coincidence.
Random action: is an action gives unknown outcome.
Outcomes set: is the set of all possible results to random action and named as sample space and symbolized by Ω.
The event: Is a possible outcome or a set of possible outcomes.

Example 1

A dice numbered 1 to 6 every number might appear once, the possible outcomes are: 1 ، 2 ، 3 ، 4 ، 5 ، 6

The set of possible outcomes is called sample space and symbolized by Ω.
So: $\Omega = \{ 1 , 2 , 3 , 4 , 5 , 6 \}$ is simple random experiment
The prime numbers are 2 , 3 , 5 and they are a subset of sample space, we write it as a set.
$E = \{ 5 , 3 , 2 \} \subset \{ 1 ، 2 ، 3 ، 4 ، 5 ، 6 \}$ is a simple event.

Example 2

Two coins are flipped together once:
i) Write the set of possible outcomes.
ii) Write the outcomes that satisfy the event: When two similar faces of coins appear together.

i) If we name the first face as (H) head and second face as (T) tail.
We will get four possible outcomes, construct the tree diagram to help showing all the possible results as shown at the tree chart.

$\Omega = \{(T,T) , (T,H) , (H,T) , (H,H)\}$

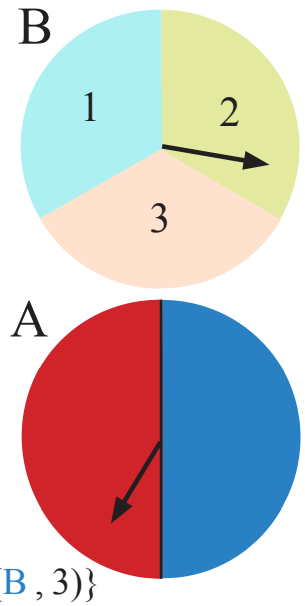
H < H , H
H , T

T < T , H
T , T

ii) The event occurs if the outcome is (H,H) ، (T,T) and it is a subset of sample space.

$E = \{(T,T) , (H,H)\} \subset \Omega$

Example 3 **Tariq started the disc spinner of A and B as shown in the figure:**



i) Write the possible outcomes.

ii) Write the outcomes that satisfy the event: Spinner of B on 3.

i) Construct the tree diagram to help showing all possible outcomes.

	Disc A	Disc B
R	1	(R , 1)
	2	(R , 2)
	3	(R , 3)
B	1	(B , 1)
	2	(B , 2)
	3	(B , 3)

Possible outcomes set: $\Omega = \{(R , 1) , (R , 2) , (R , 3) , (B , 1) , (B , 2) , (B , 3)\}$

ii) $E = \{(R , 3) , (B , 3)\}$

[7-3-2] Fundamental Counting Principle

Fundamental counting principle States that: The number of possible outcomes of a random experiment depends on two random actions and it is the product of number of outcomes of the first action (m) with the number of outcomes of second action (n).
Which means the outcomes number of both actions is $m \times n$.

Example 4 **Two coins are thrown once:**
Use fundamental counting principle to find outcomes of experiment:

i) By throwing the first coin we get 2 outcomes, so $m = 2$

By throwing the second coin we get 2 outcomes , so $n = 2$

By using fundamental counting principle: Number of experiment results = $m \times n$

So, the total number of possible outcomes is $2 \times 2 = 4$

ii) In example (3)

With disc A we get 2 possible outcomes (blue , red), so $m = 2$

With disc B we get 3 possible outcomes (1, 2, 3), so $n = 3$

By using fundamental counting principle: Number of experiment results = $m \times n$

So, the total number of possible outcomes is $2 \times 3 = 6$

iii) If you throw a dice and started a disc with four equal divisions (1, 2, 3, 4), what is the possible number of outcomes?

With the dice it is 6 (1, 2, 3, 4, 5, 6) so $m = 6$

With the disc it is 4 (1, 2, 3, 4) so $n = 4$

By using fundamental counting principle: Number of experiment results = $m \times n$

So, the total number of possible outcomes is $6 \times 4 = 24$

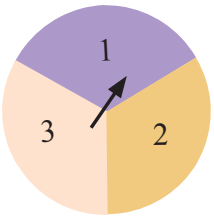
Make sure of your understanding

1 **Su·ad wants to sit on one of eight chairs numbered from 1 to 8.**

i) Write the possible outcomes set.

ii) Write the events that satisfy the event ((Her sitting on an even-numbered chair)).

Find the number of all outcomes using fundamental counting principle for each of the following:



2 Thrawing a coin and starting the disc, write the possible outcomes set.

3 Write the outcomes that satisfy the event ((Tail appears on the coin and spinner stops at number 1)).

4 Muhannad has two coats (black brown) and he has one white shirt and one blue shirt and one grey shirt. In how many ways Muhannad can wear one shirt and one coat?

Questions 1-4 are similar to examples 1-4

Solve the Exercises

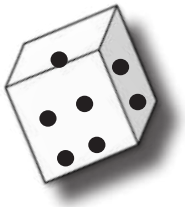
5 **Sara and Amal are waiting one of the six buses numbered from 1 to 6**

i) Write the possible outcomes set.

ii) Write the outcomes that satisfy the event ((Sara riding a bus with odd number less than 6))

iii) Write the outcomes that satisfy the event ((Amal riding a bus with an even number))

6 **Rolling the dice and starting the opposite disc:**



i) Write the possible outcomes set.

ii) Write the outcomes that satisfy the event (appearing a prime number and the spinner stopping on the red part of disc).

iii) Write the outcomes that satisfy the event (appearing a number greater than 5 on dice and spinner showing yellow on disc).

iv) Use the fundamental counting principle to find the total number of outcomes.

Solve the problems

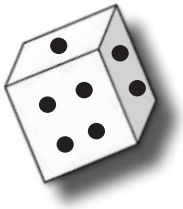
7 Internet: Riyadh wants to choose password for the router consisting of two letters from the lettersA, B, C following one of the numbers 1, 2, 3.
How many passwords can he choose by using fundamental counting principle?

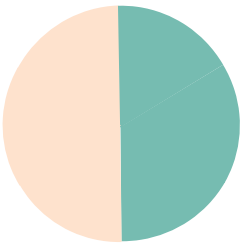
8 Food: **A customer can choose either cheese or eggs as a breakfast, he also can choose either milk or orange juice or apple juice as a drink.**

i) Write all the possible choices.
ii) Write the possible outcomes to satisfy the event "Having cheese and orangejuice".
iii) Find the number of all possible outcomes using fundamental counting principle.

Think

9 Challenge: **If you roll a dice and throw a coin and start a disc with two divisions.**





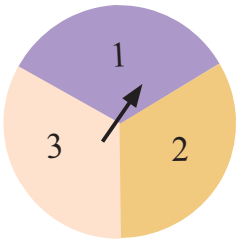
i) Write the possible outcomes.
ii) Find the number of all possible outcomes using fundamental counting principle.

10 Health: **The table containing three groups:**

Group C	Group B	Group A
Vegetables	Meat	Cheese
Fruits	Chicken	Eggs
	Fish	Yogurt

i) Find number of possible outcomes for group A with group B.
ii) Find number of possible outcomes for set A with group C.
iii) Find number of possible outcomes for set A,B and C together.

11 Games: Muhammed and Muhannad are playing the disc game, the disc started two times. If the sum of numbers is odd then Muhannad is the winner. Write the possible outcomes set for Muhammad to be the winner.



Write

How can you find the number of possible outcomes if you roll a dice and flip two coins together?

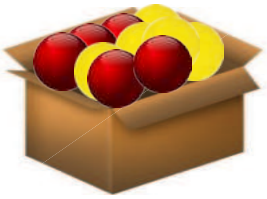
Lesson [7-4] The Event

Idea of the lesson:
* Identify the event, dependent and independent events.
* Distinguish the dependent and independent events.
* Identify compound event.

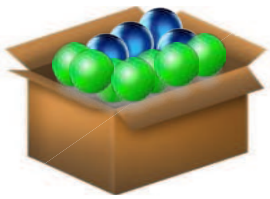
Vocabulary:
* Event
* Independent events
* Dependent events
* Compound event

Learn

You have two boxes one contains red and yellow balls and the other box contains blue and green ball, if you draw a ball from each box.



* What do we call the drawing method?
* What do we call the outcomes?
* What is the relation between the outcomes?



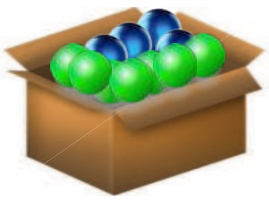
We can name the operation in learn paragraph as **Experiment.**
Event: Is one or more outcomes, events might be dependent or independent or compound.

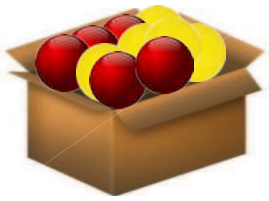
[7-4-1] Independent and Dependent Events

Dependent events: The occurrence of one of the events does affect occurrence of the other events.
Independent events: Two events are independent if the occurrence of each event doesn't affect the occurrence of the other.

Example 1

Answering learning paragraph questions:





- We call it drawing by experiment.
- We call the outcomes of this experiment are events.
- The relation between the events is:

Assume E_1 event of drawing one ball from first box (red or yellow)

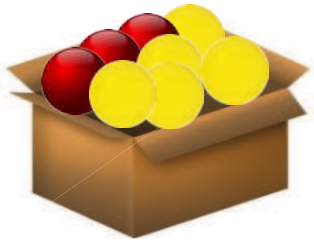
Assume E_2 event of drawing one ball from second box (blue or green)

Notice that drawing a ball from one box doesn't affect the other box.

Mean that event E_1 doesn't affect event E_2

So E_1 and E_2 are independent events.

Example 2 **A box contains 3 red 5 yellow balls. Determine if the event is dependent or independent for each of the following:**



i) Drawing one red ball then drawing one yellow ball without returning the balls.

Assume E_1 drawing a red ball, E_2 drawing a yellow ball.

Since red balls are not returned after drawing, this means there is two red and 4 yellow balls left.

So the occurance of E_1 affects E_2. Then they are dependent events.

ii) Drawing a red ball then returning it and then drawing a yellow ball.

Assume that E_1 drawing a red ball, E_2 drawing a yellow ball after returning the red one. The same number of balls remains in the box.

So, occurance of E_1 does not affect E_2, the events are independent.

Example 3 Determine if the events are dependent or independent when the head appears after flipping a coin, and then getting tail after flipping it again.

Assume E_1 is getting head in first flipping.

Assume E_2 is getting tail in second flipping.

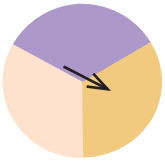
E_1 does not affect E_2.

So, they are independent events.

[7-4-2] Compound Events

Compound events: It consist of two or more simple events dependent or independent.

Example 4 Rolling a dice and starting a disc divided to coloured divisions equal in area. Is the event compound? What type of the simple events?



Rolling a dice and starting a disc is considered compound events from two simple events.

First event: Occurs numbers from 1 to 6.

Second event: Occurs specific colour. Occurrence of a number doesn't affect occurance of the colour.

So, they are independent events.

Example 5 A clothes shop announced that customers will have a free piece once they buy any piece. What type of the two events?

Representing the event of buying one piece and the event of getting a free one is a compound event.

Getting a free piece is depending on buying the first piece.

So, the events are dependent.

Make sure of your understanding

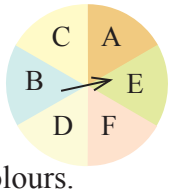
Determine whether the events are dependent or independent in the following compound events:

1 Draw a card from the below cards without returning it, then draw another one.

1 2 3 4 5 6

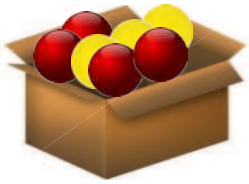
2 If one of letter cards was chosen, and the spinner was started:

A B C D



3 Rolling a dice and drawing a ball from a box contains many balls with different colours.

4 A box contains 4 red 3 yellow balls, Mohanned draw a ball randomly and Muhammad draw a ball randomly too.



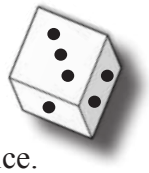
5 Drawing the first ball from the box without returning it, then drawing another ball from the same box.

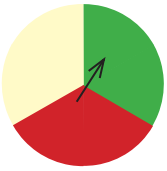
Questions 1-6 are similar to examples 1-5

6 Drawing first ball from the box and returning it then drawing another ball from the same box.

Solve the Exercises

Determine whether the two events are dependent or independent in the following compound events.





7 The spinner of disc stops at red part and getting 2 , 5 on dice.

8 Rolling two dices together and getting 6 in first dice and 3 in second dice.

9 Flipping a coin and getting head and rolling a dice and getting 5.

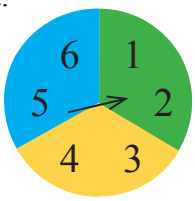
10 Rolling a dice and getting even number, and getting 1 or 4.

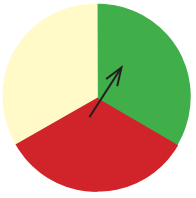
11 A can containing desserts with different flavours, 3 with lemon, 4 with strawberry, 2 with banana and 5 with orange flavours. Jumana drew two pieces without returning them to the can.

12 Taim fliped two coins together, the same head appeared on both coins.

13 Muhammad started disc A and Muhannad started disc B at the same time, the spinner of disc A stops at number 4 while spinner of disc B Stops at green part.

A





B

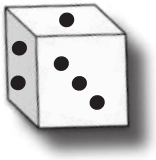
14 Drawing a red card from the cards set A B C D and getting tail after flipping a coin.

Solve the problems

15 **Fruit:** A basket contains three oranges, 5 bananas, and 4 apples, Ahmed took one fruit randomly and his sister took one fruit randomly too without returning the first one.What is the relation between two events?

16 **Clothes:** Drawer contains 5 white shirts, 4 blue shirts, 2 gray ones, a person drew one shirt out of drawer, then drew another without returning first one.
Are the events independent? Illustrate your answer.

17 **Game:** Ahmed rolled a dice and a coin once, he got number 3 on the dice and head on the coin.
Are the events independent? Clarify that.



Think

18 **Open problem:** There are 6 coloured balls in a box with 3 different colours.
Write a problem about drawing two balls randomly without returning them to the box.

19 **Discover the mistake:** Mohammed started the disc 3 times and said "if the pointer stops on 5 three times, it doesn't affect the result". And Salih said "if the spinner stops at 5 three times, it affects the result". Whose answer is correct? Illustrate your answer.

20 **weather:** The directorate of meteorology expects the possibility of raining on Tuesday is 80%, and the possibility of raining on Wednesday is 30%.
What is the relation between the two events?

Write

What is the difference between the two independent events and the two dependent events?

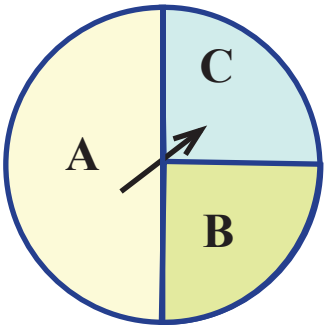
Lesson [7-5] The Probabilities

Idea of the lesson:
* Calculating probability.
* Calculating probability of complement event.

Vocabulary:
* Possibility of event
* Complement event

Learn

Mohannad took the near by disc. If the spinner was started, what is the probability of spinner to stop at each letter?



[7-5-1] Probability

probability of event E: is measuring its chances of happening by checking, and the probability of event E is writen in form P (E)

It can be expressed by a decimal fraction between 1 , 0, or fraction or a percentage.

If $P(E) = 0$, then it is impossible event , but if $P(E) = 1$, then it is certain event.

We can find the event probability P (E) by using the following relation: $P(E) = \frac{m}{n}$

Such that m is the number of outcomes satisfies the event E.

And n is the number of all possible outcomes in one experiment.

Example 1

In learn paragraph,

First method: Since the letter A represents half of the disc, then the feasible estimate is that the spinner stops on the letter A: $P(E) = \frac{1}{2}$

Since each of two letters B and C represents quarter of the disc, then the estimate that is that the spinner stops on the letter B or C:

$P(C) = \frac{1}{4}$, $P(B) = \frac{1}{4}$

Second method: From the figure we notice that the disc is formed by 4 quarters, i.e $n = 4$

The part A represents 2 quarters, i.e $m = 2$, $P(A) = \frac{2}{4} = \frac{1}{2} = 0.5 = 50\%$

The part B or part C of disc represents one quarter, i.e $m = 1$, $P(B) = P(C) = \frac{1}{4} = 0.25 = 25\%$

Example 2

A box contains 10 Green, and 3 white cards. Jumana drew a green card without returning to the box and then her sister Sally drew a white card. What is the drawing probability for each?

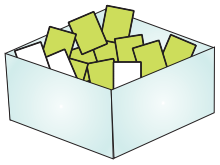
The box contains 10 green + 3 white cards which is 13 cards in total

The probability of drawing green card for Jumana is: $P(\text{green}) = \frac{\text{No. of green cards}}{\text{Total no. of cards}} = \frac{10}{13}$

Because the card was drawn did not return to the box, that means there are 12 cards in the box.

The probability of drawing a white card for Sally is:

$P(\text{white}) = \frac{\text{No. of white cards}}{\text{Total no. of cards}} = \frac{3}{12} = \frac{1}{4}$



[7-5-2] Complement Event

Compliment event: E_1 and E_2 said to be complement, if E_1 outcomes doesn't satisfy E_2 outcomes. So if the probability of their happening was: $P(E_1)$, $P(E_2)$, then $P(E_1) + P(E_2) = 1$

Example 3

E_1 and E_2 are complementary events, if $P(E_2) = \frac{2}{5}$, then find $P(E_1)$ and write it in decimal fraction and percentage.

$P(E_1) + P(E_2) = 1$ *since E_1 and E_2 are complementary events*

$P(E_1) + \frac{2}{5} = 1$ *substitute $P(E_2) = \frac{2}{5}$ in the formula*

$P(E_1) = 1 - \frac{2}{5} = \frac{3}{5}$ *relation between addition and subtraction*

$= \frac{3}{5} \times \frac{20}{20} = \frac{60}{100} = 60\%$ *to write it in percentage, we make denominator equls 100*

$= \frac{3}{5} \times \frac{2}{2} = \frac{6}{10} = 0.6$ *to write it in decimal fraction, we make denominator equls 10*

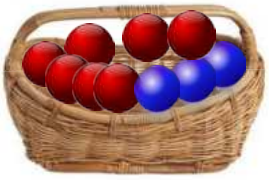
Example 4

A basket contains 3 blue and 7 red balls, a ball was drawn randomly, the probability of being blue is $\frac{3}{10}$, what is the probability of not being blue?

Assume that the probability of being blue is $P(E_1) = \frac{3}{10}$

Assume that the probability of not being blue is $P(E_2)$

The events are complementary, i.e: $P(E_1) + P(E_2) = 1$



$\frac{3}{10} + P(E_2) = 1$ *substitute $P(E_1) = \frac{3}{10}$*

$P(E_2) = 1 - \frac{3}{10} = \frac{7}{10} = 0.7 = 70\%$ *relation between addition and subtraction*

So, the probability of not being blue is $\frac{7}{10}$ or 0.7 or 70%

Make sure of your understanding

We have the below cards

1 2 3 4 5 6 7 8 **find the probability of the following:**

1 Drawing an even numbered card.

2 Drawing a non-prime card.

3 Drawing a card with a number divisible by 5.

Questions 1-6 are similar to examples 1,2

In the experiment of rolling a dice once, find probability of:

4 Getting a number divisible by 3.

5 Getting the number 7.

6 Getting the numbers between 2 and 6.

7 If the probability of getting an odd number in rolling a dice once is $\frac{1}{2}$ by rolling the dice once, what is the probability of getting an even number?

8 If the even E_1, E_2 and E_3 are complementary events, and $P(E_1) = \frac{2}{3}$, $P(E_3) = \frac{1}{4}$, find $P(E_2)$?

9 Use the near by disc and find probability of each possible outcome, then check:

Outcome	Blue	Green	Yellow
probability			

Questions 7-9 are similar to examples 3,4

Solve the Exercises

A basket contains 10 red balls, what is the probability of:

10 Drawing one red ball?

11 Drawing a ball which is not red?

12 Three red balls?

13 Balls numbered from 1 to 20, if E_1 is not one of number 4 multiples, find E_2 which represents multiples of number 4 by using two methods.

The table shows probability of getting a number after rolling the dice, find outcome of probability of each event:

14 Getting a number indivisible by 3.

15 Getting a number less than 4.

16 Getting the number 6.

17 Getting the numbers between 3 , 6 .

Event	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

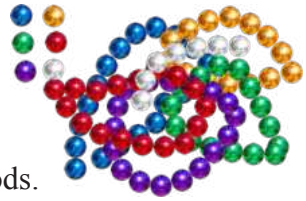
Solve the problems

18 **Game:** Tamara wants to sit on a chair among 8 chairs which numbered from1 to 8. What is the probability of sitting on an even numbered chair?

19 **Transportation:** Muhannad is waiting a bus from 5 buses numbered from 1 to 5, what is the probability of riding a bus numbered less than 4?

20 **Dice:** A dice is rolled once  , what is the probability of getting a number other than 6?

A bag contains 8 yellow, 3 white and 5 blue beads, if one bead drawn randomly:



21 What is the probability that the drawn bead is blue?

22 What is the probability that the drawn bead is yellow?

23 What is the probability that the drawn bead is not white? by two methods.

24 **Sport:** A coach expected the probability of his team winning is 85%, write the probability of losing in fraction in simple form.

25 **Weather:** Shahad expects the probability of the weather being clear tomorrow is 25%, find probability of each outcome as fraction:

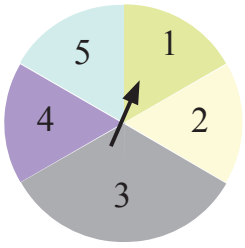
Outcome	Cloudy	Clear
Probability		

Think

26 **Critical thinking:** If you was told the probability of 4 events only, the probability of first event is $\frac{1}{4}$, and probability of second event is $\frac{1}{2}$, if the probability of third and fourth event is equal, what is the probability of the fourth event?

27 **Fruits:** There are four oranges in a basket, two oranges were drawn respectively without returning the first one and the second one, what is the probability of each event?

28 **Discover the mistake:** The disc was started twice, the probability of stopping the spinner on an odd number in the first time is $\frac{3}{5}$, and in the second time is $\frac{2}{5}$. Which one is correct?



29 **Challenge:** Write all possible events in a sample space consists of three outcomes A,B,C.

Write

An example of an event its probability is (0), and another event which its probability is (1).

Lesson [7-6] Experimental Probability and Theoretical Probability

Idea of the lesson:
* Calculating Theoretical Probability.
* Calculating Experimental Probability.

Vocabulary:
* Theoretical Probability
* Experimental Probability
* Sample Space

Learn

A football player is training to score goals in penalties, he scored 20 goals out of 25 penalties. We notice from both numbers that he scored goals more than he missed. How you can guess probability of scoring next shot?

Determine the probability in learn paragraph by an experiment,and it is called the experimental probabilities, while the theoretical probabilities provide us with the experiment outcomes without need to do it, then:

Experimental probability: You can guess the probability by number of repeating experiment several times, and then counting the number of times satisfies the event.

$$\textbf{Experimental probability} \approx \frac{\text{Number of times satisfies the event}}{\text{Total number of experiments}}$$

Theoretical probability: is used to guess probabilities of event, by using counting principles without need to repeat the experiment, and then all possible outcomes are equal in experiment

$$\text{Theoretical probability} = \frac{\text{Number of outcomes satisfies event}}{\text{All possible outcomes (The number of sample space elements)}}$$

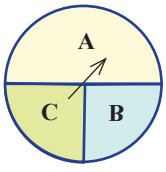
Example 1 In learn paragraph, the probability of scoring a goal in the next shot is $\frac{20}{25}$, i.e $\frac{4}{5}$

Example 2 **After starting the spinner 20 times, the outcomes are given in the table below**

i) Is this probability experimental or theoretical?

ii) Find probability of spinner stopping on letter A.

Outcome	A	B	C
N. of spinner stoppings	10	5	5



i) This is an experimental probability, because it depends on repeating experiment (20 times).

ii) The experimental probability of letter A $\approx \frac{\text{Number of times spinner stops on A}}{\text{Total number of experiments}}$

$P(A) \approx \frac{10}{20} = \frac{1}{2}$

So, the expermental probability of spinner stopping on A is: $\frac{1}{2}$ or 0.5 or 50% .

Example 3

Muhammad drew a ball from a box contains 50 balls: 20 red, 17 white and 13 yellow.

i) Is this theoretical or experimental probability?

ii) Find the probability of drawing yellow ball.

The probability is theoretical, because all outcomes are equal in the experiment. there is no need to repeat the experiment.

The number of sample space elements is 50 (The number of balls in the box)

$$\text{Theoretical probability} = \frac{\text{Number of outcomes satisfies the event}}{\text{All possible outcomes (The number of sample space elements)}}$$

$$P(A) = \frac{\text{Number of yellow balls}}{\text{Total number of balls}}$$ probability of drawing a yellow ball

$$P(A) = \frac{13}{50}$$ substitue the number of yellow balls 13, the total number of balls is 50

So the probability of drawing a yellow ball is $P(A) = \frac{13}{50}$ or 0.26 or 26%

Example 4

Use the near by table which shows outcomes of flipping two coins 8 times, and answer the following:

i) What is the theoretical probability of getting two heads?

ii) What is the experimental probability of getting two heads?

Outcomes	Frequensy
H , H	3
H , T	2
T , H	1
T , T	2

The sample space of flipping two coins once is:

{(H , H) , (H , T) , (T , H) , (T , T)}

The number of sample space elements is 4 .

i) When flipping the coin we get (H , H) once.

$$P(H, H) = \frac{\text{The number of (H, H)}}{\text{The number of sample space elements}} = \frac{1}{4}$$

So, the theoretical probability $= \frac{1}{4}$ or 0.25 or 25%

ii) The number of getting (H, H) is 3 times when we flip 8 times. (Repeating experiment)

$$P(H, H) \approx \frac{\text{Number of (H, H)}}{\text{Total number of experiments}} = \frac{3}{8}$$

So, the experimental probability $\approx \frac{3}{8}$ or 0.375 or 37.5%.

Make sure of your understanding

1 A ball was drawn randomly from a box and then returned, the following table shows the outcomes afetr 50 draws, estimate the probability of drawing red ball. What is the type of probability?

Outcome	Blue	Yellow	Red	Green
draws	20	12	13	5

Questions 1-7 are similar to examples 1-4

You rolled a dice once, find:

2 The probability of getting number greater than 2.

3 The probability of getting even number.

4 The probability of getting a prime number.

5 The probability of getting a number divisible by 6.

6 What is the type of previous probabilities, experimental or theoretical?

7 In a basketball game, a player scored 15 times out of 25, what is the probability to score in the next shot? What is type of probability? Write the probability in fraction, decimal and percemtage.

Solve the Exercises

You rolled the dice once, find:

8 The probability of getting an odd number.

9 The probability of getting a non prime number.

10 The probability of getting a number divisible by 4.

11 What is the type of previous probabilities, experimental or theoretical?

12 Mohanned drew a disc from a can containing 3 red, 4 yellow, and 4 black discs. What is the probability the drawn disc is yellow?

13 A man wanted to count the number of cars on the traffic light, among 20 cars there was 5 tracks, 3 sport cars and 12 sedan cars, What is probability of next coming car to be a truck?

In a sport education class,there are 6 students in basketball team, 8 students in football team. The teacher chooses a student randomly, find the probability of:

14 Choosing a student to play in basketball team.

15 Choosing a student to play in football team.

Solve the problems

16 **Agriculture:** Tariq has a bag containing 120 red and yellow flower seeds, 30% of these seeds was yellow flower seeds. Tariq grew the seeds in the garden, after 2 weeks he noticed the first flower growing up, what is the probability the grown-up flower is red?

17 **Sport:** In a basketball training match, Muhammad scored 13 out of 30 shots, what is the probability of scoring in the next shot? What is the type of probability?

18 **Time:** A research was applied over 250 person to know the way of knowing time. Use the near by table to find the probability of a person using his phone to know the time.

Number of users	Method
75	Hand watch
30	Clock
145	Cell phone watch

19 **Weather:** The table below represents the temperature of one week.

Day	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday
Temperature	15	29	25	21	16	18	20

What is the probability of being the temperature less than 20°C next day? Show the type of probability.

20 **Restaurant:** 10 people entered a kebab restaurant, 6 ordered kebab, what is the probability the next one will not order kebab?

Think

21 **Open problem:** A statistical study was applied over 30 persons about their favourite colour: (Blue, Red, Green, White), make a table for all possible outcomes if the experimental probability of favourite colour to be blue is $\frac{2}{5}$.

22 **Challenge:** A statistical study shows that 75 out of 200 students having a black shoes and 280 out of 400 students having white socks. What is the probability of a student having black shoes and white socks together?

23 **Discover the mistake:** A bag contains 5 yellow balls, 7 white balls, how many yellow balls we need to add to the bag, so that the probability of drawing a white ball is $\frac{1}{2}$, Jumana said "we should add 6 yellow ball", her sister Sally said "we should add two yellow balls, whose answer is correct?

Write

A problem about an experiment you have seen or lived including a question about the experimental probability.

Lesson [7-7] Problem Solving Plan (Presentation of Problem)

Idea of the lesson:

* Solving problem using (Presentation of Problem)

Learn

The length of stadium is 100m, if Mohammed runs 25m forward and 5m backward. How many times he has to repeat this run to reach the end of stadium?

UNDERSTAND

What are the given information in the problem: Length of stadium is 100m, Mohammed runs 25m forward 5m backward.

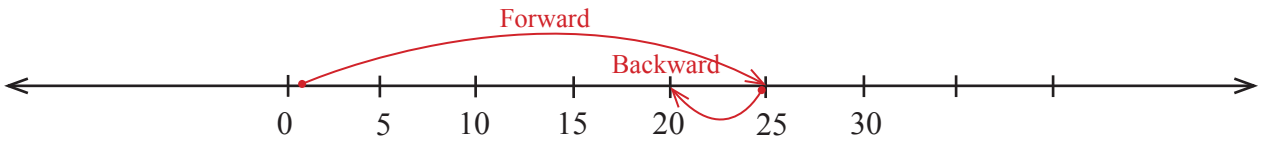
What is wanted in the problem: How many times more he has to repeat the run to reach the end of stadium.

PLAN

How can you solve the problem? By representing the problem on a number line.

SOLVE

Draw a number line and divide it to equal divisions.



25m forward , 5 backward

So in every time he covers distance ..... 25 - 5 = 20m

The total number of times ..... 100 ÷ 20 = 5

CHECK

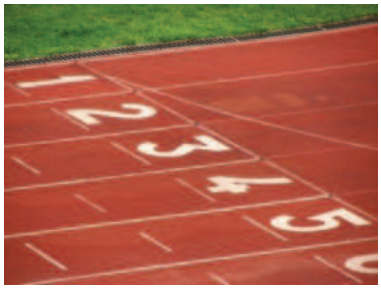
Check your answer: We multiply the distance length of running for each run by 5:

$20 \times 5 = 100$m

Problems

1 Ahmed bought a book for 25 thousands and 500 dinars, if Ahmed paid 30 thousands, how much he can get in return if the seller has coins of 1000 and 500 dinars only?

2 A player runs 8m forward and 2m backward in every time, if the race length is 60m, how many times he needs to run so he can finish the distance?



3 Jumana wanted to put 5 books on the shelf, such that the first book is Math and the last book is Arabic. In how many ways she can put them?

4 In how many ways can Mohanned, Mohammed, Ahmed and Mahmud stand on a straight line?

5 In the math test flipping a coin to solve 5 questions of true or false. Is this a good way to get good mark in the test?

Chapter Test

1 **The below table shows the marks average of some of the second intermediate students in math subject for two classes**

95	80	60	61	60	71	83	66	71	63	86	class A
85	99	77	81	84	90	67	77	65	84	90	class B

i) Represent the data by stem and leaf
ii) Which class has greatest range?
iii) Compare between two medians.

2 **Use the data 68, 73, 56, 59, 73, 68, 73, 68, 73, 56 , 56, 73 to construct a Box and Whisker, then answer the following:**

i) What is the Range?
ii) Find upper and lower quartile.
iii) How can you illustrate that one of the two Box-Whiskers is shorter than the other?

3 Flipping a coin and a rolling a dice, write all the possible outcomes by using tree diagram.

4 Use the fundamental counting principle to find the number of probabilities of question (3).

5 How do you distinguish between two events to say they are independent or dependent? Clarify that with example.

6 Rolling a dice, find probability of getting the numbers that divisible by 3.

7 If E_1 , E_2 are two complement events, and $P(E_1) = \frac{2}{9}$. Find $P(E_2)$.

8 The following table represents number of drawing a coin 15 times, estimate pobability of getting the head. What is the type of probability?

Outcome	Head	Tail
No. of times	9	6

9 When rolling a dice, find the probability of not getting number 3.

10 A box contains 5 yellow cards, 8 blue cards. What is the probability of drawing a blue card.

Exercises of chapters

Mutiple Choice

Choose The Correct Answer for each of the following:

[1-1] Ordering Operations on Rational Numbers

Use the addition and subtraction of the rational numbers to write each expression in simplest form:

1 $2\frac{1}{3} + 1\frac{1}{5} - \frac{6}{15} = \ldots$ a) $\frac{15}{47}$ b) $-\frac{15}{47}$ c) $\frac{47}{15}$ d) $-\frac{47}{15}$

2 $6.13 - 5.02 + 1.5 = \ldots$ a) 2.16 b) 2.61 c) -2.16 d) 2.61

Use the multiplication and division of the rational numbers to write each expression in simplest form:

3 $1\frac{1}{4} \times (-2\frac{1}{2}) \div 3\frac{4}{2} = \ldots$ a) $\frac{5}{8}$ b) $-\frac{5}{8}$ c) $\frac{8}{5}$ d) $-\frac{8}{5}$

4 $4.3 \times 1.2 \div (-0.4) = \ldots$ a) 12.9 b) -12.95 c) -19.20 d) -12.90

By using order of operations of rational numbers to write each expression in simplest form:

5 $\frac{-2}{6} \times \frac{12}{-3} - \frac{1}{3} \times \frac{15}{2} = \ldots$ a) $-\frac{7}{6}$ b) $\frac{6}{7}$ c) $\frac{7}{6}$ d) $-\frac{6}{7}$

6 $\frac{1}{9} \div \frac{5}{36} \times \frac{1}{-2} + \frac{7}{2} = \ldots$ a) $-\frac{31}{10}$ b) $\frac{31}{10}$ c) $\frac{10}{31}$ d) $-\frac{10}{31}$

[1-2] Scientific Notation and Negative Powers (Exponents) of Number

Calculate the negative exponents:

1 4^{-2} a) $\frac{-1}{8}$ b) $\frac{-1}{16}$ c) $\frac{1}{8}$ d) $\frac{1}{16}$

Use the order of the operations to calculate:

2 $(7)^{-2} - (-1)^4 + 1^{-7} - \frac{1}{7} = \ldots\ldots$ a) $\frac{-6}{49}$ b) $\frac{-114}{49}$ c) $\frac{6}{49}$ d) $\frac{114}{49}$

Write the numbers in the digital notations:

3 $3.4 \times 10^5 = \ldots\ldots$ a) 300040 b) 300400 c)304000 d) 340000

4 $2.51 \times 10^{-3} = \ldots\ldots$ a) 0.00251 b) 0.00215 c) 0.00125 d) 0.00512

Write the numbers in the scientific notation:

5 $52100 = \ldots\ldots$ a) 5.21×10^3 b) 5.21×10^4 c) 5.21×10^5 d) 5.21×10^6

6 $0.0035 = \ldots\ldots$ a) 3.5×10^{-6} b) 3.5×10^{-5} c) 3.5×10^{-4} d) 3.5×10^{-3}

Mutiple Choice

Choose The Correct Answer for each of the following:

[1-3] Properties of Powers (Exponents)

Multiply and Dividing and write the multiplication and dividing results as a single power:

1 $7^4 \times 7^{-4} \times 7^0 =$..... a) 7^{-1} b) 7^0 c) -7^0 d) -7^{-1}

2 $4^{-2} \times 16^{-2} \times (-32)^2 =$..... a) -2^{-4} b) 2^{-4} c) 2^{-2} d) -2^{-2}

3 $\frac{(-27)^2}{(-9)^3} =$..... a) 3^{-12} b) -3^{-12} c) -1 d) 1

4 $\frac{(-32)^0}{(-2)^3} =$..... a) -2 b) 0 c) -2^{-3} d) 2^2

Simphify and write the divison result as single power

5 $\frac{-5^3 \times 4^3}{4^2 \times 5^6} =$..... a) $\frac{5^{-3}}{2^2}$ b) $\frac{5^3}{2^{-2}}$ c) $\frac{2^2}{5^3}$ d) $-\frac{2^2}{5^3}$

6 $\frac{7^{-6} \times 7^0 \times 9^{-2}}{(-7)^2 \times 3^2 \times (-1)^0} =$..... a) $\frac{7^{-8}}{-3^6}$ b) $\frac{7^{-8}}{3^6}$ c) $\frac{3^2}{7^8}$ d) $-\frac{3^2}{7^8}$

[1-4] Recurring Decimal Fractions and Scientific Notation of Number (Using Calculator)

Use the calculator to write the addition (multiplication and divison) result in decimal fraction form:

1 $\frac{3}{8}+\frac{5}{9} =$..... a) $0.390\overline{5}$ b) $0.930\overline{5}$ c) $0.590\overline{3}$ d) $0.903\overline{5}$

2 $\frac{8}{9} \times \frac{36}{54} \div \frac{-6}{21} =$ ..... a) $2.\overline{3}$ b) $-2.07\overline{4}$ c) $\overline{3}.2$ d) $-\overline{3}.2$

Use the calculator to write the multiplication and division result in scietific notation for number:

3 $(4.2 \times 10^5)(3.6 \times 10^{-4}) =$.....

a) 1.251×10^2 b) 1.152×10^2 c) 1.512×10^2 d) 1.125×10^2

4 $\frac{6.25 \times 10^{-7}}{2.5 \times 10^6} =$.....

a) 2.5×10^{13} b) 2.5×10^{-1} c) 2.5×10^{-13} d) 2.5×10

Mutiple Choice

Choose The Correct Answer for each of the following:

[1-5] Simplifying Fractional Numerical Sentences

Simplify the following fractional numerical sentences, and write the result in simplest form:

1 $\frac{\sqrt[3]{-1}}{7} \times \frac{|-35|}{10} + \frac{2}{7} \times \frac{-21}{\sqrt[3]{8}} = ...$ a) $\frac{-7}{2}$ b) $-\frac{5}{2}$ c) $\frac{2}{7}$ d) $-\frac{2}{5}$

2 $\frac{4}{\sqrt{81}} \div \frac{|-20|}{10} - \frac{\sqrt[3]{-27}}{12} \div \frac{-3}{8} = ...$ a) $-\frac{4}{9}$ b) $\frac{4}{9}$ c) $\frac{9}{4}$ d) $-\frac{9}{4}$

3 $\frac{1}{2^3} \times 2^{-5} \div (-1)^{-4} (2)^{-2} = ...$ a) 2^{-5} b) 2^{-6} c) 2^{-10} d) 2^{10}

Simplify the following fractional numerical sentences, and write the result in scientific notation:

4 $3.2 \times 10^4 + 0.22 \times 10^5 = ...$

a) 0.54×10^4 b) 0.45×10^4 c) 0.45×10^5 d) 5.4×10^4

5 $8.3 \times 10^{-6} - 0.57 \times 10^{-3} = ...$

a) 5.671×10^{-4} b) 5.617×10^{-4} c) 5.617×10^{-3} d) 5.671×10^{-3}

[2-1] Concept of Real Numbers and Representing them on Number Line

Classify the numbers whether it is rational number or irrational number or unreal number:

1 $\sqrt{13}$ a)Rational number b)Natural number c)Irrational number d)Integer

2 $\sqrt{\frac{4}{36}}$ a)Integer b)Unreal number c)Irrational number d)Rational number

Write the following square roots in simplest form:

3 $\sqrt{\frac{2}{18}}$ a) $\frac{\sqrt{2}}{6}$ b) $\frac{2}{18}$ c) $\frac{1}{3}$ d) $\frac{2}{\sqrt{2}}$

Estimate the following square roots by approximate to the nearest tenth:

4 $\sqrt{5} \approx .....$ a) -2.2 b) 2.236 c) -2.3 d) 2.23

5 $\sqrt{7.6} \approx .....$ a) 2.7 b) 2.75 c) 2.275 d) 2.8

Arrange the following real numbers from largest to smallest:

6 $-\sqrt{17}, -4\frac{1}{5}, -4.02$ a) $-4.02, -\sqrt{17}, -4\frac{1}{5}$ b) $-4.02, -4\frac{1}{5}, -\sqrt{17}$

c) $-\sqrt{17}, -4\frac{1}{5}, -4.02$ d) $-4\frac{1}{5}, -\sqrt{17}, -4.02$

Mutiple Choice

Choose The Correct Answer for each of the following:

[2-2] Properties of Real Numbers

Write an example for each property from the following properties:

1 $a+(-a)=0,\ \forall a, -a \in R$ a) $\frac{5}{\sqrt{7}}+\frac{\sqrt{7}}{5}=0$ b) $\frac{5}{\sqrt{7}}+(-\frac{5}{\sqrt{7}})=0$

c) $\frac{5}{\sqrt{7}}+\frac{5}{\sqrt{7}}=0$ b) $\frac{5}{\sqrt{7}}+(-\frac{\sqrt{7}}{5})=0$

2 $a\times\frac{1}{a}=1,\ \forall\ a\in R, a\neq 0$ a) $\frac{1}{\sqrt{3}}\times\frac{1}{\sqrt{3}}=1$ b) $\frac{1}{\sqrt{3}}\times\frac{-1}{\sqrt{3}}=1$

c) $\frac{1}{\sqrt{3}}\times\sqrt{3}=1$ b) $\frac{1}{\sqrt{3}}\times(-\sqrt{3})=1$

Find the additive inverse for the following real numbers:

3 $7\sqrt{5}-8$ a) $7\sqrt{5}+8$ b) $-7\sqrt{5}-8$ c) $\frac{1}{7\sqrt{5}-8}$ d) $-7\sqrt{5}+8$

4 $-\sqrt{1}-\sqrt{2}$ a) $1+\sqrt{2}$ b) $\sqrt{1}-\sqrt{2}$ c) $\sqrt{2}-\sqrt{1}$ d) $-1-\sqrt{2}$

Find the multiplicative inverse for the following real numbers:

5 $-\sqrt{\frac{1}{12}}$ a) $2\sqrt{3}$ b) $\frac{-1}{2\sqrt{3}}$ c) $-2\sqrt{3}$ d) $\frac{1}{2\sqrt{3}}$

6 $-6\frac{2}{5}-3\frac{3}{5}$ a) $\frac{5}{14}$ b) $\frac{-5}{14}$ c) $\frac{1}{10}$ d) $\frac{-1}{10}$

[2-3] Simplifying the Numerical Sentences which contain Square Roots

Simplify the following numerical sentences by using the properties of real number:

1 $7\sqrt{20}-3\sqrt{45}$ a) $23\sqrt{5}$ b) $5\sqrt{5}$ c) $-23\sqrt{5}$ d) $-5\sqrt{5}$

2 $\sqrt[3]{-27}(\sqrt{7}+1^3)+7\sqrt{7}$ a) $4\sqrt{7}-9$ b) $10\sqrt{7}-3$ c) $10\sqrt{7}-9$ d) $4\sqrt{7}-3$

Simplify the following numerical sentences by using the property of displacer denominator:

3 $\frac{\sqrt{98}-\sqrt{18}}{3\sqrt{24}}$ a) $\frac{3}{2\sqrt{3}}$ b) $\frac{2}{3\sqrt{3}}$ c) $\frac{2}{3\sqrt{2}}$ d) $\frac{3}{2\sqrt{2}}$

4 $\frac{10\sqrt{6}}{\sqrt{48}}\div\frac{5\sqrt{2}}{\sqrt{49}}$ a) $\frac{2}{7}$ b) $-\frac{7}{2}$ c) $\frac{7}{2}$ d) $-\frac{2}{7}$

Mutiple Choice

Choose The Correct Answer for each of the following:

[2-4] Applications on Pythagorean Theorem

Find both square root positive and negative for the following numbers:

1 49 a) $\pm\sqrt{7}$ b) ± 7 c) $\sqrt{7}$ d) -7

2 225 a) $\sqrt{15}$ b) $\pm\sqrt{15}$ c) $\pm =15$ d) 15

3 6.25 a) ± 2.5 b) ± 5.2 c) ± 2.25 d) ± 22.5

Find the length of the right side if you know that the length of the hypotenuse and the right side is as given:

4 3 cm, 5 cm, ? a) $\sqrt{4}$ b) 4 c) $-\sqrt{4}$ d) -4

5 4 cm, $\sqrt{65}$ cm, ? a) $\sqrt{7}$ b) -7 c) $-\sqrt{7}$ d) 7

[2-5] Coordinate Plane

When you represent the following value table on the coordinate plane the geometrical shape will be:

1

X	4	-4	-4	4
Y	4	-4	4	-4

a) Recangle b) Triangle c) Square d) Trapezoid

2

X	2	-4	-3	2
Y	3	3	0	0

a) Recangle b) Parallelogram c) Square d) Trapezoid

Represent each pair from the order pairs on the coordinate plane and then find the distance between them approximated to the nearest tenth if it doesn't represent an integer:

3 {(-4,0) ، (0,5)} a) 6.3 b) 6 c) 4. 6 d) 6.4

4 {(3,3) ، (-3,-3)} a) 8.4 b) 5.8 c) 8.5 d) 8

5 {(0,0) ، (6,-5)} a) 7 b) 6 c) 7. 8 d) 6.7

Mutiple Choice

Choose The Correct Answer for each of the following:

[3-1] Addition and Subtraction of Algebraic Expressions

Solve the algebraic expression by using the addition:

1 $(\sqrt{5}\, y^2x^2 +6) + (3\sqrt{5}\, x^2y^2+ 9y+4)$ a) $4\sqrt{5}\ y^2x^2+ 10$

b) $2\sqrt{5}\ y^3x^2+ 9y+10$ c) $4\sqrt{5}\ y^3x^2+ 10y$ d) $4\sqrt{5}\ y^2x^2 +9y+10$

2 $(|-2|z^2w +4k+\sqrt{7}) + (|10|z^2w - 10k +2\sqrt{7})$ a) $8z^2w+6k+3\sqrt{7}$

b) $-12z^2w-6k+3\sqrt{7}$ c) $12z^2w - 6k +3\sqrt{7}$ d) $-12z^2w -14k+3\sqrt{7}$

Solve the algebraic expression by using the subtraction:

3 $(\sqrt{13}\, g^3h^4 + z^2+ 20) - (5\sqrt{13}\, g^3h^4 - 4z^2 +12)$ a) $4\sqrt{13}\, g^3h^4+ 5z^2 + 22$

b) $4\sqrt{13}\, g^3h^4 + 3z^2 + 8$ c) $-4\sqrt{13}\, g^3h^4- 5z^2 +20$ d) $-4\sqrt{13}\, g^3h^4 +5z^2 + 8$

4 $(|-24|yz +2x^2 +12) - (6yz -15x^2- 4)$ a) $30yz +13x^2 +16$

b) $-18yz -13x^2 + 8$ c) $18yz +17x^2+ 16$ d) $18yz -13x^2 + 16$

[3-2] Multiplying an Algebraic Term by an Algebraic Expression

Find the result of the multiplication of each of the following:

1 $(-25x^2y^2z)(4xy)$ a) $-100x^3y^3$ b) $100x^3y^3z$ c) $100x^2y^2z$ d) $-100x^3y^3z$

2 $(\frac{7}{2}z^2w^2)\ (\frac{3}{9}zw^3)$ a) $\frac{21}{18}\ z^2w^6$ b) $\frac{7}{6}z^3w^5$ c) $\frac{7}{6}\ z^3w^6$ d) $\frac{7}{6}\ z^2w^5$

3 $(\sqrt{5}\, h^2k^2)(\ (\sqrt{5}\, h^2k^3)$ a) $5h^4k^5$ b) $5k^5$ c) $5k$ d) $5k^6$

4 $(|-3|g^3h)(\ (|4|g^4h^2)$ a) $-12gh$ b) $-12\ g^7h^2$ c) $12g^7h^3$ d) $12g^7h$

5 $\frac{1}{2}m^2n^2\ (4mn+8)$ a) $4m^3n^3 + 16mn$ b) $2m^3n^3 + 4m^2n^2$

c) $4mn + 4m^2n^2$ d) $4m^3n^3 + 4m^2n^2$

Mutiple Choice

Choose The Correct Answer for each of the following:

[3-3] Multiplying of Algebraic Expressions

Find the result of the multiplication of each of the following:

1 $(x-5)(x+5)$ a) x^2+25 b) x^2-25 c) $x^2-10x+25$ d) $x^2+5x-25$

2 $(z-\sqrt{5})(z+\sqrt{5})$ a) z^2-5 b) z^2+5 c) $z^2+\sqrt{25}z+5$ d) $z^2-\sqrt{25}z-5$

3 $(|-4|gh-3)(4gh+3)$ a) $16g^2h^2+gh+9$ b) $16g^2h^2-gh+9$ c) $16g^2h^2+9$ d) $16g^2h^2-9$

4 $(\frac{1}{2}x-3)(\frac{1}{4}x^2+\frac{3}{2}x+9)$

a) $8x^3+27$ b) $\frac{1}{8}x^3+27$

c) $8x^3-27$ d) $\frac{1}{8}x^3-27$

5 $(2y-1)(4y^2+2y+1)$ a) $8y^3-1$ b) y^3-8 c) $8y^3+1$ d) y^3+8

[3-4] Dividing an Algebraic Expression by an Algebraic Term

Find the result of the dividing of each of the following:

1 $\frac{36z^2w^2}{9zw}$ a) $4z^3w^3$ b) $\frac{1}{4}zw$ c) $4zw$ d) $\frac{1}{4}z^3w^3$

2 $\frac{81g^3h^3}{3g^{-6}h^{-6}}$ a) $27g^3h^3$ b) $27g^9h^9$ c) $27g^{-3}h^{-3}$ d) $27g^{-9}h^{-9}$

3 $\frac{\sqrt{49}x^{-2}y^2}{7x^{-3}y^{-2}}$ a) xy^4 b) $7x^{-6}y^4$ c) $x^{-6}y^4$ d) xy

4 $\frac{5x^2y^2-15x^4y^2+20x^5y^3}{5x^3y^3}$ a) $x^5y^5-3x^7y^6+4x^2$ b) $\frac{1}{xy}-3xy^{-1}+4x^2$ c) $x^5y^5-3xy^5+4x^2y^6$ d) $xy-3xy+4x^2y$

5 $\frac{16m^6n^6-32m^7n^6-4m^4n^4}{4m^3n^3}$ a) $4m^9n^9-8m^{10}n^9-m^7n^7$ b) $4m^2n^2-8m^4n^3-mn$ c) $4m^3n^3-8m^4n^3-m^7n^7$ d) $4m^3n^3-8m^4n^3-mn$

Mutiple Choice

Choose The Correct Answer for each of the following:

[3-5] Factoring Algebraic Expressions

Factorize the algebraic expression of the following:

1 $81xy + 72x$ a) $9x(9y+8)$ b) $9(9xy-8)$ c) $9x(9xy-9)$ d) $9x(9xy+9)$

2 $7z^3 - z^2$ a) $7z^2(z + \frac{1}{7})$ b) $7z^2(z + 1)$

c) $7z^2(z - \frac{1}{7})$ d) $z^2(z - 1)$

3 $49g^2h^2 + 21gh$ a) $7gh(7gh+3)$ b) $7gh(7gh-3)$

c) $7(7gh+3)$ d) $7(7g^2h^2 +3gh+1)$

4 $11m^3n^2 - 44m^2n^2 - 121mn$

a) $mn(11m^2n^2 - 4mn-1)$ b) $11mn(m^2n^2 -4mn-11)$

c) $11mn(m^2n-4mn-11)$ d) $11\ (11m^2n^2 -4mn-1)$

5 $150r^3v^2 + 25rv+75r^2v^2$ a) $25rv(6r^2v+1+3\ rv)$ b) $25rv(6rv+3rv-1)$

c) $75\ rv(2rv+1)$ d) $75rv(2r^2v+13+rv)$

[4-1] Solving Two-Step First Degree Equations with One Variable in R

Solve the following equations by using addition and subtraction:

1 $5x + 20 = 4x - 10$ a) $x = 10$ b) $x = 30$ c) $x = -10$ d) $x = -30$

2 $|-12|\ z = \sqrt[3]{-8} + 13z$ a) $z = 2$ b) $z = 4$ c) $z = -2$ d) $z = -4$

3 $\sqrt[3]{27}\ x+1= |-11|+2x$ a) $x = 12$ b) $x = -10$ c) $x =10$ d) $x = -12$

Solve the following equations by using multiplication and division:

4 $\sqrt[3]{27}\ x \div |-5| = 8$ a) $x = -\frac{40}{3}$ b) $x = \frac{3}{40}$ c) $x = \frac{40}{3}$ d) $x = -\frac{3}{40}$

5 $|-6|\ h = 81 \div (-3)$ a) $h = \frac{2}{9}$ b) $h = -\frac{9}{2}$ c) $h = -\frac{2}{9}$ d) $h = \frac{9}{2}$

Mutiple Choice

Choose The Correct Answer for each of the following:

[4-2] Solving Mlti-Step First Degree Equations with One Variable in R

Solve the following equation by using the properties of real numbers:

1 $|-8|+ x = 12 -2x$ a) $x = -\frac{3}{4}$ b) $x = -\frac{4}{3}$ c) $x = \frac{4}{3}$ d) $x = \frac{3}{4}$

2 $\sqrt[3]{-125}\ x \div |-5| = 2\sqrt{7} \div 7$ a) $x = -\frac{2}{\sqrt{7}}$ b) $x = \frac{2}{7}$ c) $x = \frac{2}{\sqrt{7}}$ d) $x = -\frac{2}{7}$

3 $\frac{6y}{2 + \sqrt[3]{-27}} = \frac{y}{3}$ a) $y = -\frac{1}{17}$ b) $y = 0$ c) $y = \frac{1}{17}$ d) $y = 17$

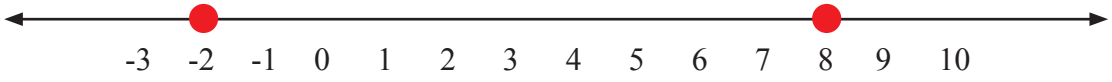
Solve the following equations:

4 $|z - 12| = 7$ a) $s = \{-19,-5\}$ b) $s = \{-19,5\}$ c) $s = \{19,-5\}$ d) $s = \{19,5\}$

5 $|3y-20| = \sqrt{36}$ a) $s = \{\frac{26}{3}, \frac{3}{14}\}$ b) $s = \{\frac{3}{26}, \frac{14}{3}\}$ c) $s = \{\frac{26}{-3}, \frac{14}{-3}\}$ d) $s = \{\frac{26}{3}, \frac{14}{3}\}$

6 The equation of a bsolute value that is represented by a number line

is:



a) $| x + 4 | = 5$ b) $| x - 4 | = 5$ c) $| x + 3 | = 5$ d) $| x - 3 | = 5$

[4-3] Solving Second Degree Equations with One Variable in R

Solve the following equation by using the square root:

1 $9y^2 = 1$ a) $s = \{\frac{1}{3}, \frac{1}{-3}\}$ b) $s = \{\frac{1}{9}, \frac{1}{-9}\}$ c) $s = \{\frac{1}{-3}, \frac{1}{-9}\}$ d) $s = \{\frac{1}{3}, \frac{1}{9}\}$

2 $\frac{1}{3}t^2 = 12$ a) $s = \{6,0\}$ b) $s = \{-6,0\}$ c) $s = \{2,-2\}$ d) $s = \{6,-6\}$

3 $n^2 - \frac{2}{8} = \frac{3}{8}$

a) $s = \{\sqrt{\frac{8}{5}}, -\sqrt{\frac{8}{5}}\}$ b) $s = \{\sqrt{\frac{1}{8}}, -\sqrt{\frac{1}{8}}\}$ c) $s = \{\sqrt{\frac{5}{8}}, -\sqrt{\frac{5}{8}}\}$ d) $s = \{\sqrt{\frac{5}{8}}, -\sqrt{\frac{8}{5}}\}$

Solve the following equation by using zero product property:

4 $x^2 - x = 0$ a) $s = \{0,-1\}$ b) $s = \{1,1\}$ c) $s = \{1,-1\}$ d) $s = \{0,1\}$

5 $\frac{1}{5}r - \frac{1}{\sqrt{5}}r^2 = 0$ a) $s = \{0, \frac{1}{5}\}$ b) $s = \{0, \frac{-1}{\sqrt{5}}\}$ c) $s = \{0, \frac{-1}{5}\}$ d) $s = \{0, \frac{1}{\sqrt{5}}\}$

Mutiple Choice

Choose The Correct Answer for each of the following:

[4-4] Solving Two-step Algebraic Inequalities in R

Solve the following inequalities in R by using the properties of addition and subtraction:

1 $3y-\sqrt[3]{8} \geq 4y+\sqrt[3]{-27}$ a) $y \leq -1$ b) $y \geq -1$ c) $y \geq 1$ d) $y \leq 1$

2 $4\left(\frac{1}{4}z+\frac{5}{14}\right) < 0$ a) $z < -\frac{10}{7}$ b) $z < \frac{10}{7}$ c) $z > \frac{10}{7}$ d) $z < -\frac{10}{7}$

Solve the following inequalities in R by using the properties of multiplication and division:

3 $\frac{4}{6}h \geq \frac{-8}{21}$ a) $S=\{h:h\in R, h\leq \frac{4}{7}\}$ b) $S=\{h:h\in R, h\leq \frac{-4}{7}\}$) $S=\{h:h\in R, h\geq \frac{-4}{7}\}$ d) $S=\{h:h\in R, h\geq \frac{4}{7}\}$

4 $\frac{1}{\sqrt{2}} \leq \frac{\sqrt{2}\,n}{9}$ a) $n \leq \frac{9}{2}$ b) $n \geq \frac{9}{2}$ c) $n \geq \frac{-9}{2}$ d) $n \leq \frac{-9}{2}$

Solve the following inequalities by using the properties of the inequalityies on the real numbers:

5 $5y+\sqrt[3]{-27} > 3y-\sqrt[3]{8}$ a) $y \leq \frac{1}{2}$ b) $y \leq \frac{-1}{2}$ c) $y \geq \frac{-1}{2}$ d) $y > \frac{1}{2}$

6 $4(\frac{1}{7}-\frac{3}{12}z) \leq 0$ a) $z \geq \frac{-4}{7}$ b) $z \leq \frac{7}{4}$ c) $z \leq \frac{4}{7}$ d) $z \geq \frac{4}{7}$

[4-5] Solving Multi-step Algebraic Inequalities in R

Solve the following inequalities by using the properties of the inequalities on the real numbers:

1 $5(y+1) \geq 8-\sqrt[3]{-125}$ a) $s=\{y:y\in R, y \leq -\frac{5}{8}\}$ b) $s=\{y:y\in R, y \leq \frac{8}{5}\}$

c) $s=\{y:y\in R, y \geq \frac{5}{8}\}$ d) $s=\{y:y\in R, y \geq \frac{8}{5}\}$

2 $\frac{1}{2}(x-\frac{4}{3})+\frac{1}{6}x < -\frac{2}{3}$ a) $x < \frac{4}{3}$ b) $x > 0$ c) $x < 0$ d) $x > \frac{-4}{3}$

3 $2(v-4\sqrt{5}) > 5(v-\sqrt{5})$ a) $v < -\sqrt{5}$ b) $v > \sqrt{5}$ c) $v > -\sqrt{5}$ d) $v < \sqrt{5}$

4 $\frac{1}{5}(z-\sqrt{3}) \geq \frac{1}{5}(\sqrt{3}-z)$ a) $z \geq \sqrt{3}$ b) $z \leq \frac{2}{\sqrt{3}}$ c) $z \leq \sqrt{3}$ d) $z \geq \frac{2}{\sqrt{3}}$

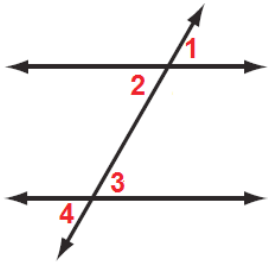
5 $\frac{12}{\sqrt[3]{-27}} -2r > \frac{1}{3} - r$ a) $r > \frac{13}{3}$ b) $r < -\frac{3}{13}$ c) $r < \frac{3}{13}$ d) $r < \frac{-13}{13}$

Mutiple Choice

Choose The Correct Answer for each of the following:

[5-1] Relationship of Angles and Straight lines (theorems)

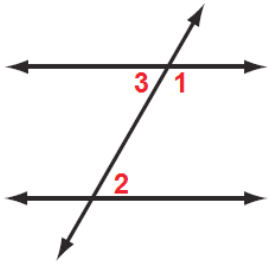
1 Relationship between the two angles 2 , 4 in the nearby figure:



a. Adjacent b. Alternated

c. Vertically opposite d. Corresponding

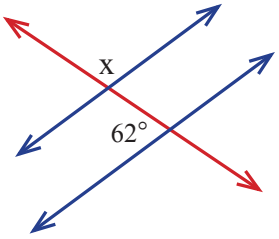
2 In the near by figure, if m $\angle$1 = 60° then m $\angle$ 2 equals:



a. 60° b. 80°

c. 90° d. 120°

3 Measurement of angle x in the near by figure:

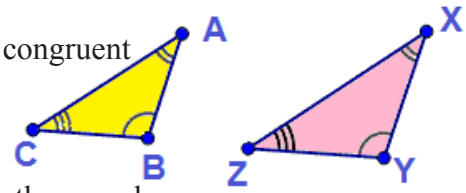


a. 118° b. 18°

c. 90° d. 88°

[5-2] Congruence of triangles

1 The two triangles in the near by figure:



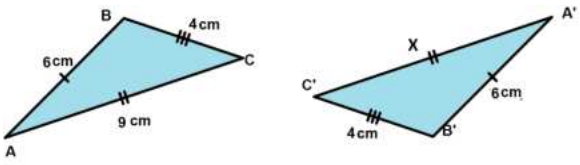
a. They are congruent because there are two sides and an angle are congruent

b. They are congruent because there are three sides are congruent

c. They are not congruent because the triangles not congruent when three angles are congruence

d. They are congruent because there are two angles and a side are congruent

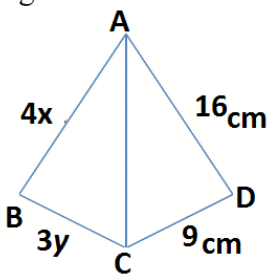
2 The two triangles, in the near by figure, are congruent, so the value of x which represents the side length is:



a. 4 cm b. 9 cm

c. 6 cm d. 2 cm

3 The value of X ,Y makes the two triangles ABC, ACD in the near by figure is congruent:



a. (x= 3 , y =4) b. (x= 4 , y =4)

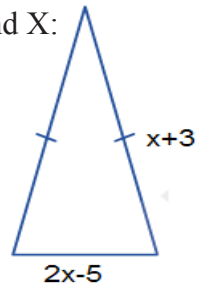
c. (x = 3 , y = 3) d. (x = 4 , y =3)

Mutiple Choice

Choose The Correct Answer for each of the following:

[5-3] Properties of triangles (Isosceles triangle, Equilateral triangle and Right-angled triangle

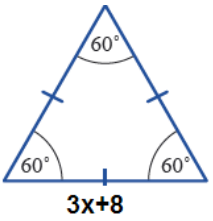
1 In the near by figure, if you knew that the perimeter of isosceles triangle is 13cm, then find X:



a. 6 b. 5

c. 4 d. 3

2 In the near by figure, the perimeter of equilateral triangle is 69cm, so the value of X is:



a. 6 b. 5

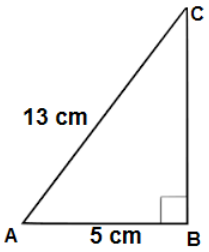
c. 4 d. 3

3 The measure of each angle in both right-angled triangle and isosceles triangle:

a. 60° b. 30°

c. 45° d. 90°

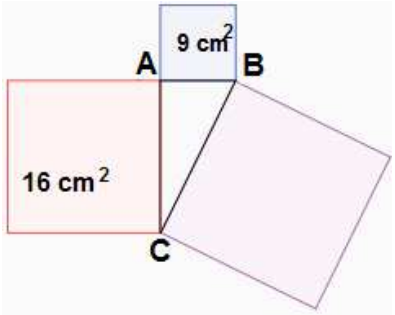
4 In the near by right-angled triangle, the length of side BC:



a. 12 b. 6 c.5 d. 9

5 In the nearby figure right-angled triangle is right angle in A. The area of the big square is:

a. 4 cm^2 b. 9 cm^2 c. 6 cm d. 25 cm^2



6 Numbers set which doesn't satisfy Pythagorean theorem if they were sides of a triangle:

a. { 3,4,5 } b. { 6,8,10 } c.{ 5,12,13 } d. { 3,5,33}

Multiple Choice

Choose The Correct Answer for each of the following:

[5-4] Parallelogram, Rhombus and Trapezoid.

1 Parallelogram area, if you know the length of its base is 10cm and the height length is 16cm, equals:

a. 800 cm^2 b. 1600 cm^2 c. 900 cm^2 d. 160 cm^2

2 Permitir of parallelogram which its two adjacent sides length is 10cm, 4cm equals:

a. 40 cm b. 20 cm c. 28 cm d. 80 cm

3 Area of trapezoid which has two paralleled sides of 6cm, 10cm and height of 5cm:

a. 110 cm^2 b. 90 cm^2 c. 100 cm^2 d. 40 cm^2

4 Area of rhombus is 1500cm^2, its height is 10cm, then its side length is:

a. 75 cm b. 150 cm c. 36 cm d. 90 cm

5 The base of parallelogram is 16cm, its height is half of base length, then the area is:

a. 144 cm^2 b. 128 cm^2 c. 80 cm^2 d. 40 cm^2

[5-5] Cylinder and Sphere (Properties, Surface Area and Volume).

1 Vertical circular cylinder, its base radius is 14cm and its height is 10cm, then its lateral area is:

a. 700cm^2 b. 820cm^2 c. 880cm^2 d. 800cm^2

2 Closed vertical circular cylinder, its base radius length is 7cm, its height is 3cm, then the total area is:

a. 176π cm^2 b. 170π cm^2 c. 190π cm^2 d. 140π cm^2

3 Vertical circular cylinder, its base radius length is 6cm and its volume is 360π cm^3, then its hight is:

a. 16 cm b. 8 cm c. 10 cm d. 5 cm

4 Radius of sphere is 3cm, then its volume is:

a. 36π cm^3 b. 18π cm^3 c. 24π cm^3 d. 72π cm^3a.

5 The radius length of sphere its surface area is 400π cm^2 :

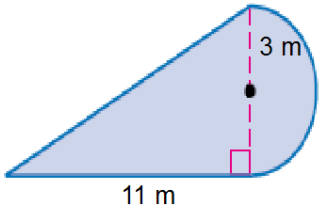
a. 12 cm b. 8 cm c. 10 cm d. 9 cm

Multiple Choice

Choose The Correct Answer for each of the following:

[5-6] Area of regular and irregular compound shapes.

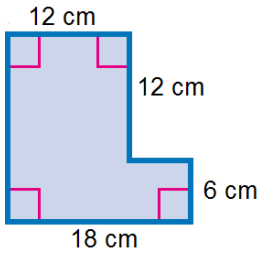
1 The near by shape of the regular compound area equals:



a. 47.13 cm^2 b. 18.13 cm^2

c. 74.13 cm^2 d. 90.13 cm^2

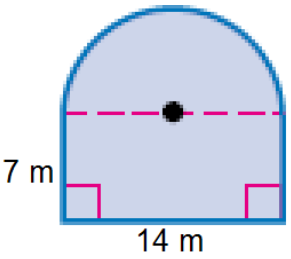
2 The near by shape of the regular compound area equals:



a. 225 cm^2 b. 252 cm^2

c. 522 cm^2 d. 512 cm^2

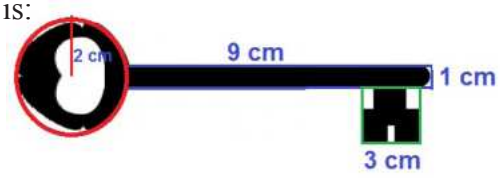
3 The near by shape of the regular compound area equals:



a. 175 cm^2 b. 170 cm^2

c. 180 cm^2 d. 215 cm^2

4 The approximate area of the key surface in the near by shape is:



a. 32 cm^2 b. 30 cm^2

c. 31 cm^2 d. 33 cm^2

5 In the near by picture, Baghdad map surrounded by a circle. The radius of this circle is 3cm, on a scale of 1:100000 the Baghdad area will approximately be gotten through the circle area:

a. 12π cm^2 b. 8π cm^2

c. 10π cm^2 d. 9π cm^2

Multiple Choice

Choose The Correct Answer for each of the following:

[6-1] Representing Function Table in the Coordinate Plane.

1 Relation whose inputs are constant and outputs are variable represented graphically in the coordinate plane on or parallel for:

a. X-axis b. Y-axis c. Origin d. X-axis and Y-axis

2 The distance that Muhammad covers in kilometres in 3 hours by train, if you know that he covers 10 kilometres per hour:

a. 5 b. 10 c. 20 d. 30

3 If the function table:

Input	-1	x	-3
Output	1	2	3

then the value of x is:

a. 3 b. 4 c. -2 d. 9

4 If the function table:

Input	-1	0	x
Output	2	3	4

then the value of x is:

a. -1 b. 1 c. 2 d. -2

[6-2] Introduction of Functions.

1 Each relation is function under condition:

a. Each input has two outputs b. Each input has one output

c. Each input has three outputs d. Every two inputs have one output

2 Which statement is correct:

a. Each function is relation b. Each relation is function

c. Function and relation have the same meaning d. Every two relations are functions

3 If the function rule 7 x -1 and set of elements {-2,0,2}, then the set of images is:

a. {-15,-1 ,13} b. {15,1 ,13} c. {15,-1,13} d. {15,-1,-13}

4 If the set of function elements {4,2,0} and the images set {3,1,-1}, then the function rule is:

a. 1-x b. -1-x c. x-1 d. 2-x

Multiple Choice

Choose The Correct Answer for each of the following:

[6-3] Linear Functions

1 Equation of the straight line is: an equation which expresses the linear function by:

a. $y = 4x-1$ b. $y = 4x^2 - 1$ c. $y = \sqrt{4(x-1)}$ d. $y = \frac{4}{x-1}$

2 Which linear function passes through the origin:

a. y=2x-1 b. y=2x+1 c. y=12-x d. y=2x

3 The linear function which passes through the second and fourth quadrants is:

a. y=13-x b. y = 13+x c. y = -3x-1 d. y = -3x

4 After multiplying a natural number by 2 then 4 was subtracted from it, then the final result was 50, What is the linear equation for the natural number?

a. 2x - 4 = 50 b. 24- x = 50 c. 42-x = 50 d. 24+x = 50

[6-4] Reflection and Rotation in the Coordinate Plane

1 If the reflection of the point (-2,3) is (-2,-3), then reflection line is:

a. X-axis b. Y-axis c. Oblique axis d. Both of them

2 The straight line AB, A(1,3), B(2,4), if a reflection of this line was done on the X-axis, so A´(1,-3), then B´is:

a. (2,-4) b. (-2 ,4) c. (-2 ,-4) d. (2,4)

3 Triangle ABC، A(1,1), B(1,3), C(3,2), a reflection of this triangle was done on the X-axis, then the result was C´ (3,-2), A´(1,-1) then B´:

a. (-1,-3) b. (-1,3) c. (1,-3) d. (1,3)

4 The image of point (-2,4) under the rotation by 90° clock wise about the origin:

a. (-2,-4) b. (4,-2) c. (-4,2) d. (4,2)

5 The image of point (-1,3) under the rotation by 90° anti-clockwise about the origin:

a. (1,-3) b. (-3,-1) c. (1,0) d. (3,-1)

Multiple Choice

Choose The Correct Answer for each of the following:

[6-5] Translation in the Coordinate Plane

1 Translating of the point (2,5), 3 units right:

a. (0,5) b. (5,5) c. (5,0) d. (2,8)

2 Translating of the point (3,6), one unit up:

a. (3,7) b. (3,5) c. (4,7) d. (4,6)

3 Translating of the point (3,-3) two units to right and one unit up:

a. (2,5) b. (-5,-2) c. (5,-2) d. (-5,2)

4 Translating the triangle ABC, A(1,1), B(3,1), C(2,3), and becomes A′ (1,3),B′ (3,3), then what is the value of C′:

a. (5,2) b. (2,5) c. (6,2) d. (2,6)

5 What are the coordinates of the point (X,4) by translating m unit down:

a.(x-m,y) b.(x+m,y) c.(x,4+m) d.(x,4-m)

[7-1] Measure of Central Tendency and Range

1 The near by table represents one of the data by the stem and leaf display:

a.4, 5, 6, 7, 8, 10, 12 b.4, 5, 6, 8, 12

c.4, 5, 6, 7, 7, 10,12 d. 4, 5, 6, 7, 7, 8, 10, 12.

Stem	leaf
0	4 5 6 7 7 8
1	0 2

2 Which of these back-to-back stem and leaf displays represents the data in the following tables:

A	3	11	21	33	17
B	10	9	30	11	3

a.

leaf B	Stem	leaf A
3 9	0	3
1 0	1	1 7
	2	1
0	3	3

b.

leaf B	Stem	leaf A
3	0	3
1 0	1	7
0	2	1
0	3	3

c.

leaf B	Stem	leaf A
3 0	0	3
1 0	1	1 7
0	2	1
0	3	3

d.

leaf B	Stem	leaf A
9	0	3
1	1	7
0	2	1
0	3	3

Multiple Choice

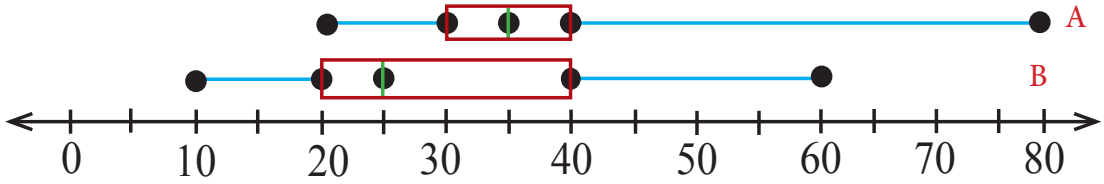
Choose The Correct Answer for each of the following:

[7-2] Representation of Data by Box-Whisker

1 Data by Box-Whisker are divided to:

a. Two divisions b. Three divisions c. Four divisions d. Five divisions

2 The following Box-Whisker shows a comparison in kg between the weight of two animals A , B, the value of the range of quartile for two types are:



a. A=20, B=10 b. A=40, B=10 c. A=20,B=30 d. A=10, B=20

3 The near by table shows increasing in height in various plants in centimetres for upper quartile and lower quartile:

a. 40,50 b. 55,51
c. 47,75 d. 41,79

Hight of plants		
80	42	55
72	68	78
65	40	52

[7-3] Random Experiment

1 A random experiment based on two random acts, the outcome number of the first act is 6 and the outcome number of the second act is 2, so the two acts outcomes are:

a. 8 b. 4 c. 3 d. 12

2 The near by table shows that two coins were thrown, which of the following sets represent appearance of two various heads on the two coins?

First piece	H	H	T	T
Second piece	H	T	H	T

a. {(H,T)} b. {(T,H)} c. {(H,T),(T,H)} d.{(H,T),(T,T)}

3 A restaurant offers 3 types of meals in 4 types of spices, these meals may offered with chicken or not, calculate the possible choices of meals.

a. 6 b. 12 c. 24 d. 36

[7-4] The Event

1 If E_1 represents the event (drawing red ball), E_2 represents the event (drawing green ball) without returning the red ball to the box contains 5 red balls and 6 green balls. The two events E_1, E_2:

a.Non compound event b.Compound event c.Independent d.Dependent

2 It was announced that there is a probability to get a free shirt if you buy one piece from the materials shown in the near by table.
Assume: the event E_1 buying skirt, E_2 getting shirt

a.Compound event b. Simple event
c.Independent d. Dependent

Shirt colour	Piece type
White Black Red	Skirt Trousers Shoes Handbag

Multiple Choice

Choose The Correct Answer for each of the following:

[7-5] The Probabilities

1 A box contains 4 red balls, 7 white balls, 5 blue balls, two white balls were drwan respectively without returning the first one, then the probability (the second white ball) is:

a. $\frac{1}{15}$ b. $\frac{7}{16}$ c. $\frac{6}{15}$ d. $\frac{2}{15}$

2 If E_1 , E_2 are two complementary events, then:

a. $P(E_1) \times P(E_2) = 1$ b. $P(E_1) \div P(E_2) = 1$

c. $P(E_1) + P(E_2) = 1$ d. $P(E_1) - P(E_2) = 0$

3 The percentage that represents probability of drawing randomly a non even card from cards numbered 1, 3, 5, 7, is:

a. 75% b. 50% c. 25% d. 100%

[7-6] Experimental Probability and Theoretical Probability

1 Use the near by table which shows the outcomes of flipping a coin 10 times, the theoretical probability of getting a head and tail is:

a. $\frac{3}{10}$ b. $\frac{7}{10}$

c. $\frac{1}{2}$ d. $\frac{1}{3}$

Outcomes	Frequency
H , H	2
H , T	3
T , H	4
T , T	1

2 Use the same information in question (1), the experimental probability of getting a head and tail is:

a. $\frac{3}{10}$ b. $\frac{7}{10}$ c. $\frac{1}{3}$ d. $\frac{1}{6}$

3 Muhammad started a disc spinner for 200 times, he got the green colour for 50 times, the probability of getting the green colour in the next time is:

a. 0.23 b. 0.24 c. 0.25 d. 0.30

4 A football score 21 successful goals out of 26 penalties, which percentage is more probability to score a successful goal in the next penalty:

a. 50% b. 60% c. 70% d. 80%

Contents